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Derivation and Validation of a Mechanically Consistent Continuum Damage Model for Brittle Composite Materials

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Motivation

A generic framework to model the stiffness degradation of a brittle material reads [1]:

$$\phi(\boldsymbol{\varepsilon}, \mathbf{D}) = \frac{1}{2}\boldsymbol{\varepsilon}:\mathbb{C}^0:\boldsymbol{\varepsilon} + \phi^d(\boldsymbol{\varepsilon}, \mathbf{D})$$

with A second rank symmetric damage tensor $\mathbf{D}$

- The damage effect $\phi^d(\boldsymbol{\varepsilon}, \mathbf{D})$:
  - Is often formulated using the effective stress concept together with different equivalence requirements, examples are [2]:

$$\mathbb{C} = \mathbb{M}(\mathbf{D})^{-1}:\mathbb{C}^0:\mathbb{M}(\mathbf{D})^{-T} \quad \text{(Energy equivalence)}$$

$$\mathbb{C} = \mathbb{M}(\mathbf{D})^{-1}:\mathbb{C}^0 \quad \text{(Strain equivalence)}$$

  - Problem: The latter is not symmetric and there is no rigorous way to choose $\mathbb{M}(\mathbf{D})$
  - A more sophisticated model for brittle materials [1]:

$$\mathbb{M}(\mathbf{D}) = \frac{\xi}{2}(\mathbf{1}\otimes\mathbf{D} + \mathbf{D}\otimes\mathbf{1}) + \frac{1-\xi}{2}\left[\mathbf{1}\,\overline{\otimes}\,\mathbf{D} + \mathbf{D}\,\overline{\otimes}\,\mathbf{1}\right]_S$$

Most of the damage effect models violate the damage growth criterion [3]:

An increase in damage should always lead to a decrease in strain energy (or equivalently a decrease in stiffness):

$$\phi(\boldsymbol{\varepsilon}, \mathbf{D}) \geq \phi(\boldsymbol{\varepsilon}, \mathbf{D} + \mathbf{dD}), \qquad \forall\boldsymbol{\varepsilon}, \forall\mathbf{dD}, \mathbf{D} \in PSym$$

Derivation of a mechanically consistent damage effect

Derivation of a mechanically consistent damage effect fulfilling the damage growth criterion by applying the theory of invariant tensor functions [4]:

Starting Point:

- The damage effect $\phi^d(\boldsymbol{\varepsilon}, \mathbf{D})$ is a polynomial with degree two in $\mathbf{D}$ and has only quadratic terms in $\boldsymbol{\varepsilon}$
- It is assumed that the undamaged material has an orthotropic symmetry

→ Damage effect $\phi^d(\boldsymbol{\varepsilon}, \mathbf{D})$ with 145 independent material parameters

The Reduction Process:

- Reduce damage to in-plane damage $\mathbf{D} = D_{11}\mathbf{e}_1 \otimes \mathbf{e}_1 + D_{22}\mathbf{e}_2 \otimes \mathbf{e}_2$
- Fulfil the damage growth criterion [3]

$$\phi^d_{\mathrm{iso}}(\boldsymbol{\varepsilon}, \mathbf{D}) = \xi_0 \mathrm{Tr}(\boldsymbol{\varepsilon}^2\mathbf{D}) + \mathrm{Tr}(\mathbf{D})[\xi_1\mathrm{Tr}(\boldsymbol{\varepsilon}^2) + \xi_2\mathrm{Tr}(\mathbf{M}^2\boldsymbol{\varepsilon}^2) \ldots$$
$$\ldots + \xi_3\mathrm{Tr}(\boldsymbol{\varepsilon})^2 + \xi_4\mathrm{Tr}(\mathbf{M}\boldsymbol{\varepsilon})^2 + \xi_5\mathrm{Tr}(\mathbf{M}^2\boldsymbol{\varepsilon})^2]$$

- Only 6 material parameters $\xi_0, \ldots \xi_5$ left
- With a slight loss in accuracy further reduction to 1 material parameter ξ_0 possible

