Adaptive Multi-Fidelity & Multi-Scale Modelling of Damage in Composites

ICCM 23
INTERNATIONAL CONFERENCE ON COMPOSITE MATERIALS
BELFAST 2023
30 JULY – 4 AUGUST

Tong-Earn (T.E.) Tay
Professor, Dept of Mechanical Engineering,
National University of Singapore

V.B.C. Tan – colleague
L.H. Poh - colleague
Karthikayen Raju – Postdoc
K.H. Leong – PhD student
K.M. Yeoh – PhD student

Past Students/Postdocs:
Zhi Jie – Tongji University
Boyang Chen – TU Delft
Lu Xin – Tokyo University

Outline

Introduction

- Hi-Fidelity Modeling of Progressive Damage in Composites
- Smeared Crack Models (SCM)
- Discrete Crack Models (DCM)
- Separable Cohesive Elements (SCE)

Numerical Models

- Adaptive Discrete-Smeared Crack Model (A-DiSC)
- Adaptive Multi-Fidelity Model
- Multiscale Modeling with Direct FE^2

Applications

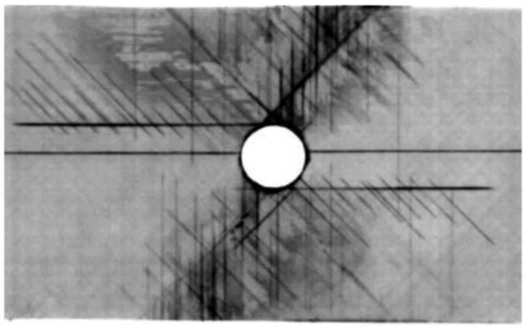
- Open-hole Tension
- Static Indentation
- Low-Velocity Impact and Compression After Impact

Outlook & Conclusion

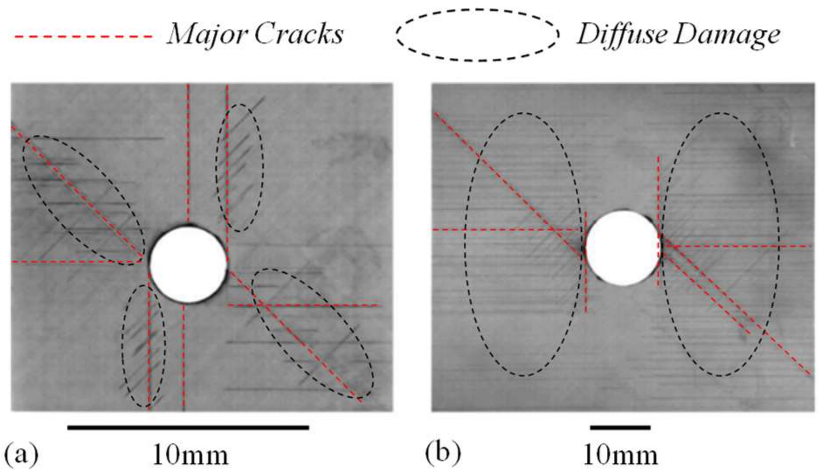
- Some thoughts for future research

NUS National University of Singapore

Discrete and Diffused Cracks Co-Exist in Composites



Hallett SR *et al., 2009*



Distinct matrix cracks

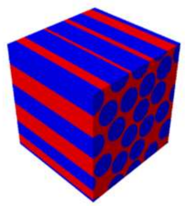
Diffused microcracks

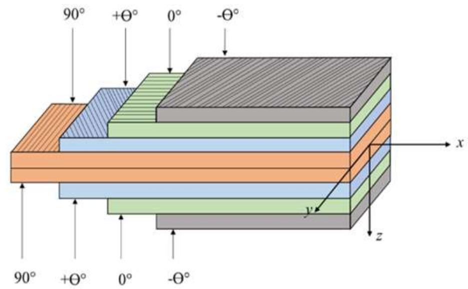
Delaminations

Fibre Damage

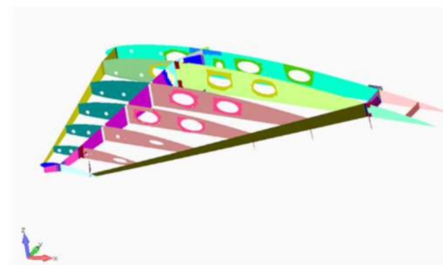
*Hallett SR, Green BG, Jiang WG, Wisnom MR. An experimental and numerical investigation into the damage mechanisms in notched composites. *Composites Part A: Applied Science and Manufacturing*. 2009;40(5):613-24.

Modeling Challenges


Micro-scale



Meso-scale


Macro-scale

Interlaminar damage:
Cohesive elements

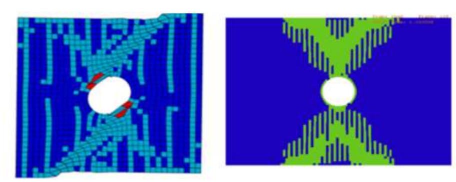
Intralaminar damage:
Cracks → Smeared *or* Discrete

Discrete Crack Methods (DCM)

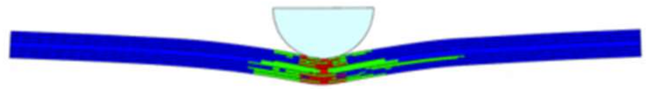
- X-FEM
- Phantom Node Method (PNM)
- Augmented FEM (AFEM)
- Floating Note Method (FNM)

Smeared Crack Method (SCM)

Damage is smeared out over the element - degradation of the stiffness



Example of a Smeared Crack + Cohesive Delamination Model
Barely-Visible Impact Damage (BVID) and Compression After Impact (CAI)



Impact Event

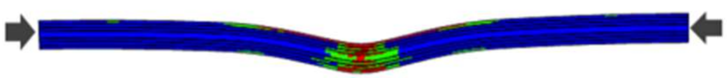
Global Buckling

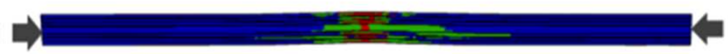
Layup $[0_2/45_2/\ 90_2/\ -45_2]_s$

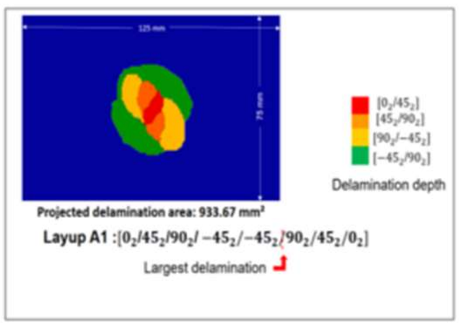
Compression After Impact

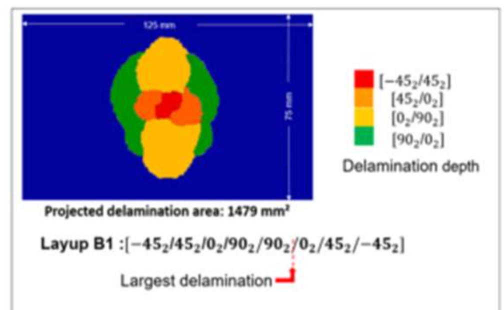
Sub-Laminate Buckling

Layup $[-45_2/45_2/\ 0_2/\ 90_2]_s$







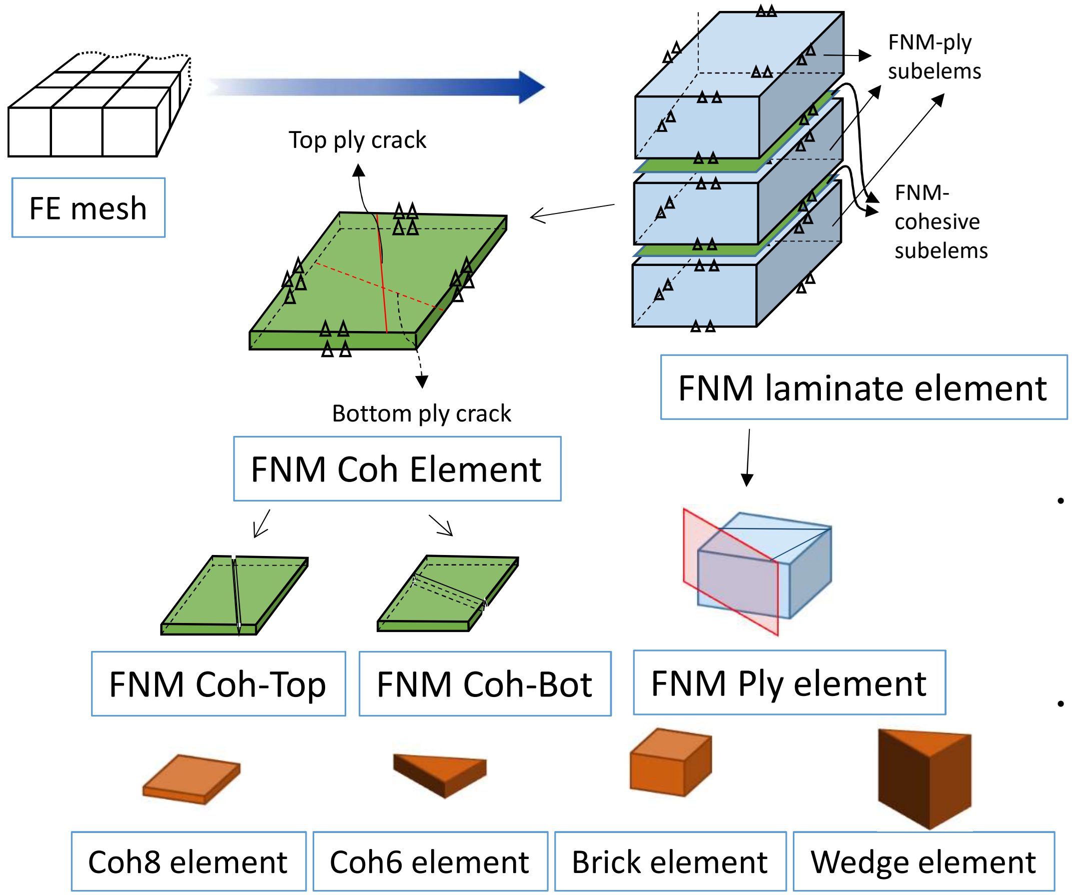


- Abir M.R., Tay T.E., Ridha M., Lee H.P., "On the relationship between failure mechanism and compression after impact (CAI) strength in composites", *Composite Structures*, (2017), vol 182, p242-250.
- Abir M.R., Tay T.E., Ridha M., Lee H.P., "Modelling damage growth in composites subjected to impact and compression after impact", *Composite Structures*, (2017), vol 168, p13-25.

