

# Advanced Multifield Models for Wave Propagation Analysis in Smart Composite Panels

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## Introduction

Commercial plate models tend to underestimate the stresses to-the-thickness directions. Taking the Classical Laminate Theory (CLT) as an example:

### CLT

$$\begin{aligned} u_x &= u_{x0} + \varphi_x z \\ v_y &= v_{y0} + \varphi_y z \\ w_z &= w_{z0} \end{aligned} \quad \begin{aligned} \varepsilon_{zz} &= \frac{\partial w_z}{\partial z} = 0 \\ \varepsilon_{xz} &= \frac{\partial u_x}{\partial z} + \frac{\partial w_z}{\partial x} = \varphi_x \end{aligned} \quad \begin{aligned} &\text{- zero to-the-thickness strain} \\ &\text{- constant shear strain} \end{aligned}$$

➔ Higher order plate models are required to predict the non-negligible to-the-thickness behaviour in laminated structures.

## Numerical Model

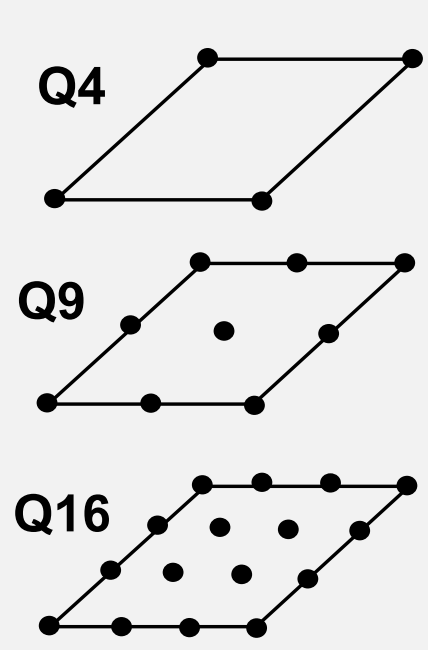
FE Model based on the Carrera Unified Formulation (CUF), developed by the MUL2 Group

$$U(u, v, w, \phi) = N_i(x, y) \cdot F_\tau(z)$$

$$N_i(x, y)$$

$$F_\tau(z)$$

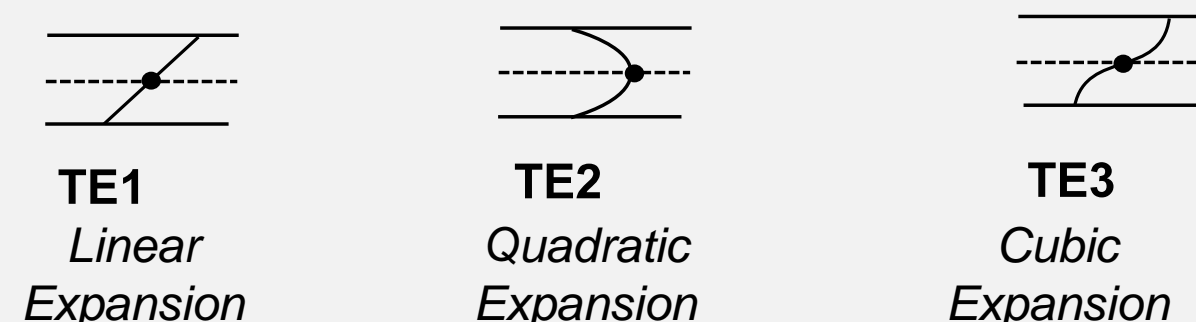
- 2D element mesh
- Can be refined in the commercial FEM tools by either refining the mesh or increasing the order of the used shape function
- FE model kinematics
- Can be refined to a high order with:
  - Equivalent Single Layer (ESL) expansion
  - Taylor Expansion Polynomials
  - Layer-Wise expansion
  - Lagrange Polynomials