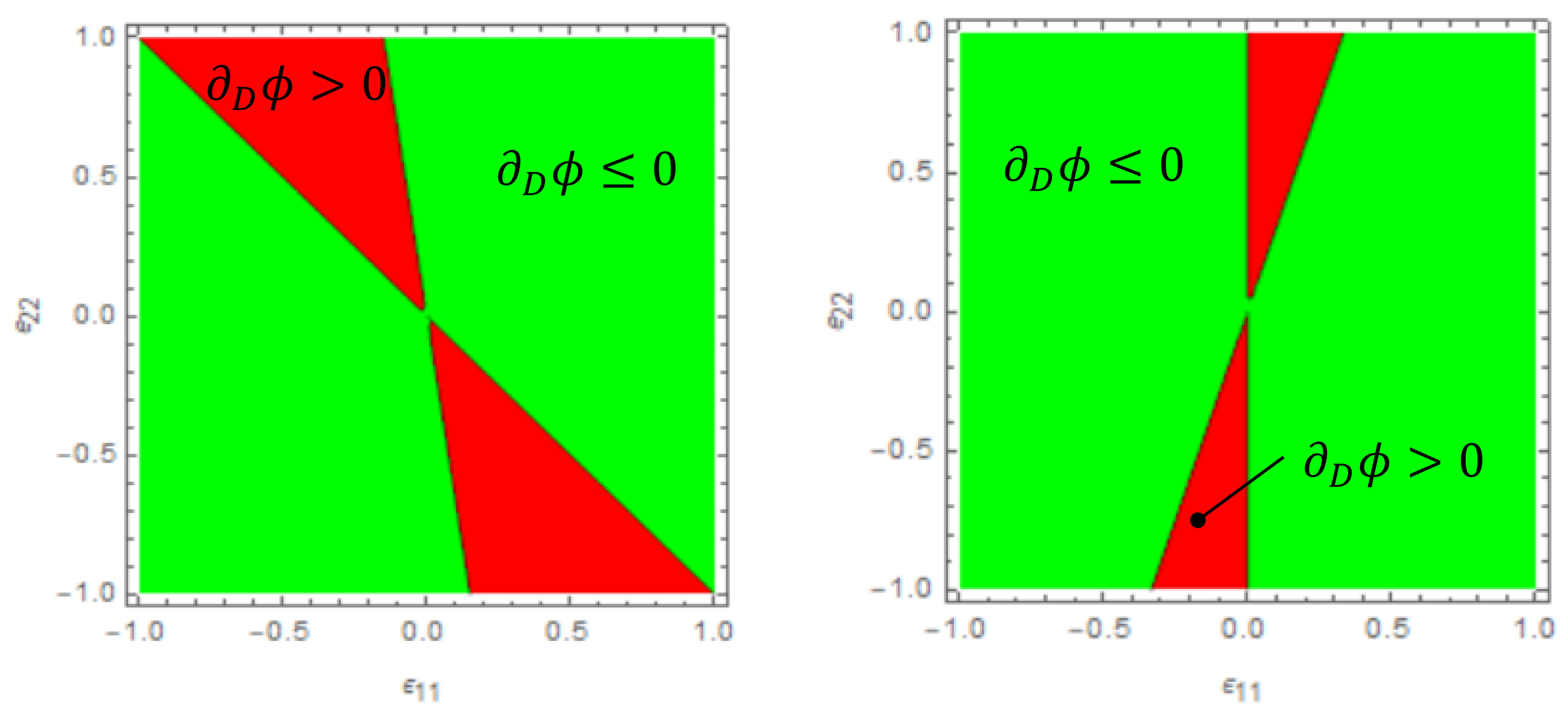

Fig 1: Model from Chaboche and Maire [1]: regions with a positive (red) and negative (green) damage growth rate. Example for isotropic $\mathbb{C}^0$ with $\xi = 1, \nu = 0.3$ (left) and $\xi = 0.5, \nu = 0.499$ (right)

