Development and Implementation of a Strain Rate-Dependent Constitutive Model for Fibre-reinforced Composites

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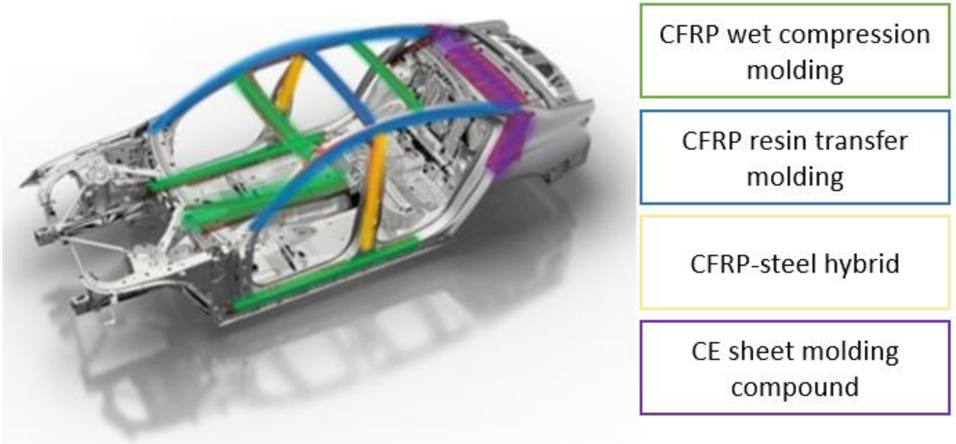
Motivation

- Fiber-reinforced plastic (FRP) composites are strong candidates for energy absorbing applications in vehicles.
- Vehicle crashworthiness is typically assessed through full-scale virtual structural analysis followed by crash testing.
- Computer-aided engineering (CAE) impact simulation models are developed to simulate a crash event and predict the energy absorption capabilities of a vehicle structure.
- In order to accurately predict the impact performance of composite structures, CAE simulation models must consider the material strain rate-dependent behaviour and possible process-induced defects.

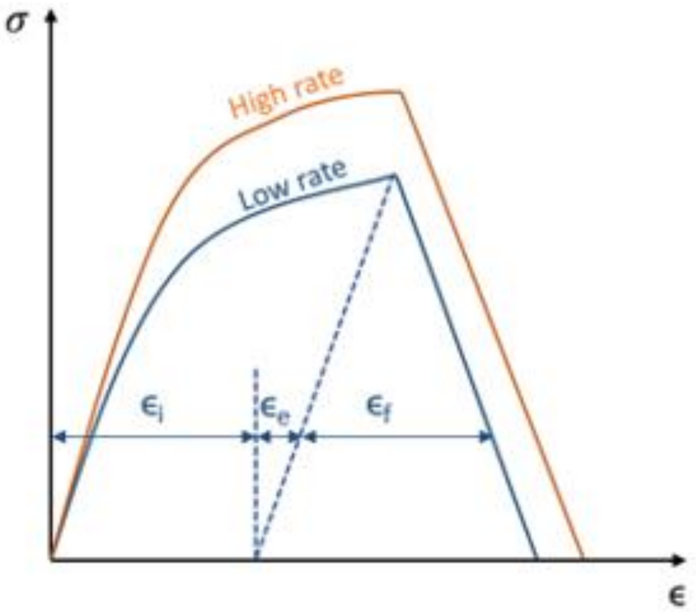


Body-in-white of BMW-7 Series highlighting parts made from FRPs. [1]

[1] G. Gardiner, "Is the BMW 7 Series the future of auto composites?," 7 10 2016. [Online]. Available https://www.compositesworld.com/articles/is-the-bmw-7-series-the-future-of-autocomposites. [Accessed 25 11 2020].