NUS National University of Singapore

Discrete Crack Model

Example: The Floating Node Method (FNM) and Separable Cohesive Elements (SCE)



Boyang Chen – TU Delft

Lu Xin – Tokyo U

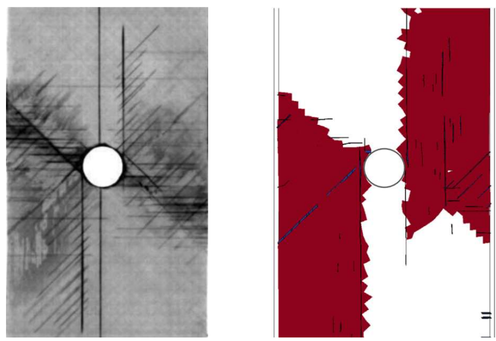
- Lu X., Chen B.Y., Tan V.B.C., Tay T.E., “A separable cohesive element for modelling coupled failure in laminated composite materials”, *Composites Part A: Applied Science & Manufacturing*, (2018), vol 107, p387-398.
- Chen B.Y., Pinho S.T., De Carvalho N.V., Baiz P.M., Tay T.E., “A floating node method for the modelling of discontinuities within a finite element”, *Engineering Fracture Mechanics*, (2014), vol 127, p104–134.

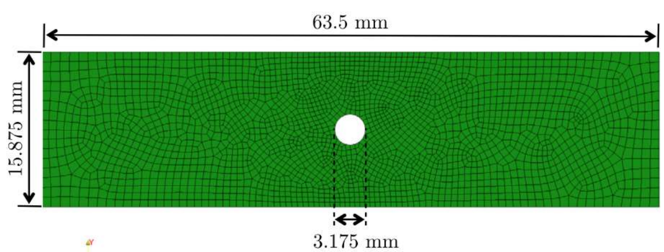
Discrete Crack Model

Applied to OHT and unnotched laminates

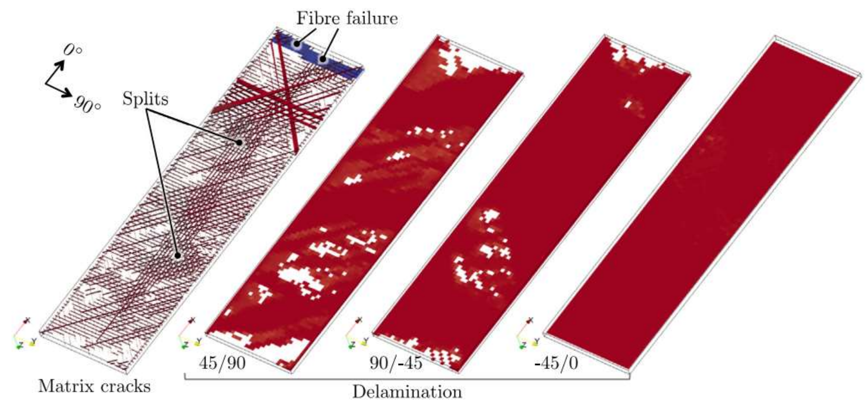
Boyang Chen – TU Delft

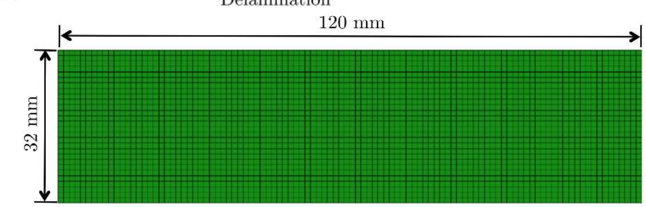
Holed laminate $[45_4/90_4/-45_4/0_4]_s$





Unnotched laminate $[45_4/90_4/-45_4/0_4]_s$





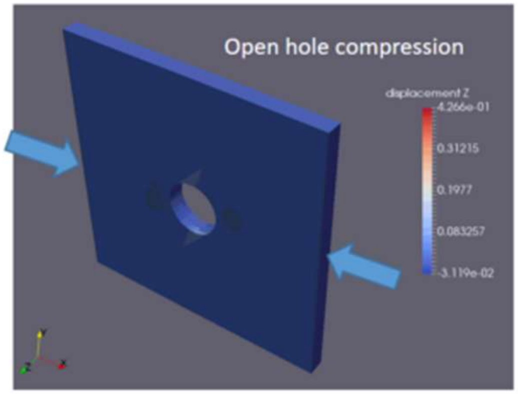
- Chen B.Y., Tay T.E., Pinho S.T., Tan V.B.C., "Modelling the tensile failure of composites with the floating node method", *Computer Methods in Applied Mechanics and Engineering*, (2016), vol 308, p414-442.

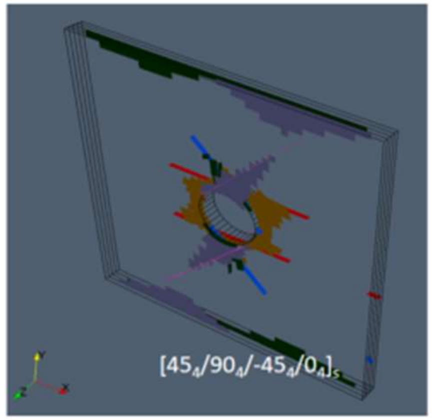
NUS National University of Singapore

Discrete Crack Model

Applied to OHC with geometrical non-linearity

Jie Zhi – Postdoc

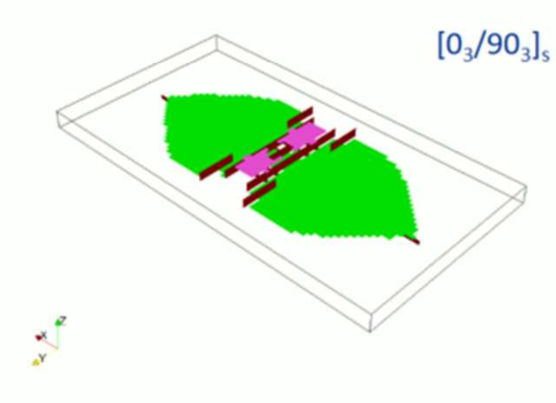




- Zhi J., Tay T.E., "Interrogating failure mechanisms of notched composites through a discrete crack modeling approach", *Composites Science & Technology*, (2020), vol 196, 108203.

Discrete Crack Model

Applied to low velocity impact of composites



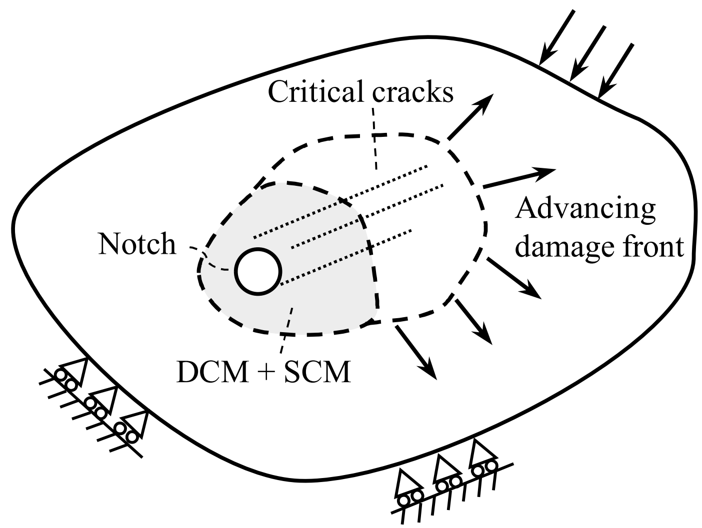
- Zhi J., Chen B.Y., Tay T.E., "Geometrically nonlinear analysis of matrix cracking and delamination in composites with floating node method", *Computational Mechanics*, (2019), vol 63, no 2, p201-217.

Adaptive Discrete-Smeared Crack (A-DiSC)

Discrete Crack Model (DCM):
- Closer fidelity to physics of fracture.
- Computationally intense.
- Not practical for larger structures.

Smeared Crack Model (SCM):
- Efficient representation of numerous micro-cracks.
- May have convergence and distortion issues
- Generally unable to model interaction with delamination accurately.

A-DiSC / Coupled Model:
- Fidelity to physics of fracture where and when needed, i.e. at active damage fronts.
- Possibly less computationally intense for larger structures.



- Lu X., Ridha M., Tan V.B.C., Tay T.E., "Adaptive discrete-smeared crack (A-DiSC) model for multi-scale progressive damage in composites", *Composites Part A: Applied Science & Manufacturing*, (2019), vol 125, 105513

Adaptive Discrete-Smeared Crack (A-DiSC)

Lu Xin – Tokyo U

A-DiSC Transition Criterion

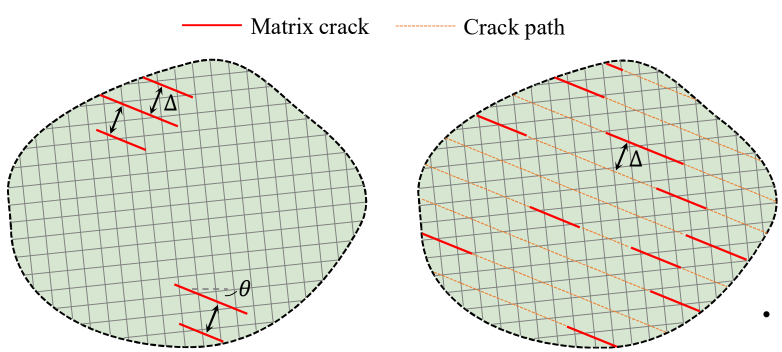
Pure DCM ↔ Coupled DCM/SCM

Energy Consistency for Model Transition

1. *When to transit from DCM to coupled DCM/SCM?*

When matrix cracks are "saturated"

