



# An Ordinary State-based Peridynamic Model for Progressive Damage Analysis of Carbon Fiber Reinforced Thermoplastics Material Based on GPU Parallel Scheme

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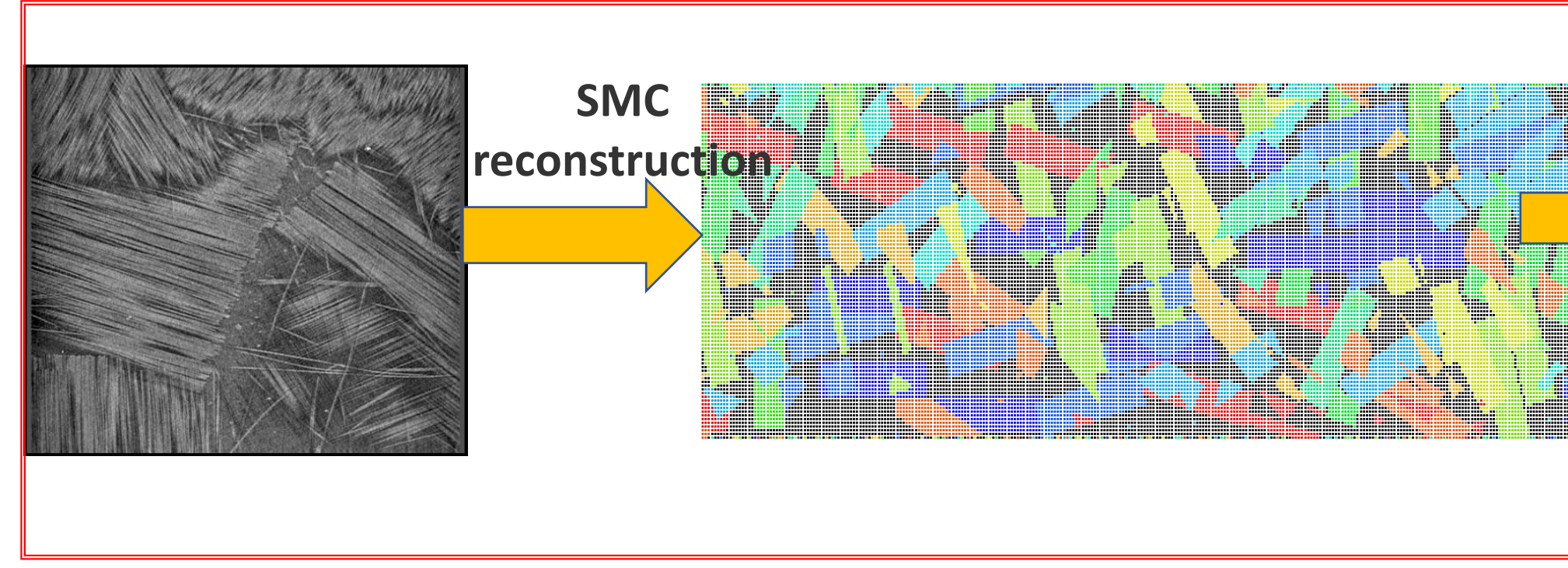