Linear element

quadratic element

cubic element



TE1  
Linear Expansion

TE2  
Quadratic Expansion

TE3  
Cubic Expansion

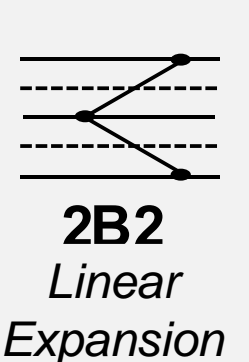
### Layer-Wise expansion

#### Lagrange Polynomials

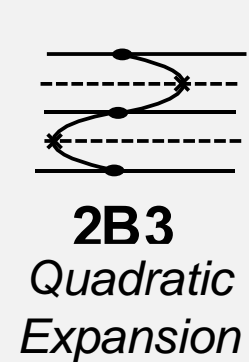
$$L_1 u_1 + L_2 u_2$$

$$L_1 u_1 + L_2 u_2 + L_3 u_3$$

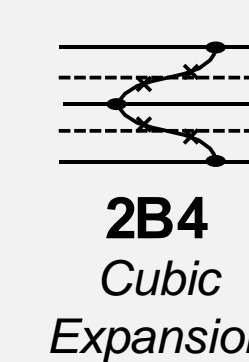
$$L_1 u_1 + L_2 u_2 + L_3 u_3 + L_4 u_4$$



2B2  
Linear Expansion



2B3  
Quadratic Expansion



2B4  
Cubic Expansion

## Objectives

Modelling Lamb wave propagation in smart composite panels

- ❖ Orthotropic material  $[0,90,90,0]_s$  with surface mounted piezoelectric transducers
- ❖ Coupled electro-mechanical finite element two-dimensional (2D) plate models
- ❖ Study the effect of using advanced higher order kinematics on the Time-Of-Flight (TOF)
- ❖ Reduce the computational cost by the implementation of Node Dependent kinematics