Motivation

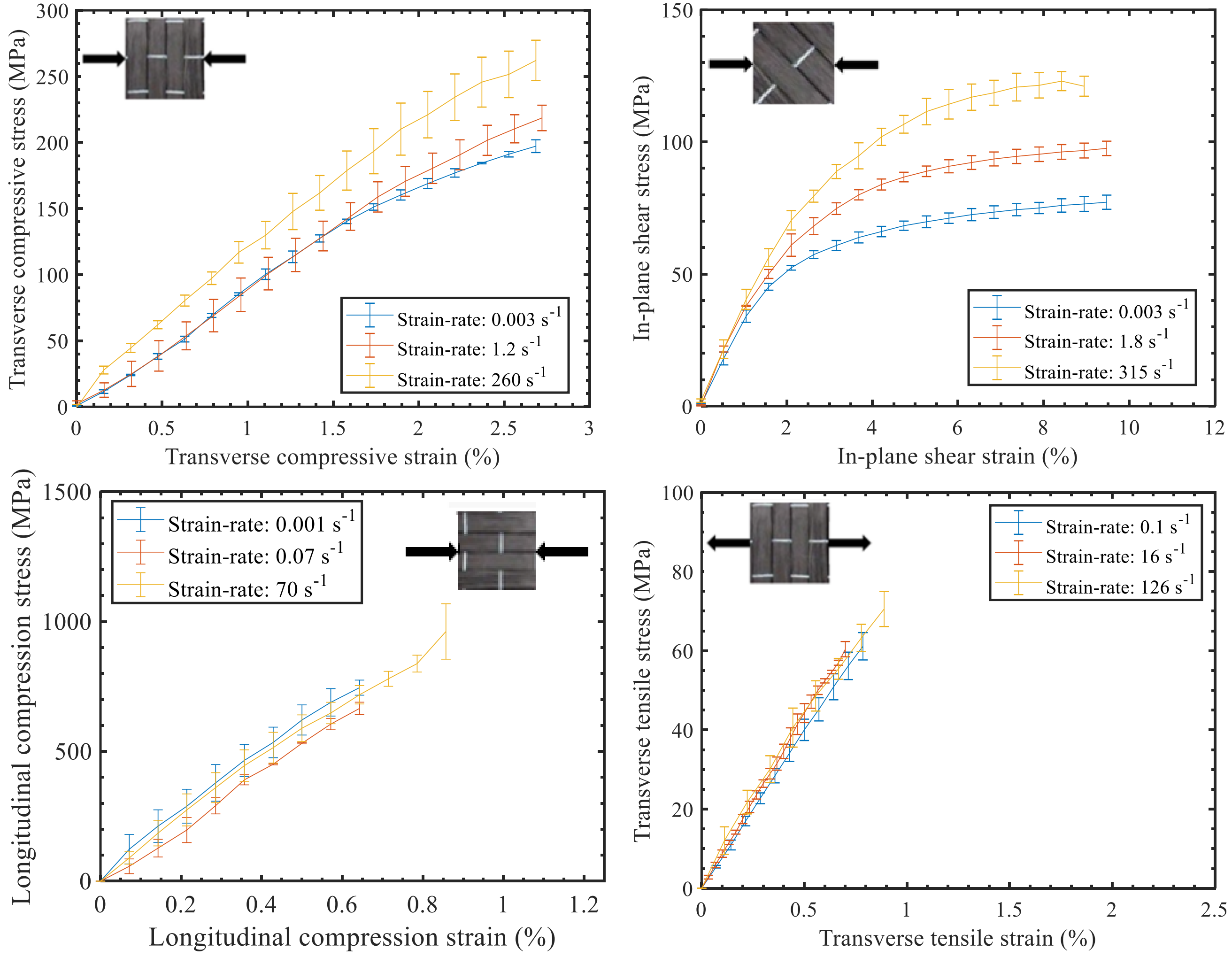
- Limitations with existing material models
  - Require calibration of non-physical parameters, e.g., *MAT_054 and *MAT_058 in LS-DYNA [2].
  - Ignore strain rate effects on all stages of deformation e.g., pre-peak, ultimate strength, and post-peak response [3], [4].
  - Most physics-based models were developed for 3D solid elements which is not suitable for full-scale vehicle simulations [5], [6].