## 1. Background & Contributions

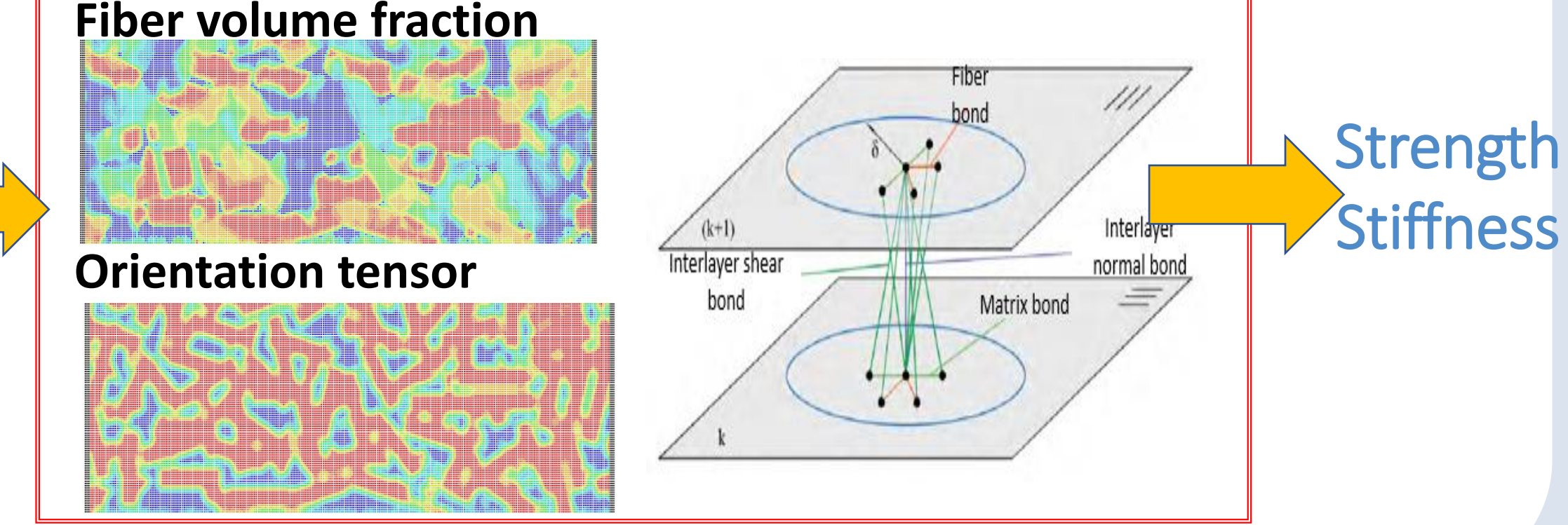
Randomly oriented strands in Carbon fiber reinforced thermoplastic sheet molding compound (CFRTP-SMC) material induce in-plane isotropic mechanical properties as well as a multi-scale structure and high heterogeneity<sup>[1][2]</sup> which makes mechanical properties and progressive damage analysis of CFRTP-SMC very challenging.

Silling<sup>[3]</sup> proposed a non-local theory of continuum mechanics called the Peridynamic (PD) theory. This approach can avoid the requirement of continuity of displacement for the derivatives of displacement in discontinuous areas.