Crack spacing $\Delta = 0.5\ mm$



2. *Are there a few larger "critical" cracks which may still grow?*

Characteristic length $l_c = 2\ mm$

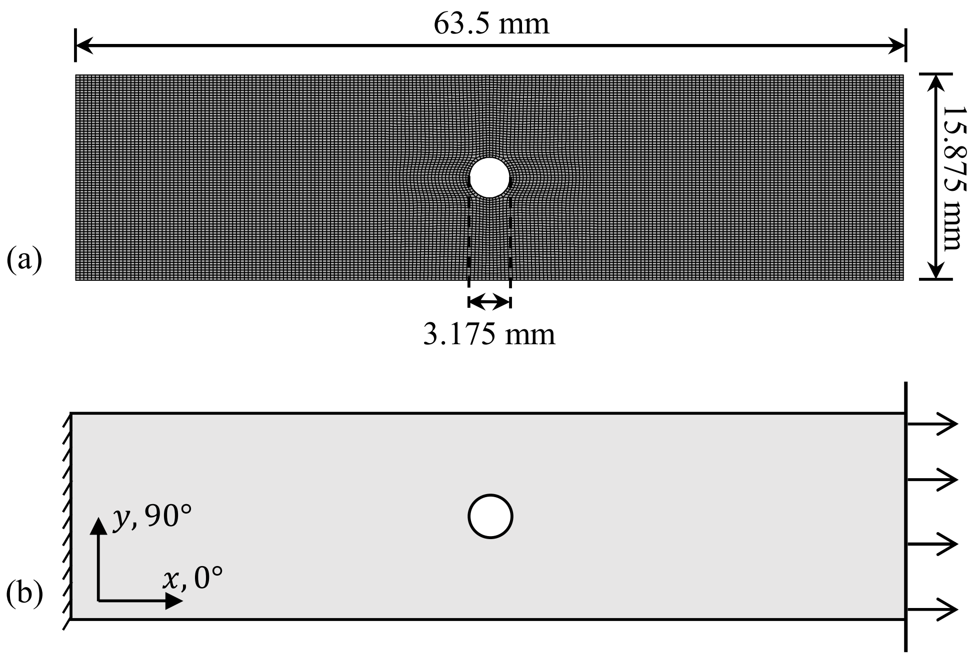
- Lu X., Ridha M., Tan V.B.C., Tay T.E., "Adaptive discrete-smeared crack (A-DiSC) model for multi-scale progressive damage in composites", *Composites Part A: Applied Science & Manufacturing*, (2019), vol 125, 105513

NUS National University of Singapore

Adaptive Discrete-Smeared Crack (A-DiSC)

Lu Xin – Tokyo U

❖ Open-hole-tension (OHT)



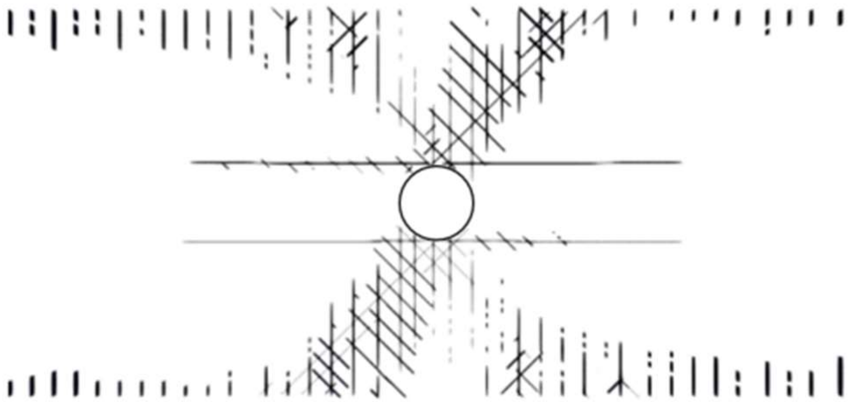
Layup: $[45_4/90_4/\text{-}45_4/0_4]_s$
Materials: IM7/8552 *
0.2mm mesh
0.5mm crack spacing

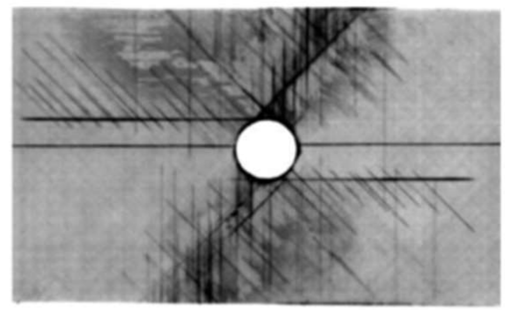
Models:

1. DCM (benchmark)
2. A-DiSC

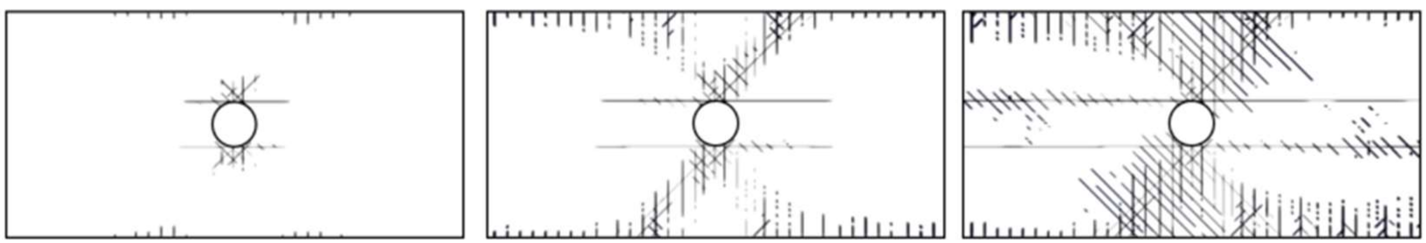
*B.G. Green, M.R. Wisnom, S.R. Hallett, An experimental investigation into the tensile strength scaling of notched composites, *Compos. Part A Appl. Sci. Manuf.*, 38 (2007) 867-878.

❖ Pure Discrete Crack Model



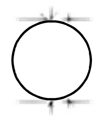


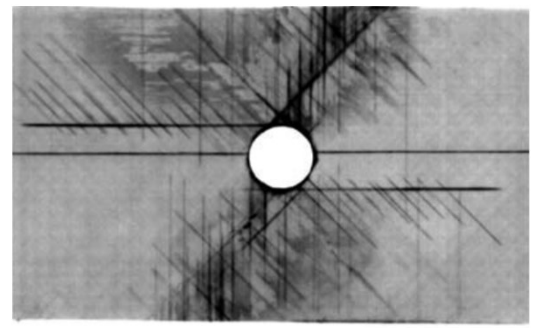
S.R. Hallett *et al., 2009*



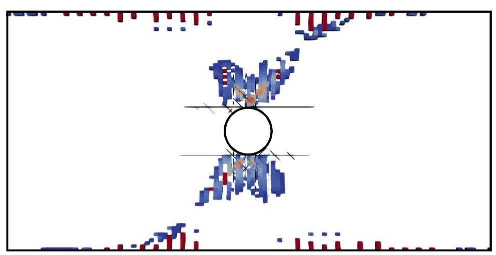
❖ Adaptive Discrete-Smeared Crack Model

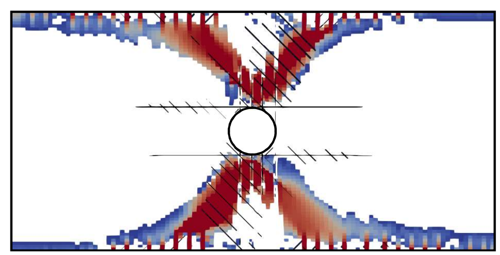
90° ply





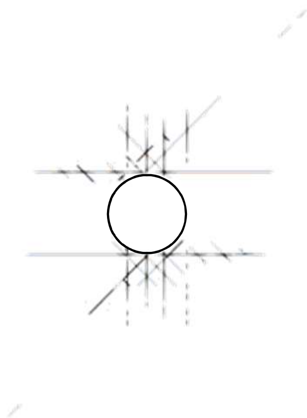
S.R. Hallett *et al., 2009*

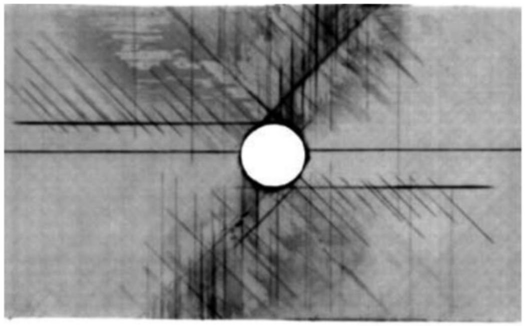




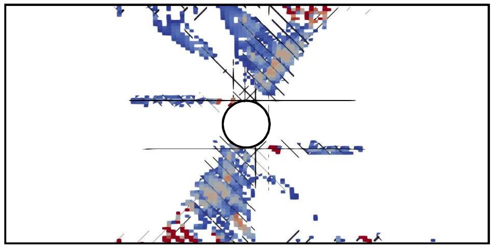
❖ Adaptive Discrete-Smeared Crack Model

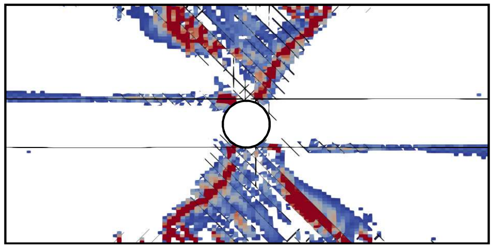
-45° ply (internal)