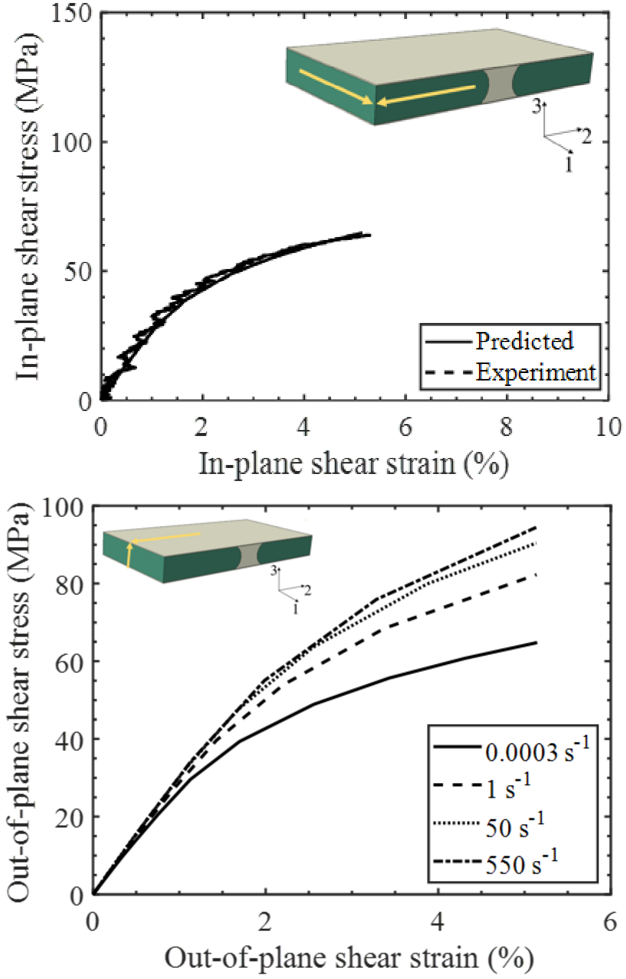
Generalized strain rate-dependent stress-strain response of a FRP lamina.

[2] Hallquist, J.O. (2006) LS-DYNA Theory Manual. Livermore Software Technology Corporation (LSTC), Livermore.
[3] Chang, F. K., & Chang, K. Y. (1987). A progressive damage model for laminated composites containing stress concentrations. *Journal of Composite Materials*, *21*(9), 834-855.
[4] Williams, K. V., Vaziri, R., & Poursartip, A. (2003). A physically based continuum damage mechanics model for thin laminated composite structures. *International Journal of Solids and Structures*, *40*(9), 2267-2300.
[5] Tan, W., & Liu, B. (2020). A physically-based constitutive model for the shear-dominated response and strain rate effect of carbon fibre reinforced composites. *Composites Part B: Engineering*, *193*, 108032.
[6] Vogler, M., Rolfes, R., & Camanho, P. P. (2013). Modeling the inelastic deformation and fracture of polymer composites–Part I: Plasticity model. *Mechanics of Materials*, *59*, 50-64.

Recent Research Contributions – UD-NCF Composite



[7] Rouf, K., Worswick, M., & Montesano, J. (In-preparation). Effect of strain rate on the in-plane mechanical response of a unidirectional non-crimp fabric carbon fiber/snap-cure epoxy composite.



[8] Rouf, K., Worswick, M. J., & Montesano, J. (2023). Experimentally verified dual-scale modelling framework for predicting the strain rate-dependent nonlinear anisotropic deformation response of unidirectional non-crimp fabric composites. *Composite Structures*, *303*, 116384.