## Benchmark Problem

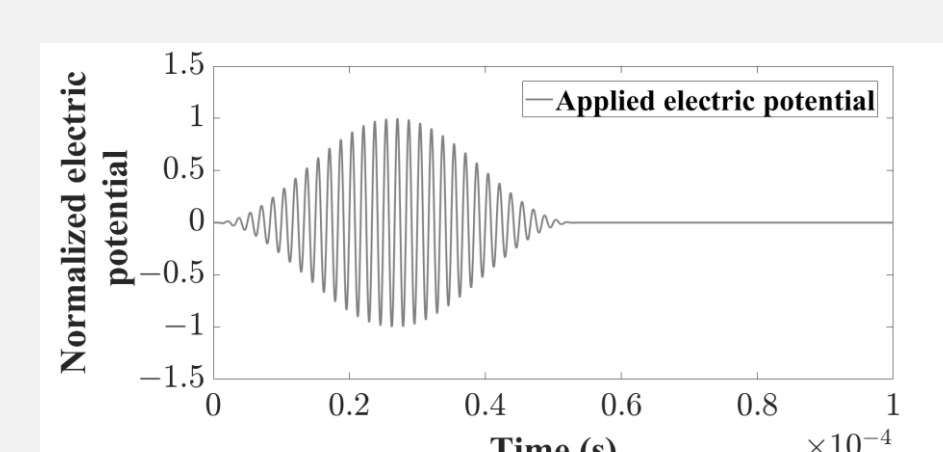


Figure 1: Actuation signal at 600 kHz

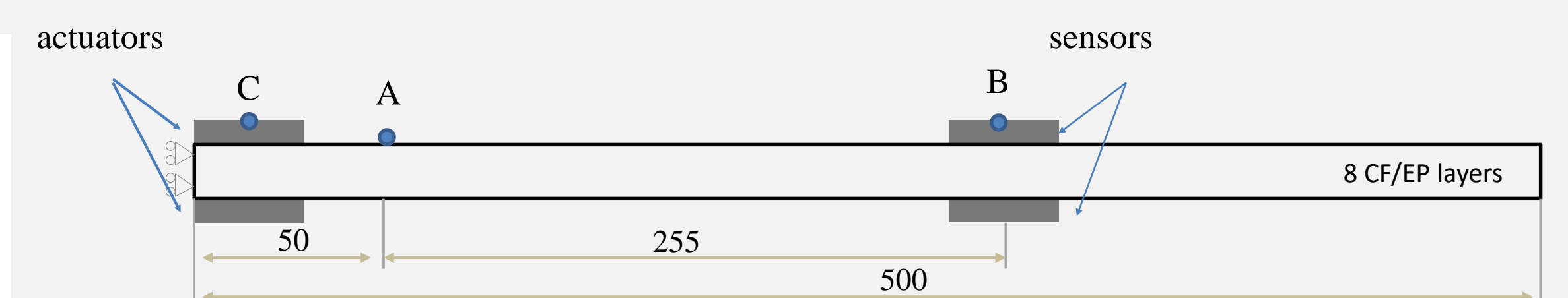


Figure 2: Benchmark problem with the actuators and sensors positioned, in mm

## Results

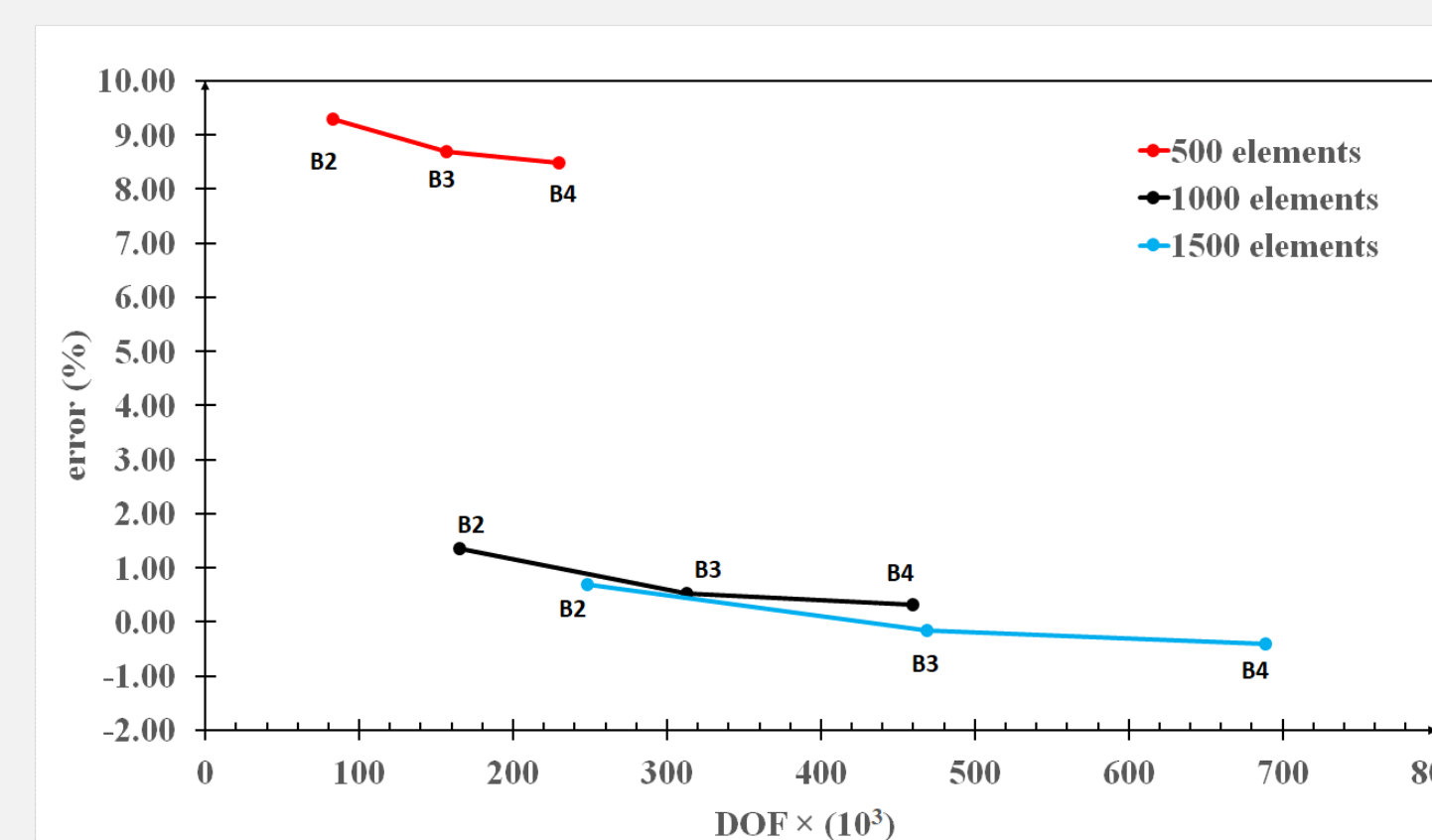


Figure 3: Antisymmetric wave propagation under refined elements and model kinematics