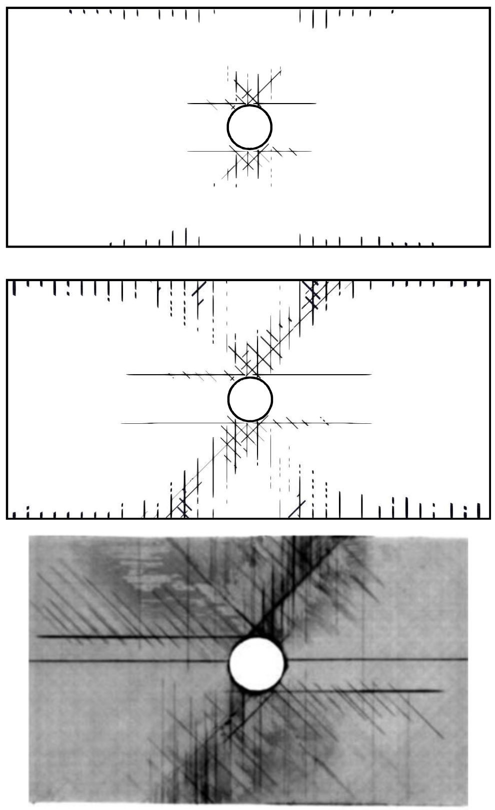
S.R. Hallett *et al., 2009*





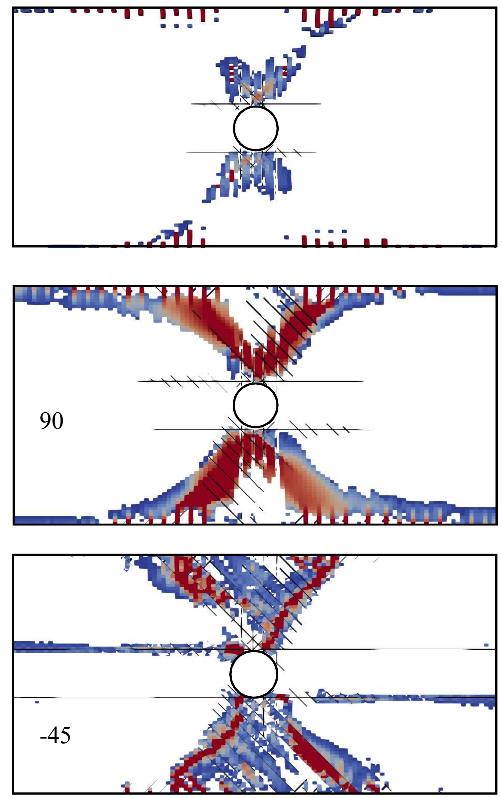
❖ Comparison

Full 3D Discrete Crack Model



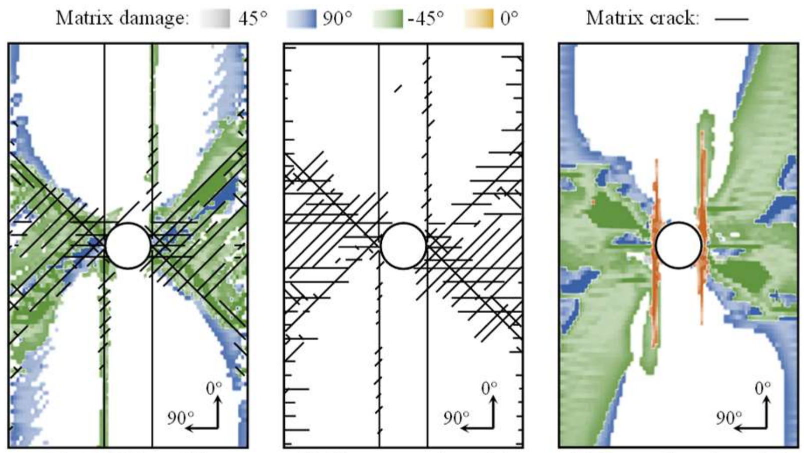
S.R. Hallett *et al., 2009*

Adaptive Discrete-Smeared Crack Model

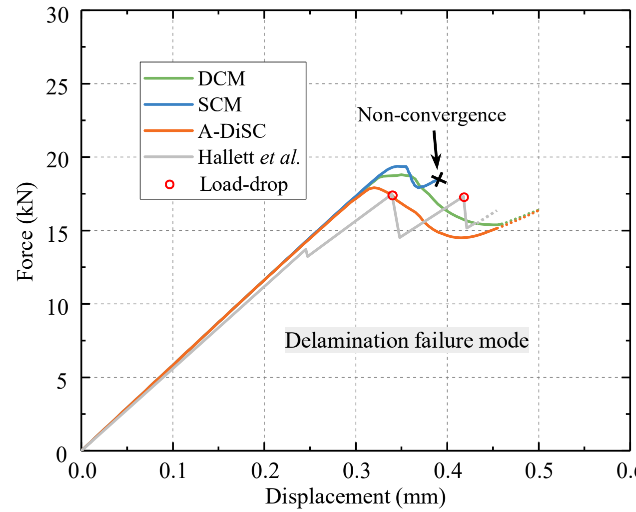


❖ Comparison

Lu Xin – Tokyo U



(a) A-DiSC model. (b) Discrete crack model. (c) Smeared crack model.



Models	DoFs	Iterations	CPU time (s)
SCM	412704	N.A.	N.A.
DCM	705012	4089	1066660
A-DiSC ($\rho = \rho_s$, $l_c = 2$ mm)	538839	3681	966364
A-DiSC ($\rho = 0.3\ \rho_s$, $l_c = 2$ mm)	446202	3726	825149

ρ_s: saturated matrix crack density

Up to 36.7% saving

Up to 22.6% saving

- Lu X., Ridha M., Tan V.B.C., Tay T.E., "Adaptive discrete-smeared crack (A-DiSC) model for multi-scale progressive damage in composites", *Composites Part A: Applied Science & Manufacturing*, (2019), vol 125, 105513

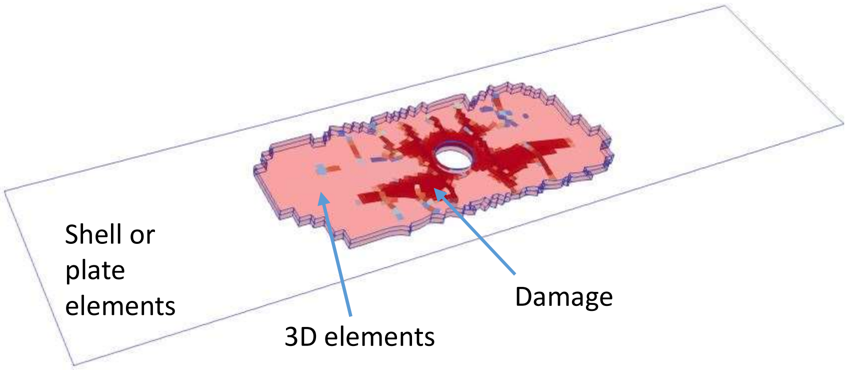
NUS National University of Singapore

Adaptive Multi-Fidelity (AMF) Model

Discrete Crack Model (DCM):
- Closer fidelity to physics of fracture.
- Computationally intense.
- Not practical for larger structures.

Smeared Crack Model (SCM):
- Efficient representation of numerous micro-cracks.
- May have convergence and distortion issues
- Generally unable to model interaction with delamination accurately.

A-DiSC / Coupled Model:
- Fidelity to physics of fracture where and when needed, i.e. at active damage fronts.
- Possibly less computationally intense for larger structures.



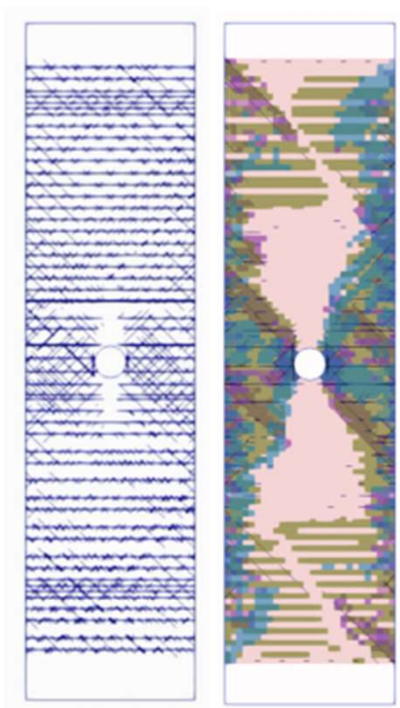
Adaptive Multi-Fidelity (AMF) Model:
- Fidelity to physics of fracture where and when needed, i.e. at active damage fronts.
- Shell→3D→Shell elements adaptivity
- Less computationally intense for larger structures.

Leong K.H., Zhi J., Lee H.P., Tan V.B.C., Tay T.E., “Adaptive multi-fidelity (AMF) modelling of progressive damage in notched composite laminates”, Composites Part A: Applied Science & Manufacturing, https://doi.org/10.1016/j.compositesa.2021.106790 (2022), vol 154, 106790.

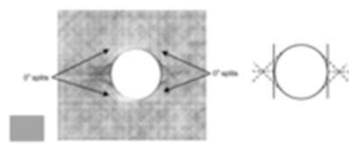
Adaptive Multi-Fidelity (AMF) Model

K.H. Leong – PhD student

Open hole tension (OHT) $[45/90/-45/0]_{4s}$



Full 3D AMF

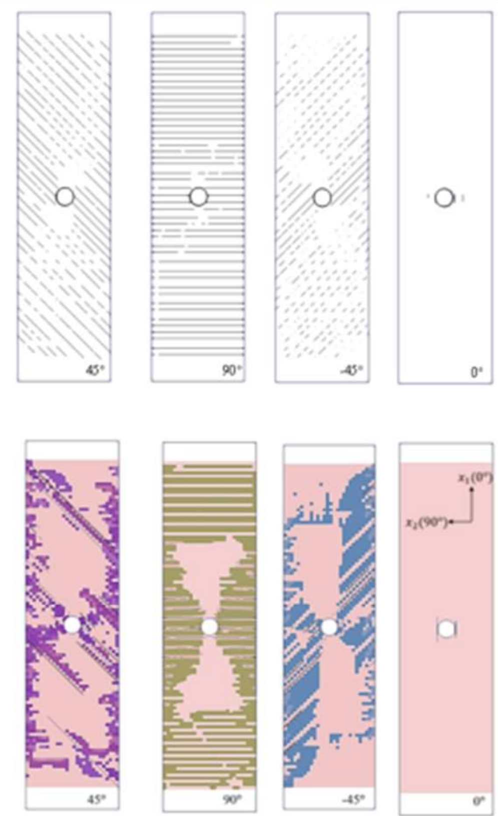


experiment

0°
90°

- Matrix crack
- Brick
- Equivalent single layer shell
- Smeared shell in −45° plies
- Smeared shell in 45° plies
- Smeared shell in 90° plies