Introduction

Objective

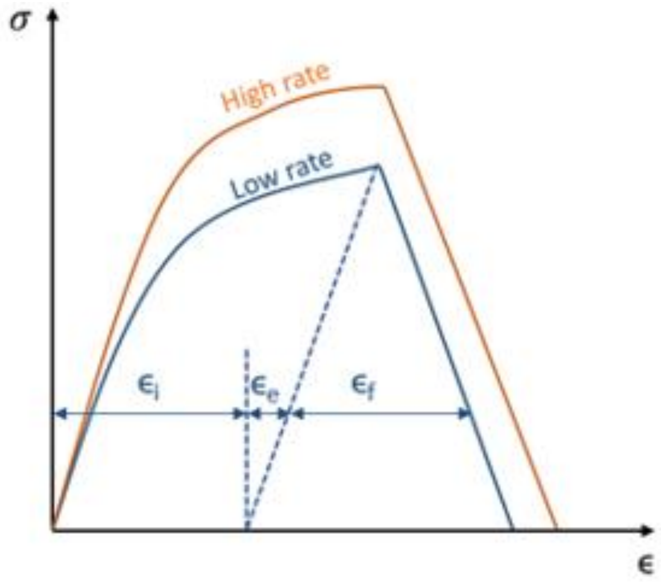
Develop a lamina-based material constitutive model that is suitable for shell elements, whose properties can be measured through physical or virtual experiments.

Features

- Elastic response
- Inelastic response
- Strain-rate dependency
- In-situ effects
- Failure and fracture response

Assumptions

- The additive split of elastic, inelastic and fracture strain.
- Pre-peak nonlinearity is captured using a plasticity model (causation is not considered).



Generalized strain rate-dependent stress-strain response of a FRP lamina.

Constitutive Model Development: Linear Elastic Response

- The theoretical formulation of the deformation model is based on the mathematical framework of invariant theory [6], [9].
- Model derived for plane-normal stress conditions to make it suitable for the shell elements.

Linear elastic formulation

Elastic-free energy density for a transversely isotropic material [6], [9].

$$\psi(\underline{\varepsilon},\underline{A}) = \frac{1}{2}\lambda\left(tr\underline{\varepsilon}\right)^2 + \mu_T tr\left(\underline{\varepsilon}\right)^2 + \alpha\left(\tilde{a}\underline{\varepsilon}\tilde{a}\right)tr\underline{\varepsilon} + 2(\mu_L - \mu_T)\left(\tilde{a}\underline{\varepsilon}^2\tilde{a}\right) + \frac{1}{2}\beta\left(\tilde{a}\underline{\varepsilon}\tilde{a}\right)^2$$

$$\mathrm{C} = \begin{bmatrix} \lambda + 2\alpha + \beta + 4\mu_L - 2\mu_T & \lambda + \alpha & \lambda + \alpha & 0 & 0 & 0 \\ \lambda + \alpha & \lambda + 2\mu_T & \lambda & 0 & 0 & 0 \\ \lambda + \alpha & \lambda & \lambda + 2\mu_T & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_L & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_L & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_T \end{bmatrix}$$

[6] Vogler, M., Rolfes, R., & Camanho, P. P. (2013). Modeling the inelastic deformation and fracture of polymer composites–Part I: plasticity model. Mechanics of Materials, 59, 50-64.
[9] Boehler, J. P. (1987). Applications of tensor functions in solid mechanics (Vol. 292). J. P. Boehler (Ed.). New York: Springer.

Constitutive Model Development: Inelastic Response

Yield function

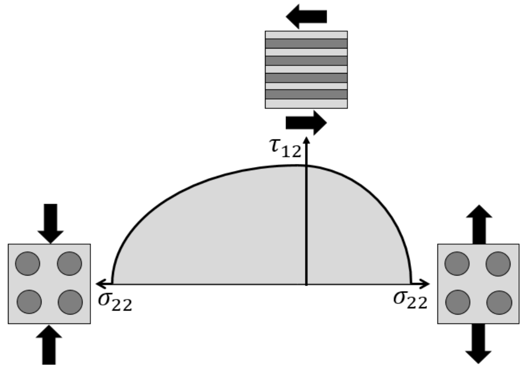
f$(\underline{\sigma}, \bar{\varepsilon}_p, A)$ = $\alpha_1 I_1 + \alpha_2 I_2 + \alpha_3 I_3$ - 1 ≤ 0