Results

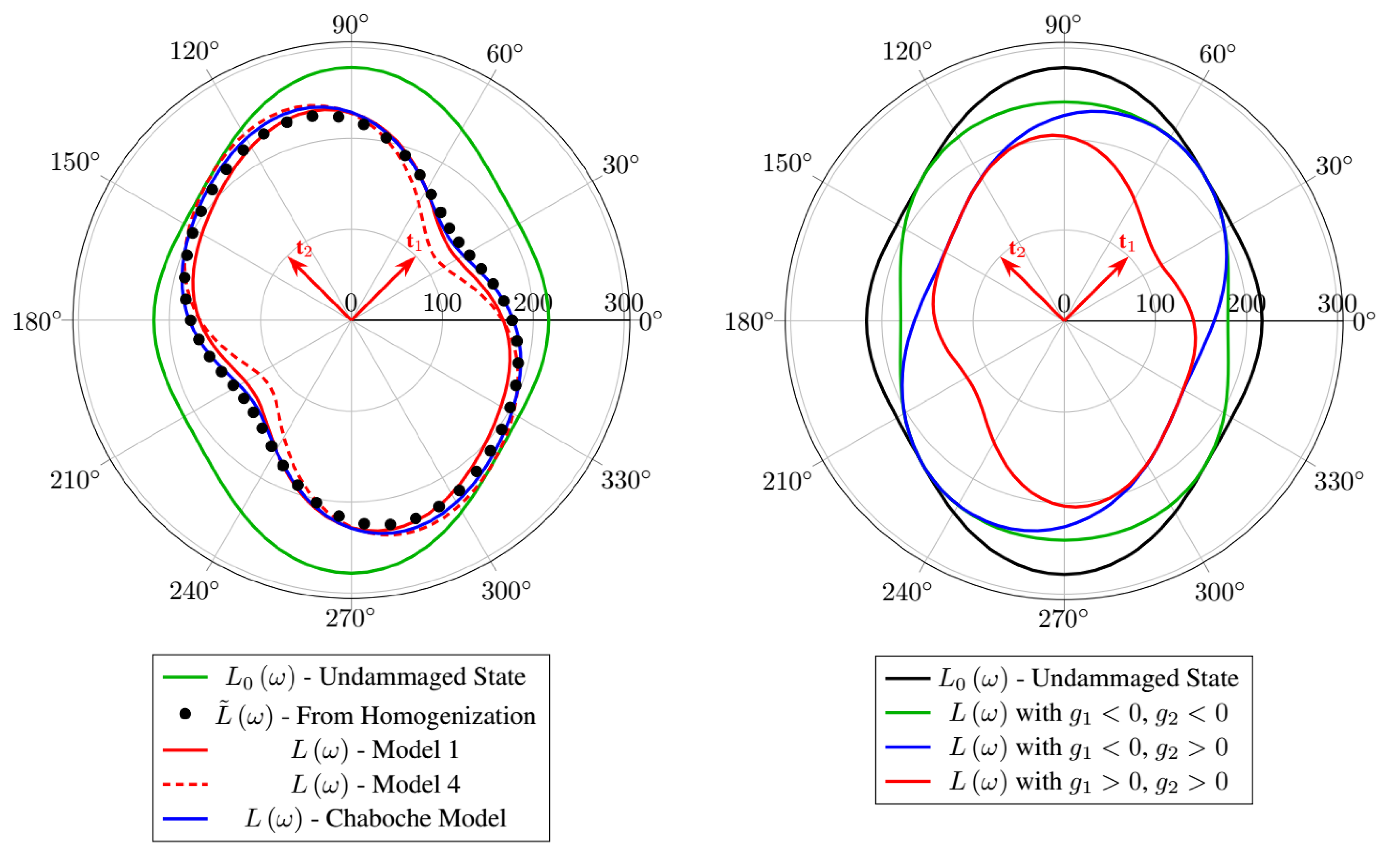

Fig 2: Model validation using virtual test data from a homogenization approach assuming a dilute crack concentration in a homogeneous anisotropic matrix (left), damage deactivation if crack set i ($g_i \leq 0$) is closed (right).

Outlook

In the next step the model will be further adapted and validated for the application to brittle composite materials (Ceramic Matrix Composites):

- Complement the model with a damage evolution
- Validation by experimental data acquired by ultrasonic measurements on damaged samples

References

[1] Chaboche J.L., and Maire J.F. Aerosp. Sci. Technol. 6 (2002), 131-145.
[2] Coredebois, J.-P., and Sidoroff, F, Mécanique des Solides Anisotropes, 295 (1982), 761–774.
[3] Wulfinghoff, S. et. al., Int. J. Solids and Struct. 121 (2017), 21-32.
[4] Zheng, Q., Appl. Mech. Rev., 47 (11), 545–587 .