Matrix failure patterns of each ply



Full 3D

AMF

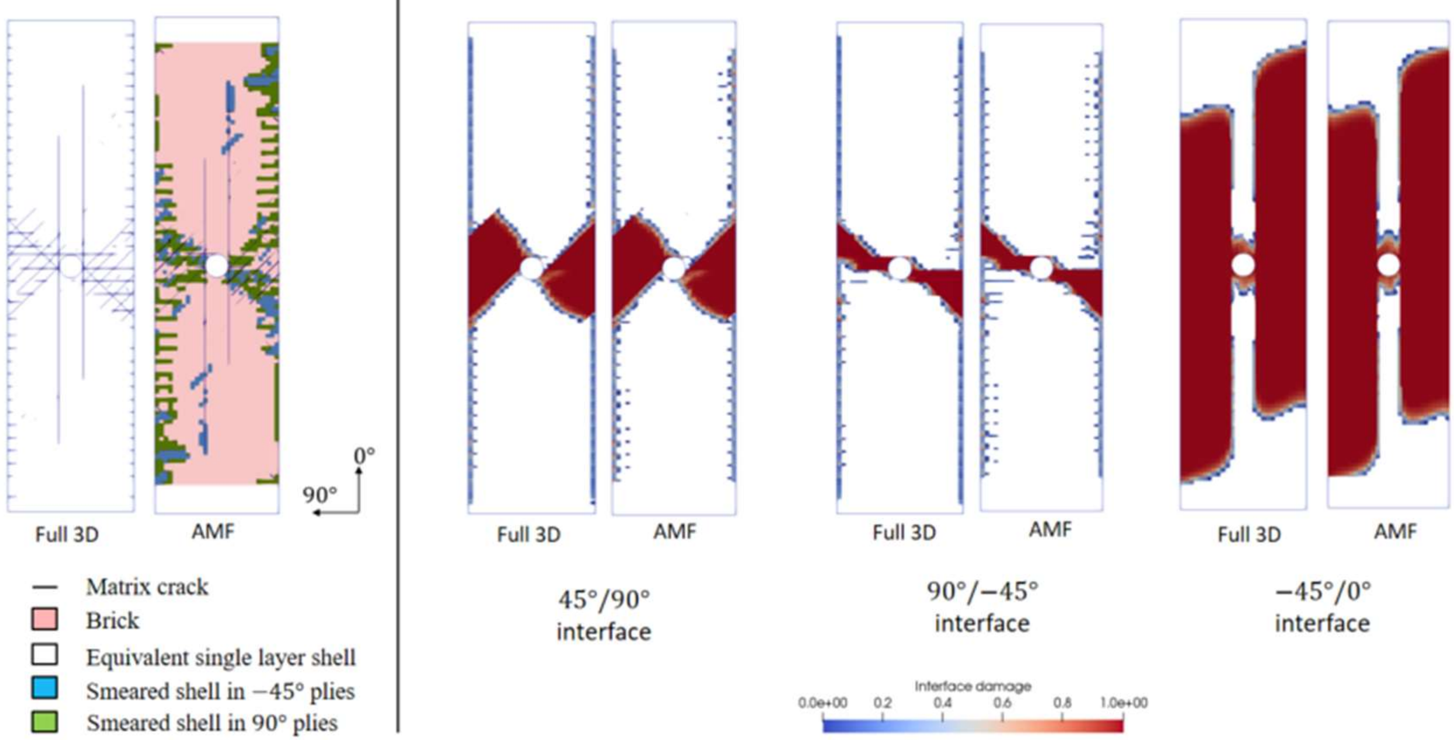
Leong K.H., Zhi J., Lee H.P., Tan V.B.C., Tay T.E., "Adaptive multi-fidelity (AMF) modelling of progressive damage in notched composite laminates", Composites Part A: Applied Science & Manufacturing, (2022), vol 154, 106790.

NUS National University of Singapore

Adaptive Multi-Fidelity (AMF) Model

Open hole tension (OHT)

Layup $[45_4/90_4/\text{-}45_4/0_4]_s$



Adaptive Multi-Fidelity (AMF) Model

Computational time
8 CPUs (Xeon(R) CPU E5-2697 @ 2.30GHz)

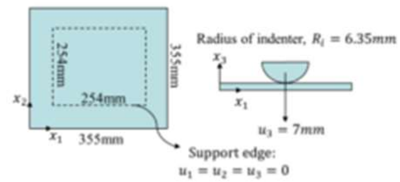
OHT $[45_4/90_4/-45_4/0_4]_s$			
Total CPU time (hours)	10.5	8.7	-17.1%
Wallclock time (hours)	11.6	9.6	-16.5%
OHT $[45/90/-45/0]_{4s}$			
Total CPU time (hours)	330.8	245.5	-25.7%
Wallclock time (hours)	333.5	252.7	-23.2%

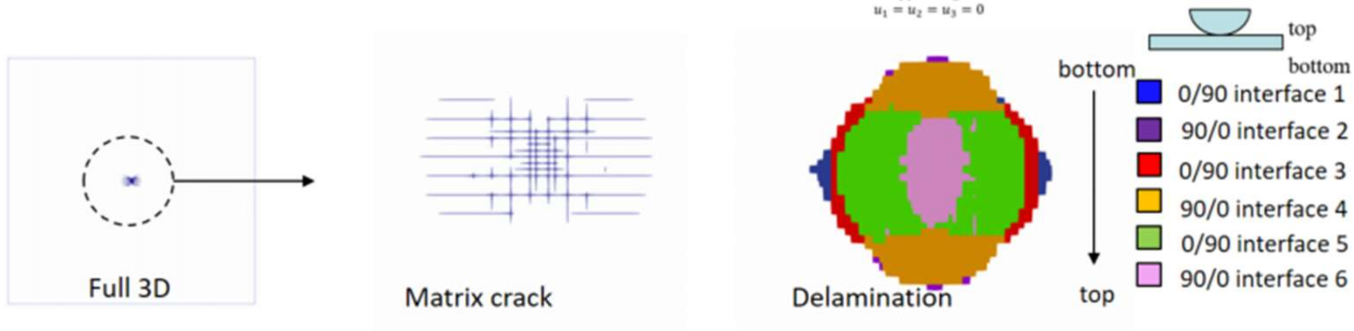
NUS National University of Singapore

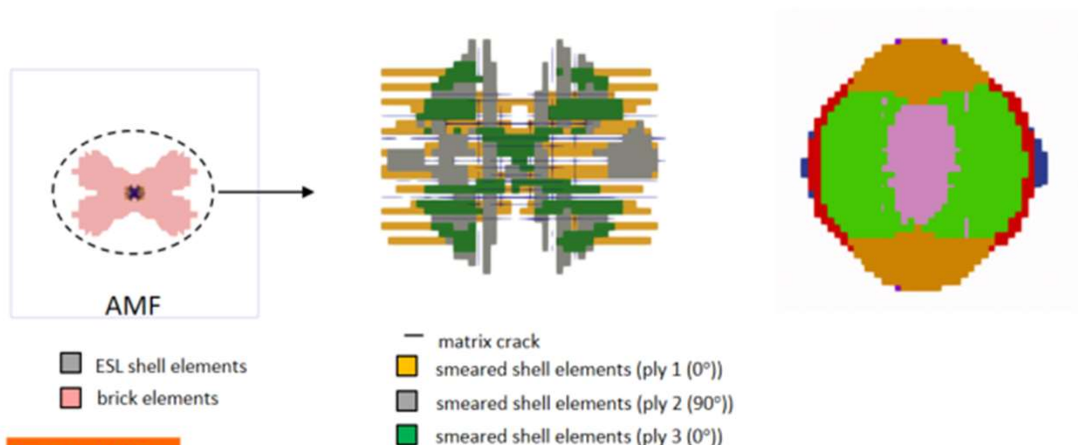
Adaptive Multi-Fidelity (AMF) Model

Static indentation test

Cross-ply layup $[0_2/90_2/0_2/90_2]_s$

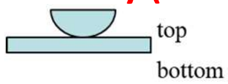


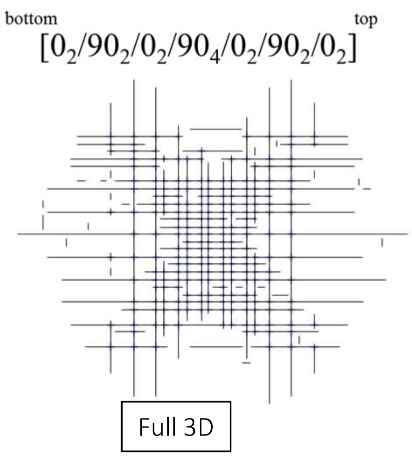


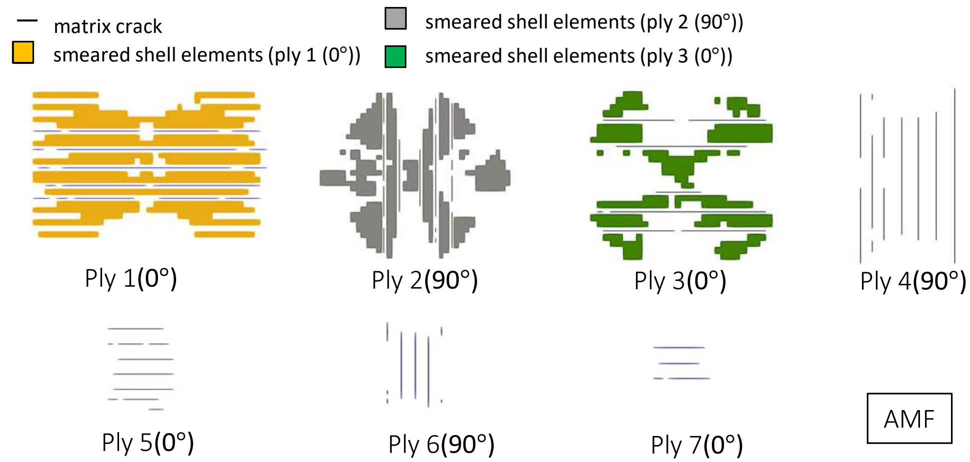


Adaptive Multi-Fidelity (AMF) Model

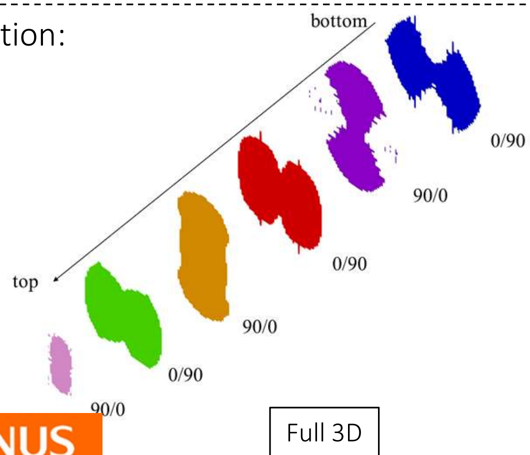
Matrix cracks:

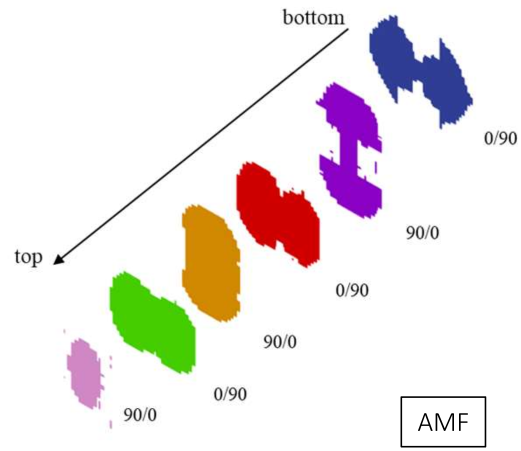






Delamination:





NUS National University of Singapore

Adaptive Multi-Fidelity (AMF) Model

Computational time
-8 CPUs (Xeon(R) CPU E5-2697 @ 2.30GHz)

Static indentation **$[0_2/90_2/0_2/90_2]_s$**			
	Full 3D	AMF	
Total CPU time (hours)	71.6	30.5	→ -57.4%
Wallclock time (hours)	73.3	33.1	→ -54.7%

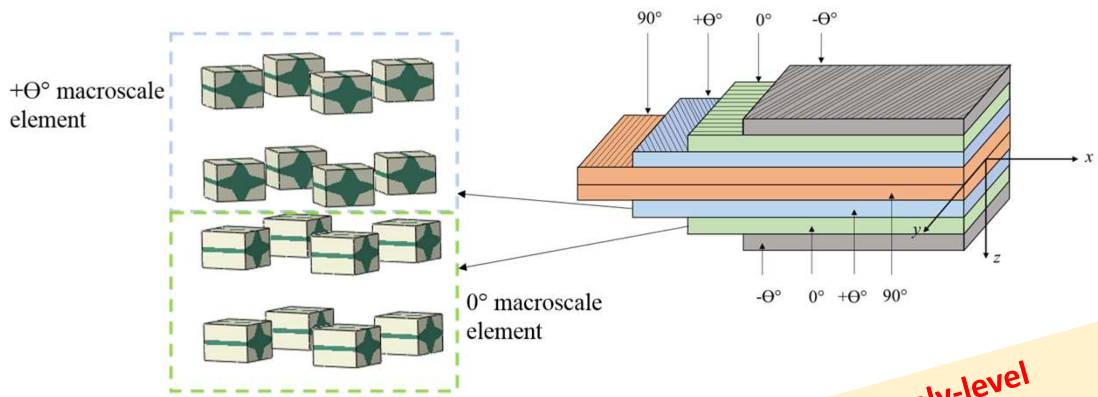
Related Publications (partial list)

- ✓ Abir M.R., Tay T.E., Ridha M., Lee H.P., "On the relationship between failure mechanism and compression after impact (CAI) strength in composites", ***Composite Structures***, (2017), vol 182, p242-250.
- ✓ Abir M.R., Tay T.E., Ridha M., Lee H.P., "Modelling damage growth in composites subjected to impact and compression after impact", ***Composite Structures***, (2017), vol 168, p13-25.
- ✓ Lu X., Chen B.Y., Tan V.B.C., Tay T.E., "A separable cohesive element for modelling coupled failure in laminated composite materials", ***Composites Part A: Applied Science & Manufacturing***, (2018), vol 107, p387-398.
- ✓ Chen B.Y., Pinho S.T., De Carvalho N.V., Baiz P.M., Tay T.E., "A floating node method for the modelling of discontinuities within a finite element", ***Engineering Fracture Mechanics***, (2014), vol 127, p104–134.
- ✓ Chen B.Y., Tay T.E., Pinho S.T., Tan V.B.C., "Modelling the tensile failure of composites with the floating node method", ***Computer Methods in Applied Mechanics and Engineering***, (2016), vol 308, p414-442.
- ✓ Zhi J., Tay T.E., "Interrogating failure mechanisms of notched composites through a discrete crack modeling approach", ***Composites Science & Technology***, (2020), vol 196, 108203.
- ✓ Zhi J., Chen B.Y., Tay T.E., "Geometrically nonlinear analysis of matrix cracking and delamination in composites with floating node method", ***Computational Mechanics***, (2019), vol 63, no 2, p201-217.
- ✓ Lu X., Ridha M., Tan V.B.C., Tay T.E., "Adaptive discrete-smeared crack (A-DiSC) model for multi-scale progressive damage in composites", ***Composites Part A: Applied Science & Manufacturing***, (2019), vol 125, 105513.
- ✓ Lu X., Guo X.M., Tan V.B.C., Tay T.E., "From Diffuse Damage to Discrete Crack: A Coupled Failure Model for Multi-stage Progressive Damage of Composites", ***Computer Methods in Applied Mechanics and Engineering***, (2021), vol 379, 113760.
- ✓ Leong K.H., Zhi J., Lee H.P., Tan V.B.C., Tay T.E., "Adaptive multi-fidelity (AMF) modelling of progressive damage in notched composite laminates", ***Composites Part A: Applied Science & Manufacturing***, (2022), vol 154, 106790.
- ✓ Leong K.H., Zhi J., Lee H.P., Tan V.B.C., Tay T.E., "Adaptive Multi-Fidelity (AMF) modelling of delamination migration under bending", ***Composite Structures***, (2023), vol 305, 116549.

NUS National University of Singapore

Concurrent Multiscale Modeling of Composites with Direct FE^2

Vincent B.C. Tan – colleague



No need for ply-level failure criteria?

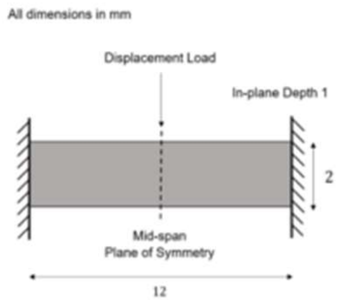
- FE^2 simulations allows for the analyses of structures made from heterogeneous materials without the need for homogenized material models
- Direct FE^2 reduces the two separate and concurrent simulations at macro- and mirco-scales required in FE^2 to a single simulation
- Direct FE^2 can be carried out on directly on commercial FE codes
- Features already on commercial codes (e.g., finite deformation, viscoplasticity, damage models) remain available without any user intervention

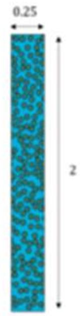
- Tan V.B.C., Raju K., Lee H.P., "Direct FE^2 for concurrent multi level modelling of heterogeneous structures", *Computer Methods in Applied Mechanics and Engineering*, (2019), 112694.
- Raju K., Tay T.E., Tan V.B.C., "A review of the FE2 method for composites", *Multiscale and Multidiscip. Model. Exp. and Des*., (2021), vol 4, 1-24.

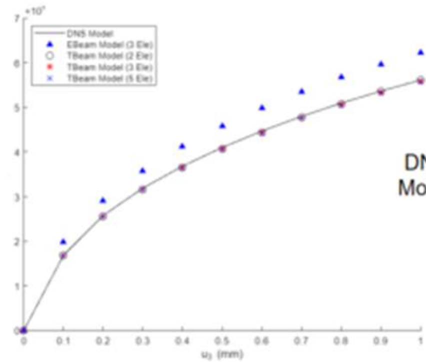
NUS National University of Singapore

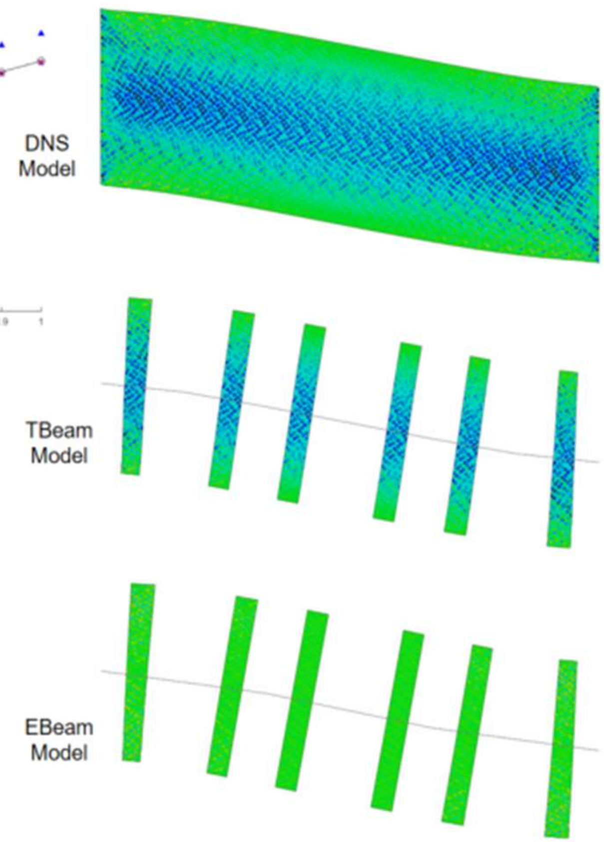
Multiscale FE^2 modelling of composites using shear-flexible beam elements

K.M. Yeoh – PhD student





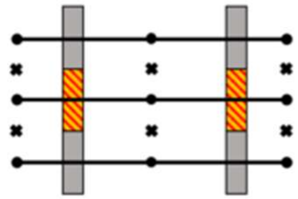




Model	CPU Time (s)	Reduction (%)
DNS	863.8	-
TBeam (2 beam ele)	113.9	86.8
TBeam (3 beam ele)	222.8	74.2
TBeam (5 beam ele)	332.8	61.5