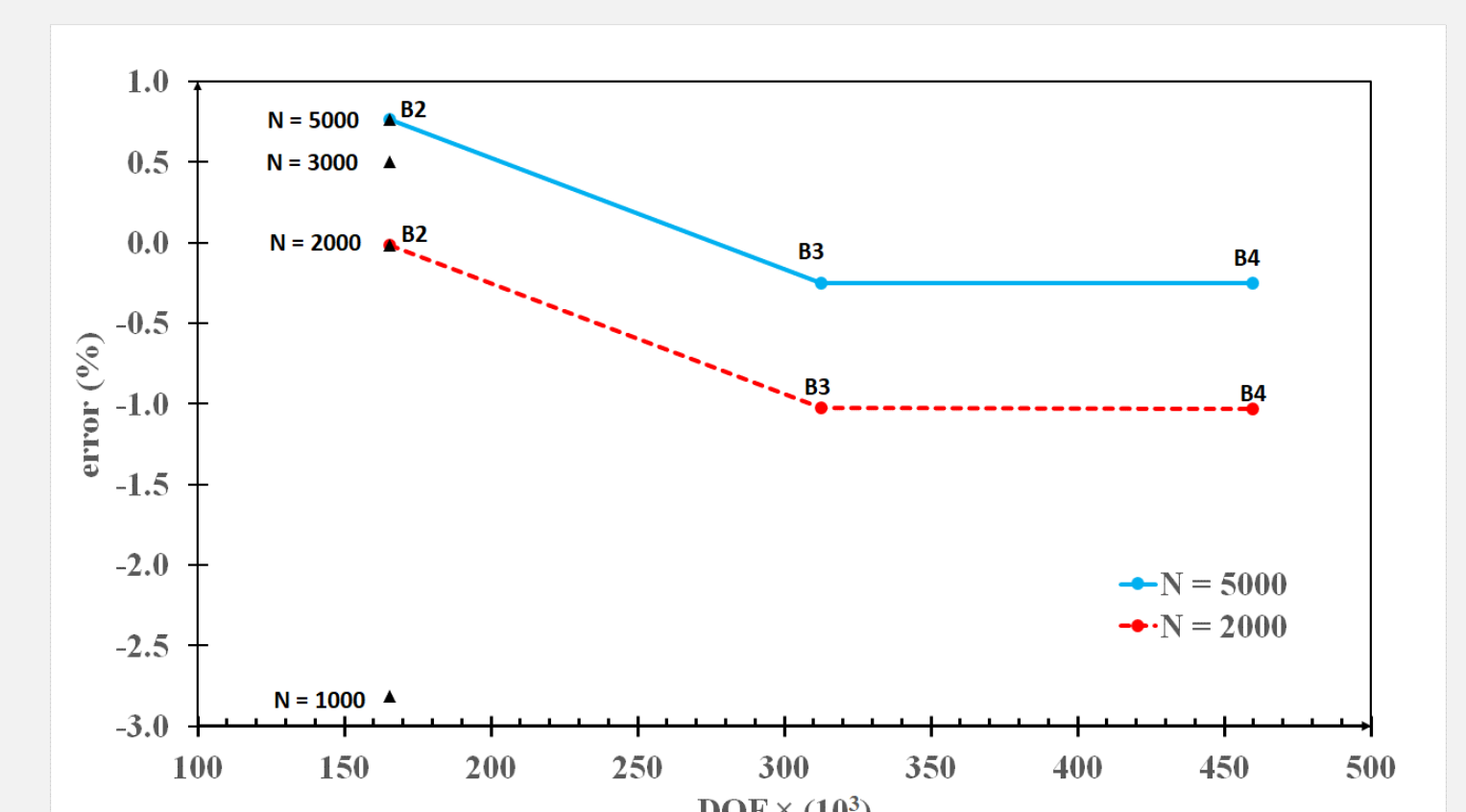


Figure 4: Symmetric wave propagation showing the number of timestep and model kinematics effect on the obtained error