### Two-scale SMC model



### PD model for SMC



## 2. Details about Two-scale SMC model & SMC model reconstruction

### Micro-Scale

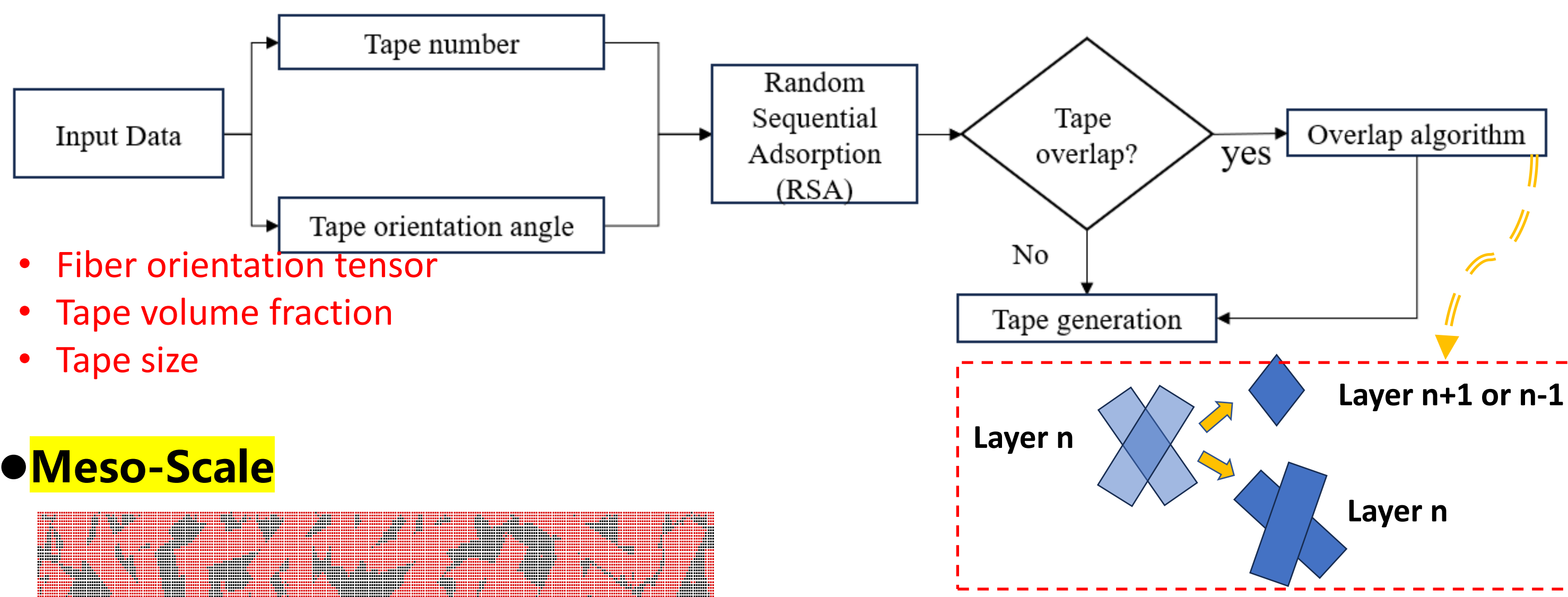
- Fiber orientation tensor

$$a_{ij} = \oint_S p_i p_j \psi(P) dP$$

$$a_{ijkl} = \oint_S p_i p_j p_k p_l \psi(P) dP$$