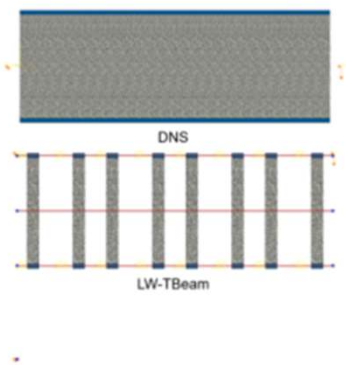
NUS National University of Singapore

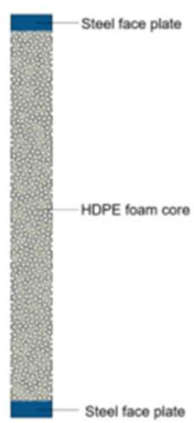
Foam core sandwich beams with steel facesheets

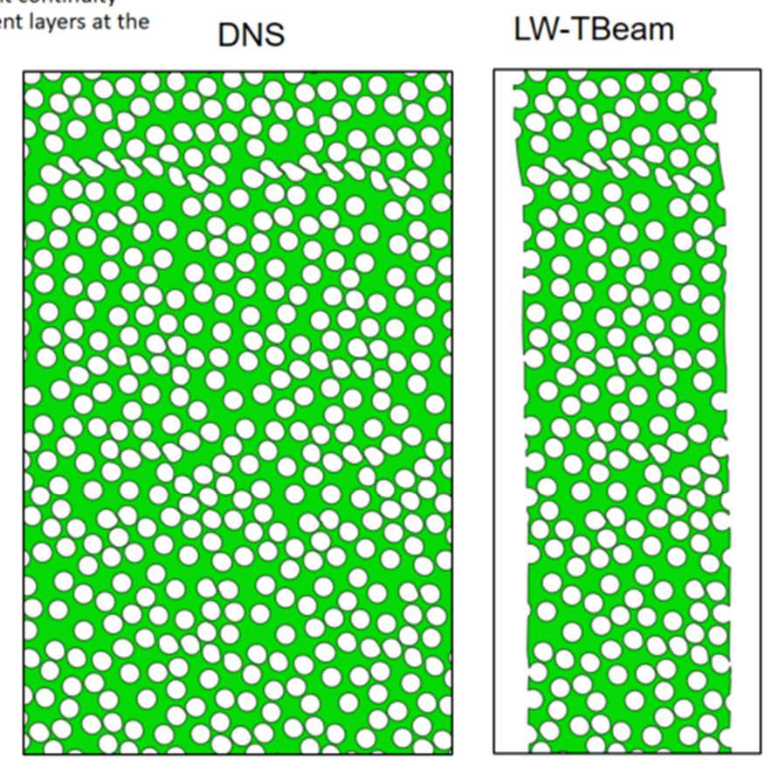


- Each layer is modelled with an individual macroscale beam element
- Modified shear angle constraints to account for layerwise nature
- Ensure displacement continuity between the different layers at the interface

K.M. Yeoh – PhD student





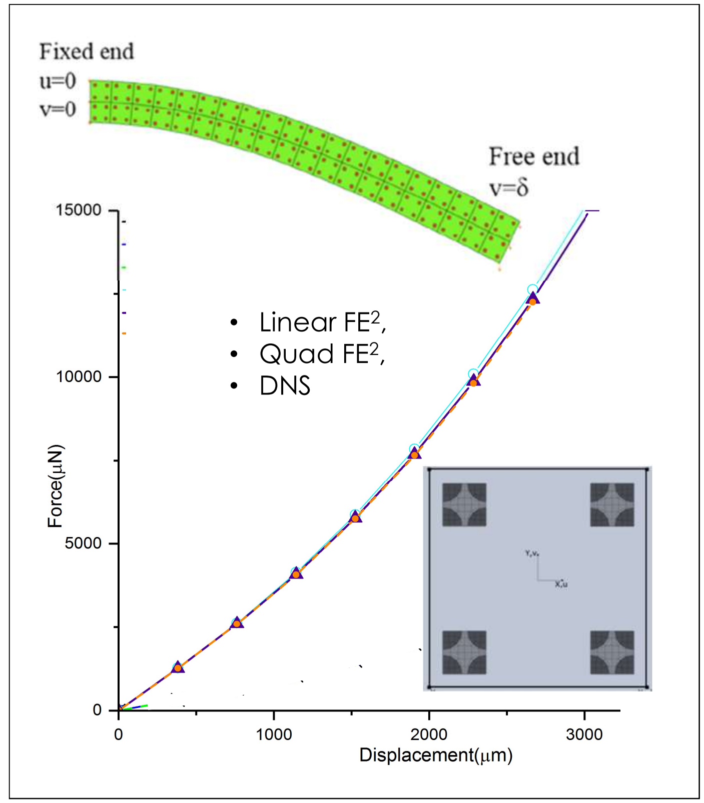


Direct FE^2 research

Slide courtesy of Vincent Tan

1. **Direct FE2 for concurrent multilevel modelling of heterogeneous structures, (2020) CMAME, 360, 112694.**
2. A review of the FE2 method for composites, (2021) Multiscale and Multidisciplinary Modeling, Exp and Des, 4 (1).
3. Direct FE2 for simulating strain-rate dependent compressive failure of cylindrical CFRP, (2021) Comp Part C, 5, 100165.
4. Analysis of nonlinear shear and damage behaviour of angle-ply laminates with Direct FE2, (2021) CST, 216, 109050.
5. Transient multi-scale analysis with micro-inertia effects using Direct FE 2 method, (2021) Comp Mech, 67 (6).
6. Multiscale analysis of thermal problems in heterogeneous materials with Direct FE2 method, (2021) IJNME, 122 (24).
7. Multiscale computational homogenisation of shear-flexible beam elements: a Direct FE2 approach, (2022) Comp Mech, 70 (5).
8. Direct FE2 for concurrent multilevel modelling modeling of heterogeneous thin plate structures, (2022) CMAME, 392, 114658.
9. Direct FE2 modeling of heterogeneous materials with a micromorphic computational homogenization framework, (2022) CMAME, 393, 114837.

First report of Direct FE^2 using CFRP beam as example. Demonstrated large deformation and material nonlinearity.

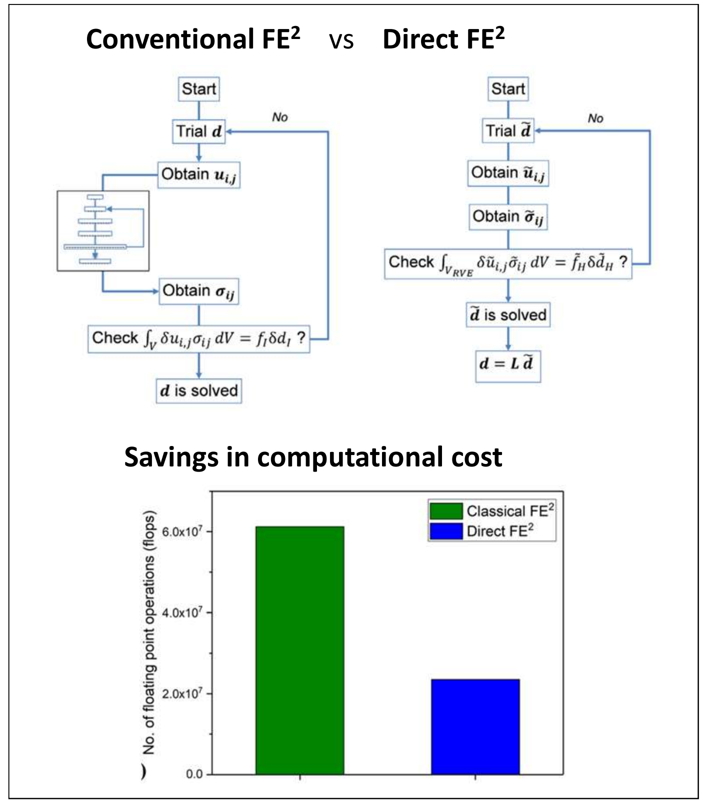


Direct FE^2 research

Slide courtesy of Vincent Tan

1. Direct FE2 for concurrent multilevel modelling of heterogeneous structures, (2020) CMAME, 360, 112694.
2. **A review of the FE2 method for composites, (2021) Multiscale and Multidisciplinary Modeling, Exp and Des, 4 (1).**
3. Direct FE2 for simulating strain-rate dependent compressive failure of cylindrical CFRP, (2021) Comp Part C, 5, 100165.
4. Analysis of nonlinear shear and damage behaviour of angle-ply laminates with Direct FE2, (2021) CST, 216, 109050.
5. Transient multi-scale analysis with micro-inertia effects using Direct FE 2 method, (2021) Comp Mech, 67 (6).
6. Multiscale analysis of thermal problems in heterogeneous materials with Direct FE2 method, (2021) IJNME, 122 (24).
7. Multiscale computational homogenisation of shear-flexible beam elements: a Direct FE2 approach, (2022) Comp Mech, 70 (5).
8. Direct FE2 for concurrent multilevel modelling modeling of heterogeneous thin plate structures, (2022) CMAME, 392, 114658.
9. Direct FE2 modeling of heterogeneous materials with a micromorphic computational homogenization framework, (2022) CMAME, 393, 114837.

Reviewed applications of FE^2 in composites and showed how the monolithic scheme of Direct FE^2 reduces computational cost.