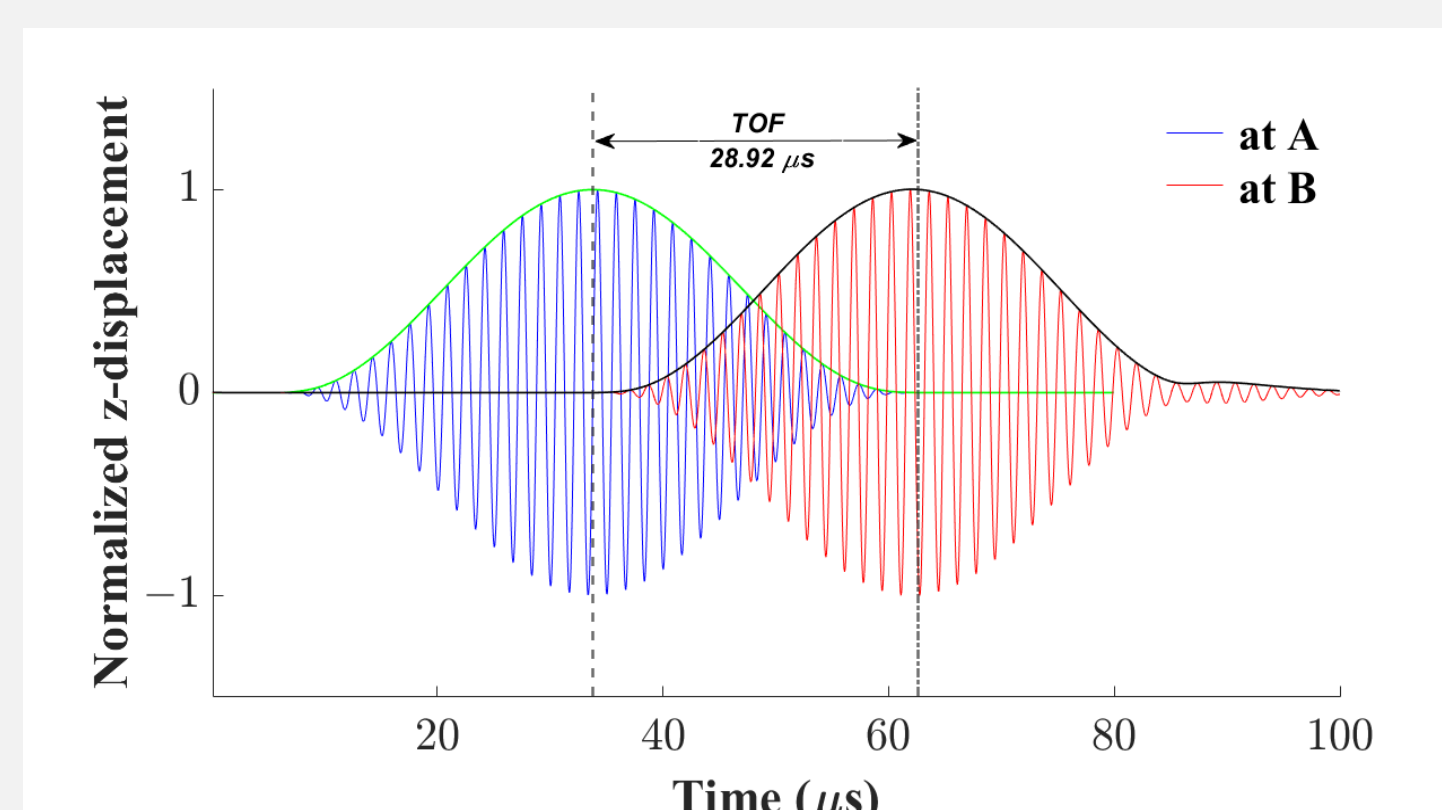


Figure 5: Wave propagation and TOF between points A and B

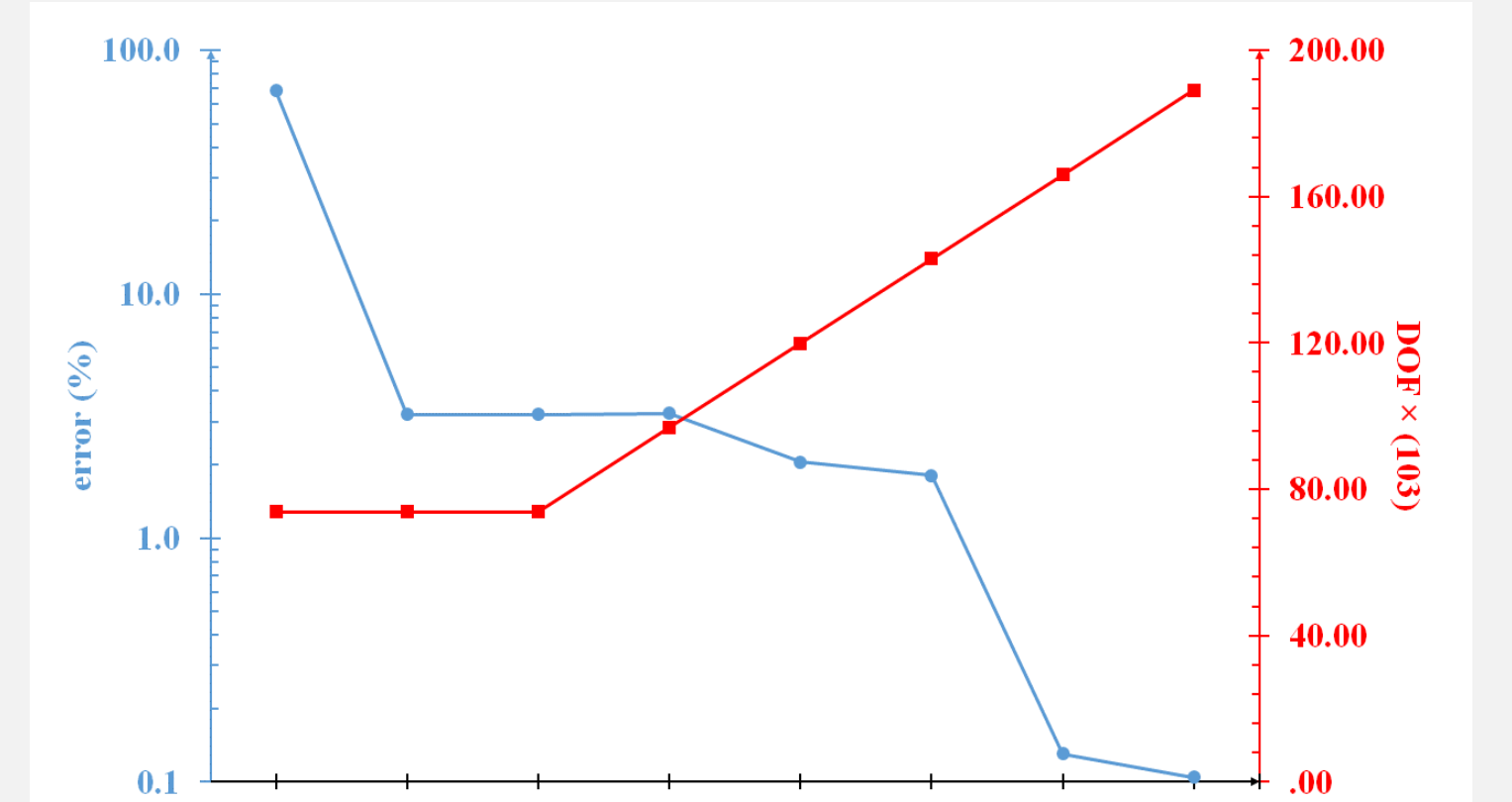


Figure 6: NDK results for a propagating  $A_0$  Lamb wave

## Conclusions

- Higher order model kinematics give satisfactory results
- It is essential to perform time step refinement
- Low order model kinematics produce positive error resulting in a stiffer structure

- Less timestep refinement produces a negative error
- The above errors may cancel out, but the problem is case dependent
- Using NDK for  $A_0$  reduces the computational cost by 60% with an error less than 0.15% compared to a fully refined model

## Funding

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## References

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- [2] A. de Miguel, A. Pagani and E. Carrera, "Higher-order structural theories for transient analysis of multi-mode Lamb waves with applications to damage detection"
- [3] C. Willberg, S. Duczek, J. Vivar Perez, D. Schmicker and U. Gabbert, "Comparison of different higher order finite element schemes for the simulation of lamb waves"