- Fiber volume fraction

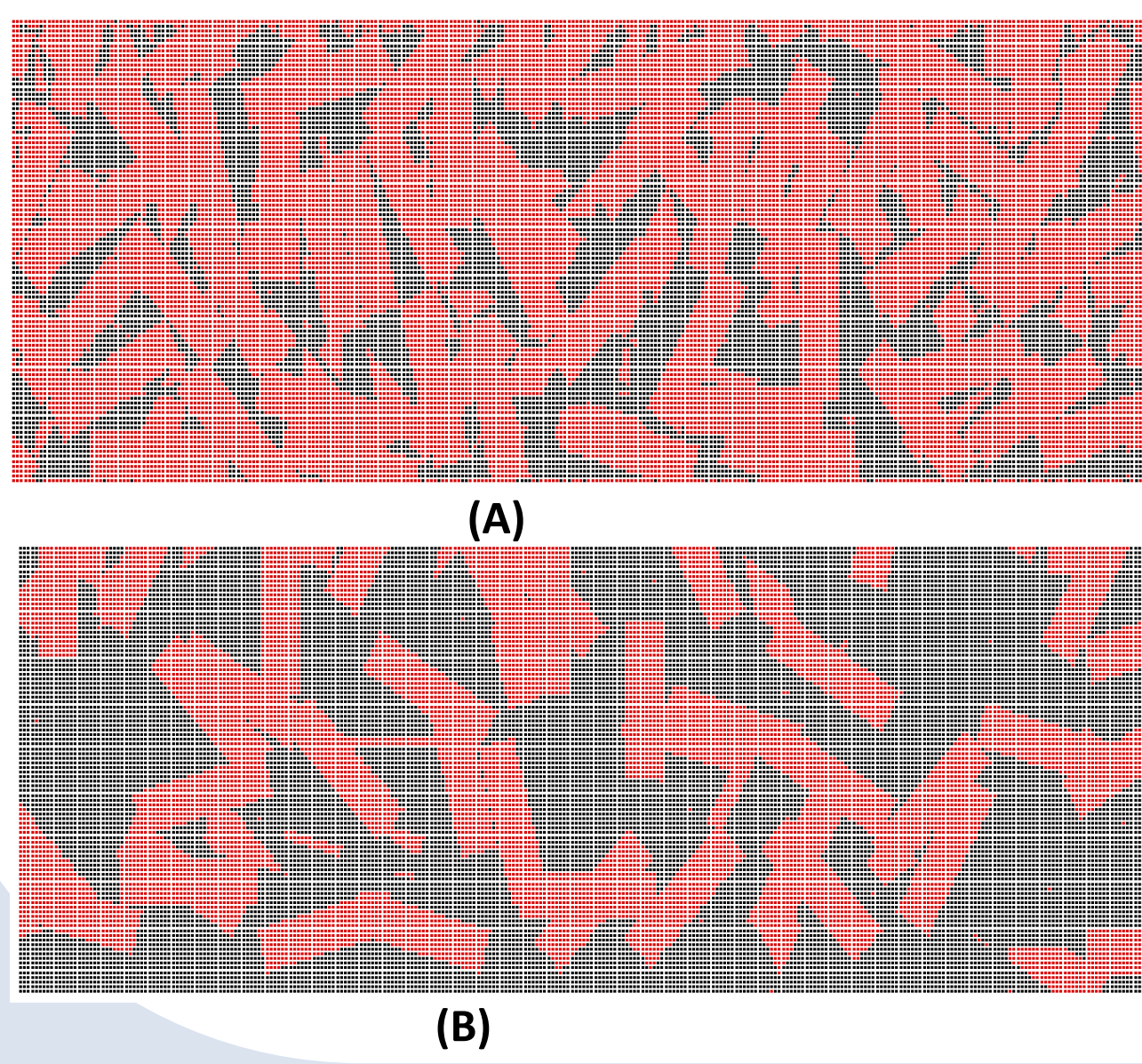
### SMC reconstruction



### Probability density function of FOD

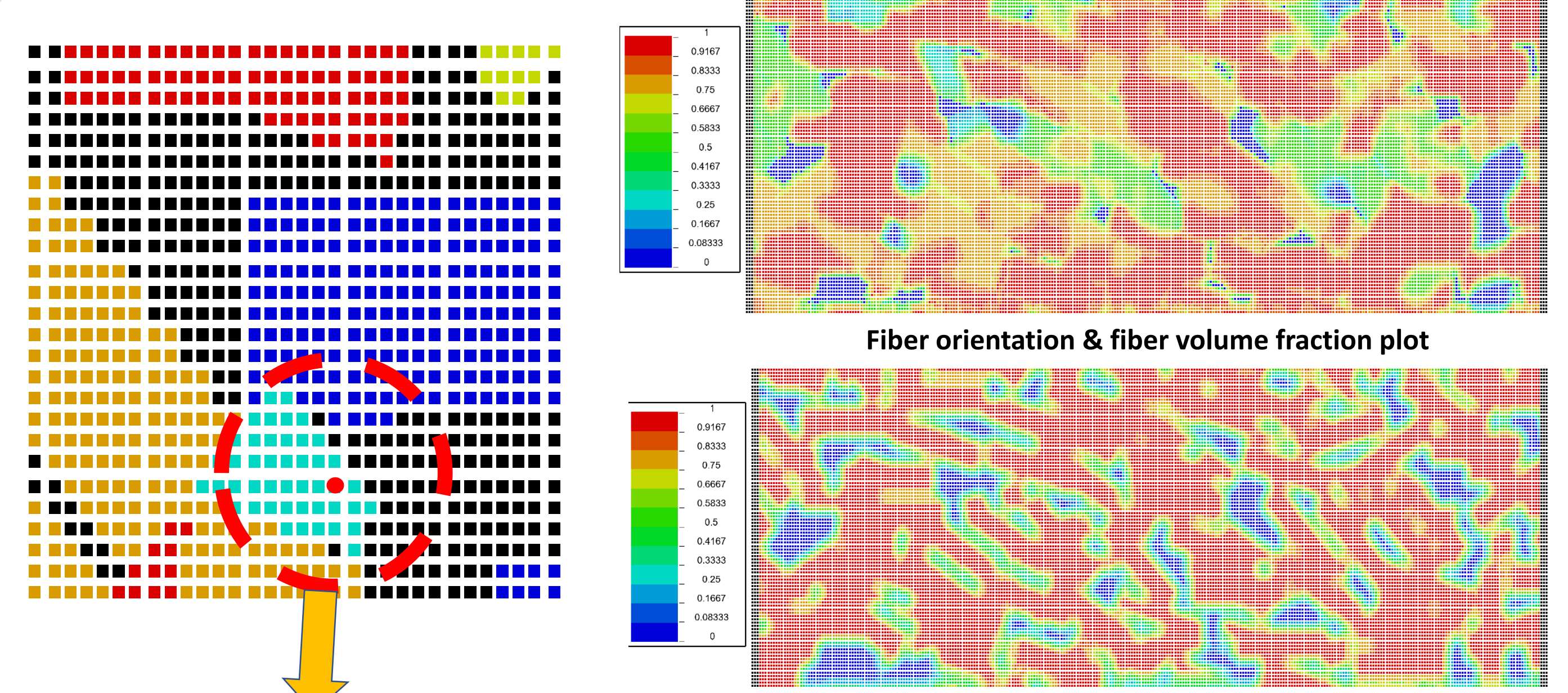
$$\psi(P) = \frac{1}{4\pi} + \frac{15}{8\pi} b_{ij} f_{ij}(P) + \frac{315}{32\pi} b_{ijkl} f_{ijkl}(P) + \dots$$