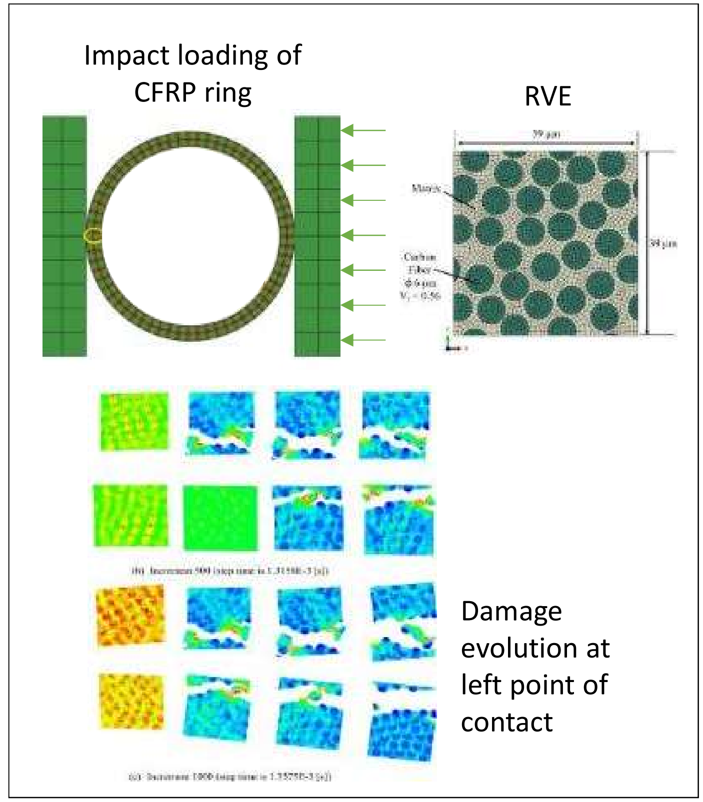
Direct FE^2 research

Slide courtesy of Vincent Tan

In Collaboration with Jun Koyanagi,
Tokyo University of Science.

1. Direct FE2 for concurrent multilevel modelling of heterogeneous structures, (2020) CMAME, 360, 112694.
2. A review of the FE2 method for composites, (2021) Multiscale and Multidisciplinary Modeling, Exp and Des, 4 (1).
3. **Direct FE2 for simulating strain-rate dependent compressive failure of cylindrical CFRP, (2021) Comp Part C, 5, 100165.**
4. Analysis of nonlinear shear and damage behaviour of angle-ply laminates with Direct FE2, (2021) CST, 216, 109050.
5. Transient multi-scale analysis with micro-inertia effects using Direct FE 2 method, (2021) Comp Mech, 67 (6).
6. Multiscale analysis of thermal problems in heterogeneous materials with Direct FE2 method, (2021) IJNME, 122 (24).
7. Multiscale computational homogenisation of shear-flexible beam elements: a Direct FE2 approach, (2022) Comp Mech, 70 (5).
8. Direct FE2 modeling of heterogeneous materials with a micromorphic computational homogenization framework, (2022) CMAME, 393, 114837.
9. Direct FE2 for concurrent multilevel modelling modeling of heterogeneous thin plate structures, (2022) CMAME, 392, 114658.

Using substantially larger RVEs, evolution of microstructural damage in rate-sensitive carbon-epoxy composite was modeled.



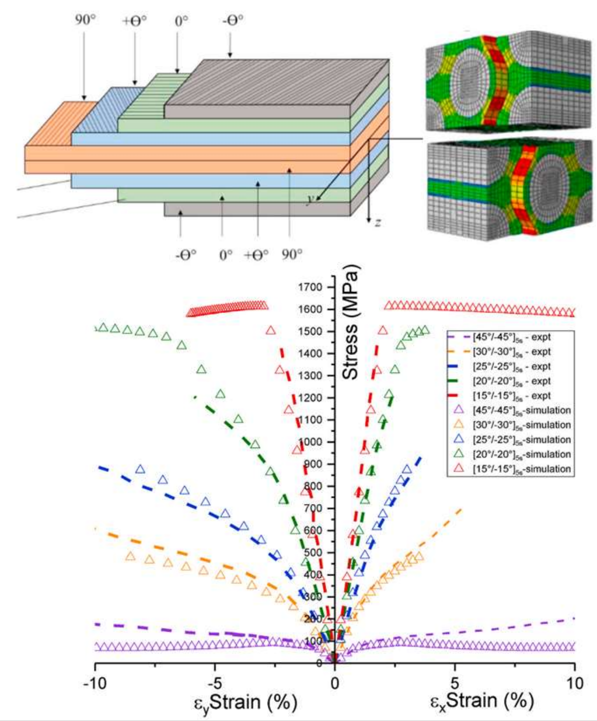
Direct FE^2 research

Slide courtesy of Vincent Tan

1. Direct FE2 for concurrent multilevel modelling of heterogeneous structures, (2020) CMAME, 360, 112694.
2. A review of the FE2 method for composites, (2021) Multiscale and Multidisciplinary Modeling, Exp and Des, 4 (1).
3. Direct FE2 for simulating strain-rate dependent compressive failure of cylindrical CFRP, (2021) Comp Part C, 5, 100165.
4. **Analysis of nonlinear shear and damage behaviour of angle-ply laminates with Direct FE2, (2021) CST, 216, 109050.**
5. Transient multi-scale analysis with micro-inertia effects using Direct FE 2 method, (2021) Comp Mech, 67 (6).
6. Multiscale analysis of thermal problems in heterogeneous materials with Direct FE2 method, (2021) IJNME, 122 (24).
7. Multiscale computational homogenisation of shear-flexible beam elements: a Direct FE2 approach, (2022) Comp Mech, 70 (5).
8. Direct FE2 for concurrent multilevel modelling modeling of heterogeneous thin plate structures, (2022) CMAME, 392, 114658.
9. Direct FE2 modeling of heterogeneous materials with a micromorphic computational homogenization framework, (2022) CMAME, 393, 114837.

DFE^2 enabled interaction of fibre rotation, matrix plasticity and cracks to predict σ vs ε plots of angle-plys matching experiments

Stress-strain plots – DFE2 vs experiments

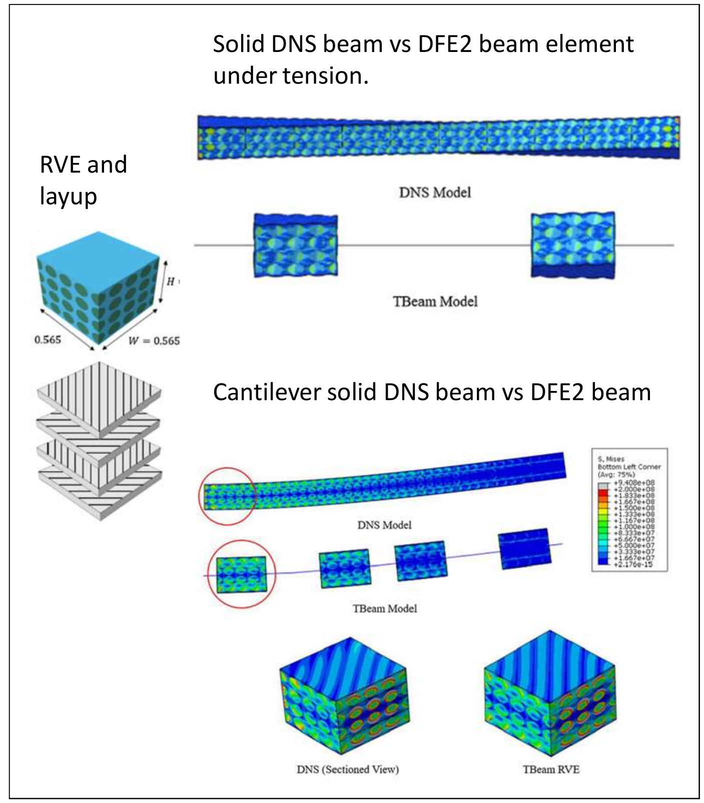


Direct FE^2 research

Slide courtesy of Vincent Tan

1. Direct FE2 for concurrent multilevel modelling of heterogeneous structures, (2020) CMAME, 360, 112694.
2. A review of the FE2 method for composites, (2021) Multiscale and Multidisciplinary Modeling, Exp and Des, 4 (1).
3. Direct FE2 for simulating strain-rate dependent compressive failure of cylindrical CFRP, (2021) Comp Part C, 5, 100165.
4. Analysis of nonlinear shear and damage behaviour of angle-ply laminates with Direct FE2, (2021) CST, 216, 109050.
5. Transient multi-scale analysis with micro-inertia effects using Direct FE 2 method, (2021) Comp Mech, 67 (6).
6. Multiscale analysis of thermal problems in heterogeneous materials with Direct FE2 method, (2021) IJNME, 122 (24).
7. **Multiscale computational homogenisation of shear-flexible beam elements: a Direct FE2 approach, (2022) Comp Mech, 70 (5).**
8. Direct FE2 for concurrent multilevel modelling modeling of heterogeneous thin plate structures, (2022) CMAME, 392, 114658.
9. Direct FE2 modeling of heterogeneous materials with a micromorphic computational homogenization framework, (2022) CMAME, 393, 114837.

DFE^2 has been extended to shear deformable beam elements with examples on CFRP beams.



Future Research Topics / Questions

- Thermoplastic Composites:

  Competition between Mechanisms: Discrete Damage and Plasticity

- OHT Modeling:

  Is it possible to predict thick laminates behavior without hi-fidelity modeling?
  ANN?
  Physics-Informed Reduced-Order Modeling?
  Double-Double (DD) Laminates with Homogenization

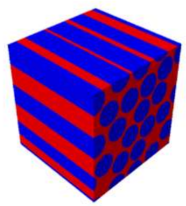
- Adaptive DNS-FE^2 modeling for Progressive Failure?

- Development of an Explicit Multi-Fidelity Model for Large Structures?

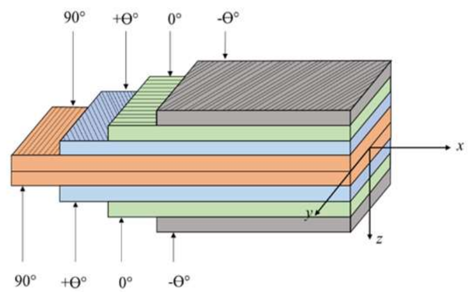
NUS National University of Singapore

Conclusion

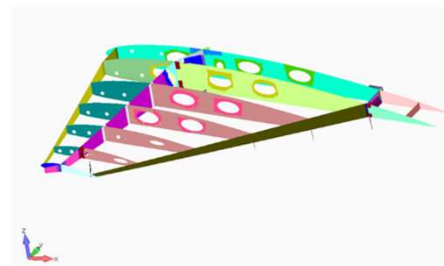
Multi-Fidelity Multi-Scale Modeling of Progressive Damage



Micro-scale



Meso-scale



Macro-scale

Concurrent Direct FE^2

Discrete Crack Methods

Smeared Crack Methods

Adaptivity — Coupled Models — Coupled Models

Towards Fundamental Properties ⟷ Towards Larger Structures

Thank You

Contact: **mpetayte@nus.edu.sg**