$\alpha_1 I_1$: For transverse shear yielding
$\alpha_2 I_2$: For longitudinal shear yielding
$\alpha_3 I_3$: For uniaxial transverse tension and compression yielding

$$\underline{\sigma} = \underline{\sigma}^{pind} + \underline{\sigma}^{reac}$$

$$\underline{\sigma}^{reac} = \frac{1}{2}(tr\underline{\sigma} - \tilde{a}\underline{\sigma}\tilde{a})\underline{I} - \frac{1}{2}(tr\underline{\sigma} - 3\tilde{a}\underline{\sigma}\tilde{a})\underline{A}$$

$$\underline{\sigma}^{pind} = \underline{\sigma} - \frac{1}{2}(tr\underline{\sigma} - \tilde{a}\underline{\sigma}\tilde{a})\underline{I} - \frac{1}{2}(tr\underline{\sigma} - 3\tilde{a}\underline{\sigma}\tilde{a})\underline{A}$$



Schematic representation of the yield surface in σ_{22}- τ_{12} space

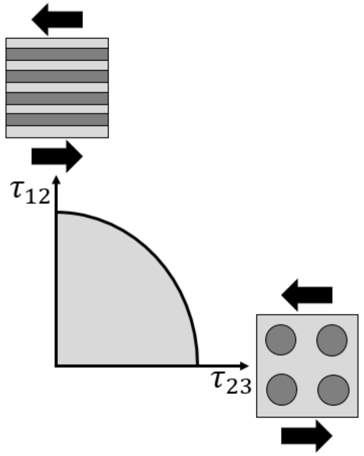
Plastic potential function (For non-associated flow rule)

g$(\underline{\sigma}, \underline{A})$ = $\beta_1 I_1 + \beta_2 I_2$ - 1 ≤ 0

Plastic potential parameters are based on the shear plastic strains [10].

Equivalent plastic strain

$$\bar{\varepsilon}_p = \sqrt{\frac{1}{2}(\underline{\varepsilon}_p : \underline{\varepsilon}_p)}$$



Schematic representation of the yield surface in τ_{23}- τ_{12} space

[10] Dean, A., Safdar, N., & Rolfes, R. (2019). A co-rotational based anisotropic elasto–plastic model for geometrically non-linear analysis of fibre reinforced polymer composites: formulation and finite Element implementation. *Materials*, *12*(11), 1816.

Constitutive Model Development: Strain Rate Dependency

Logarithmic strain rate function for transverse compression modulus

$$S_{T_E}(\dot{\varepsilon}) = \left(1 + C_{T_E} \ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ref}}\right)\right),\ E_{22}(\dot{\varepsilon}) = S_{T_E}(\dot{\varepsilon}) * E_{22}$$

Logarithmic strain rate function for shear modulus

$$S_{S_E}(\dot{\varepsilon}) = \left(1 + C_{S_E} \ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ref}}\right)\right),\ G_{12}(\dot{\varepsilon}) = S_{S_E}(\dot{\varepsilon}) * G_{12}$$