### Meso-Scale



	Tape volume fraction (%)	Fiber orientation tensor	Tape size (mm)
(A)	60%	$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$	5*6
(B)	30%	$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$	5*18
(C)	80%	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.2 \end{bmatrix}$	5*6

## 3. Details about Peridynamic(PD) model for SMC



### Classical Continuum Mechanics(CCM)

- Tape volume fraction\_local
- Fiber orientation tensor\_local

$$[D]_{CCM\_local}$$

$$W_{CCM} = \frac{1}{2} \epsilon_{local}^T D_{local} \epsilon_{local}$$

### Local Damage(Damage factor)

$$\phi(x_{(k)}, t) = 1 - \frac{\int_H \mu(x_{(j)} - x_{(k)}, t) dV_{(j)}}{\int_H dV_{(j)}}$$

### Peridynamic theory(PD)<sup>[5][6]</sup>

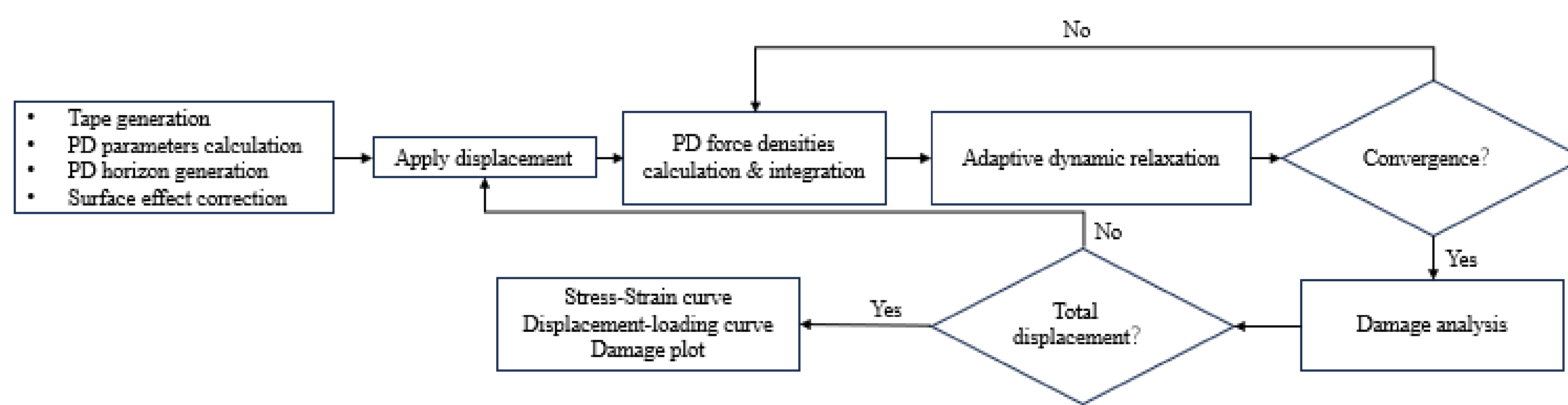
$$\omega = \frac{1}{2} \mathbf{u}_{local}^T \mathbf{k}_{local} \mathbf{u}_{local}$$

$$\mathbf{u}_{local} = \{u_i, v_i, \theta_{iz}, u_j, v_j, \theta_{jz}\}^T,$$

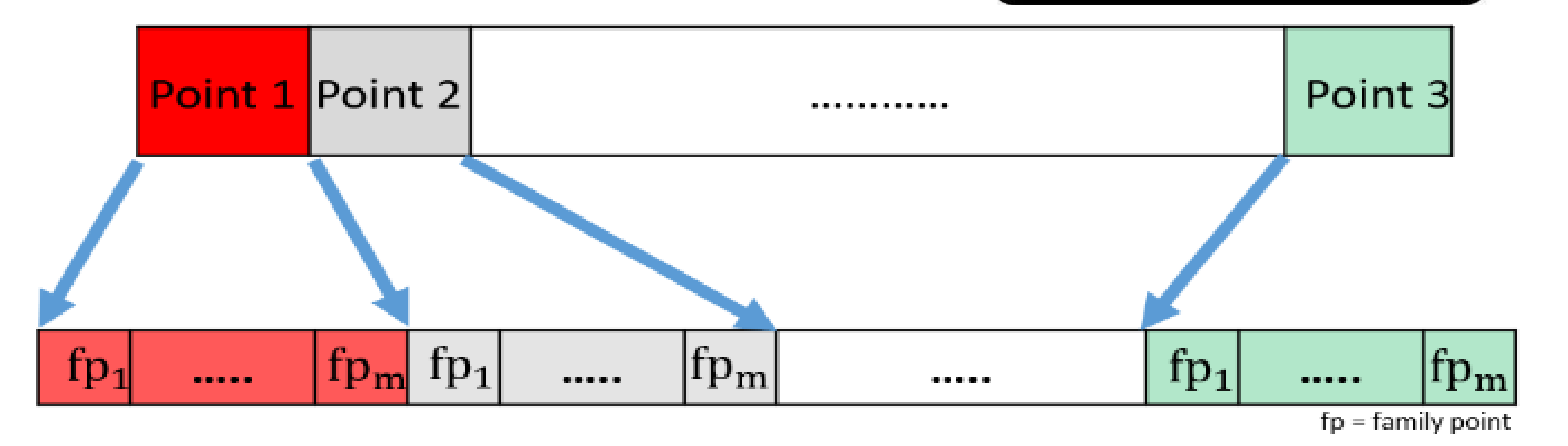
$$\mathbf{k}_{local} = \frac{1}{\xi^3} \begin{bmatrix} \xi^2 c_{ax} & 0 & 0 & -\xi^2 c_{ax} & 0 & 0 \\ 0 & 12c_{bz} & 6\xi^2 c_{bz} & 0 & -12c_{bz} & 6\xi^2 c_{bz} \\ 0 & 6\xi^2 c_{bz} & 4\xi^2 c_{bz} & 0 & -6\xi^2 c_{bz} & 2\xi^2 c_{bz} \\ -\xi^2 c_{ax} & 0 & 0 & \xi^2 c_{ax} & 0 & 0 \\ 0 & -12c_{bz} & -6\xi^2 c_{bz} & 0 & 12c_{bz} & -6\xi^2 c_{bz} \\ 0 & 6\xi^2 c_{bz} & 2\xi^2 c_{bz} & 0 & -6\xi^2 c_{bz} & 4\xi^2 c_{bz} \end{bmatrix}$$

$$W_{PD} = \frac{1}{2} \int \omega_{h(x_i)} dV_{x_j}$$

## 4. Details about Numerical simulation & CUDA parallel scheme



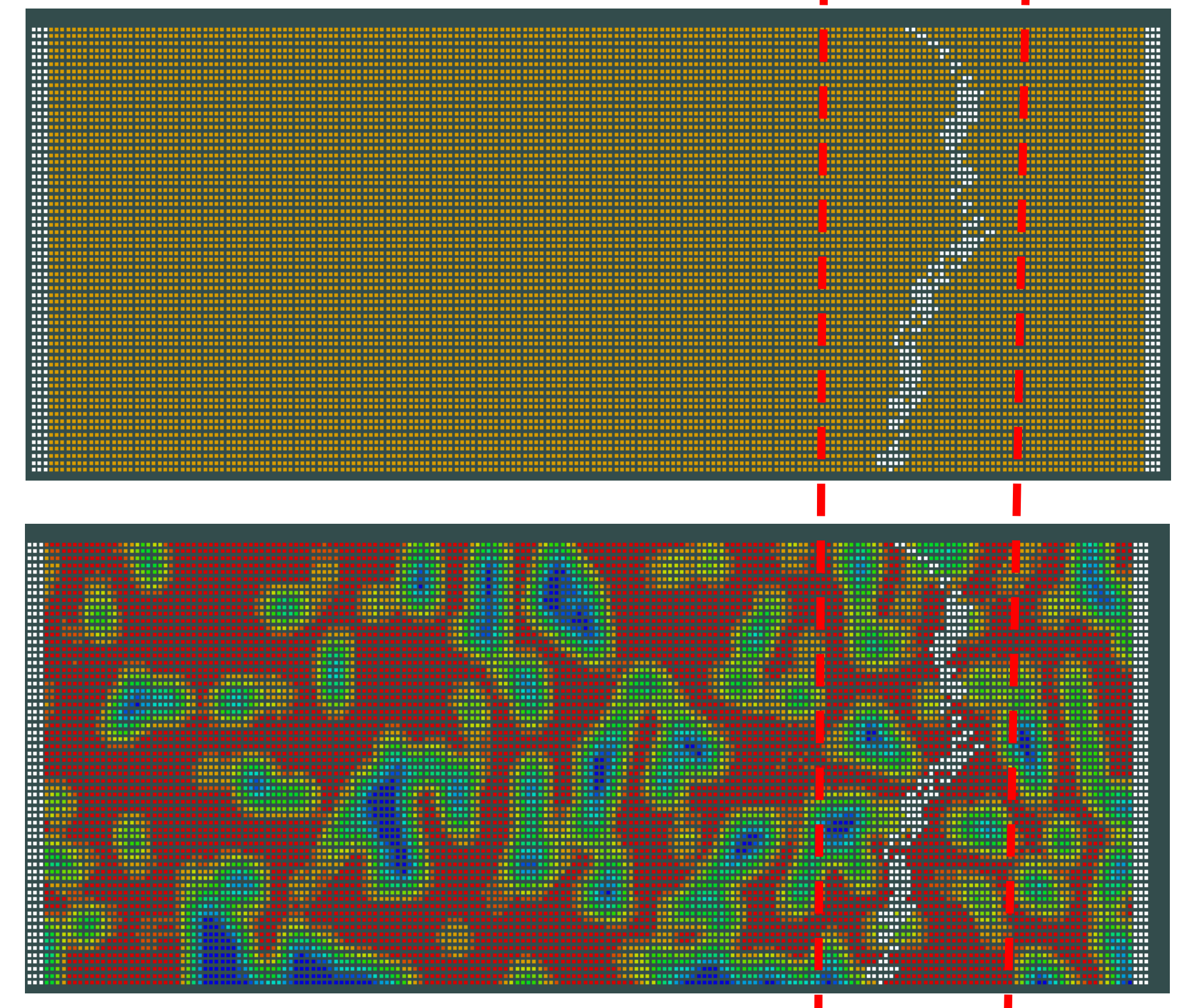
### Data Structure



## 5. Results & Discussions

Specimen Size(mm)	Fiber orientation tensor
90*15*0.2	$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$

#	Tape size(mm)	Tape volume fraction (%)	Tensile moduli(GPa)	Strength moduli(MPa)	(PD)Tensile moduli(GPa) Avg & CV	(EXP)Tensile moduli(GPa) Avg & CV <sup>[7]</sup>	(PD)Strength moduli(MPa) Avg & CV	(EXP)Strength moduli(MPa) Avg & CV <sup>[7]</sup>
0	5*6	79.2%	30.3	208	35.8(0.11)	39.3(0.015)	304(0.19)	391(0.06)
		91.7%	41.7	312				
		85.2%	35.2	307				
		87.1%	35.7	332				
		88.8%	36.1	360				
1	5*18	81.7%	32.4	309	36.2(0.08)	42.4(0.08)	377(0.11)	528(0.08)
		84.9%	39.1	382				
		88.2%	37.5	379				
		80.7%	34.6	402				
		89.3%	40.0	415				
2	5*30	84.5%	39.6	405	41.6(0.05)	48.4(0.04)	418(0.05)	537(0.06)
		90.2%	44.7	446				
		78.2%	39.5	421				
		86.2%	42.7	392				
		85.7%	41.3	428				



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