Logarithmic strain rate function for transverse compression yield stress

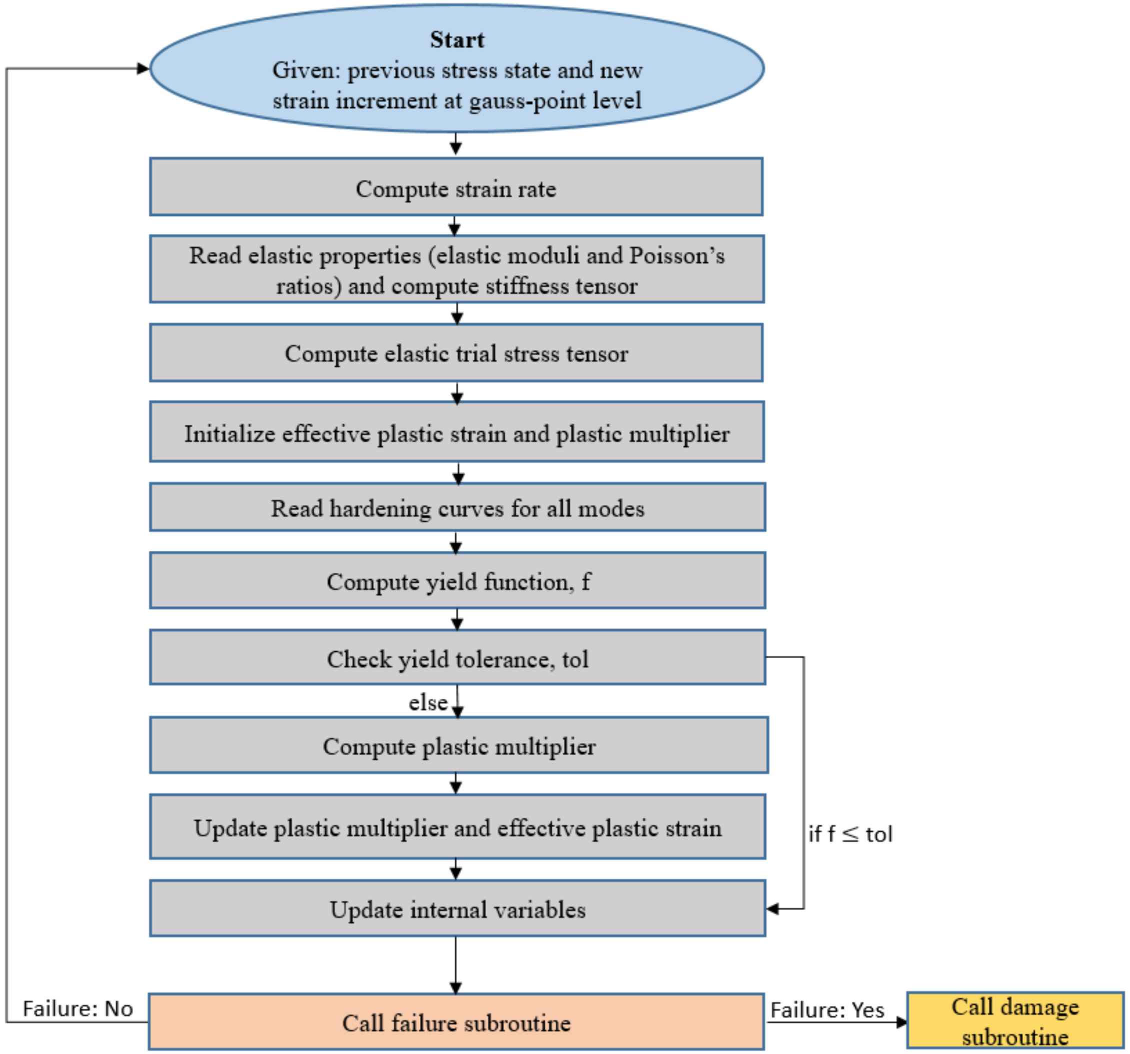
$$S_{T_Y}(\dot{\varepsilon}) = \left(1 + C_{T_Y} \ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ref}}\right)\right),\ \sigma_{22y}(\dot{\varepsilon}) = S_{T_Y}(\dot{\varepsilon}) * \sigma_{22y}$$

Logarithmic strain rate function for shear yield stress

$$S_{S_Y}(\dot{\varepsilon}) = \left(1 + C_{S_Y} \ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ref}}\right)\right),\ \sigma_{12y}(\dot{\varepsilon}) = S_{S_Y}(\dot{\varepsilon}) * \sigma_{12y}$$

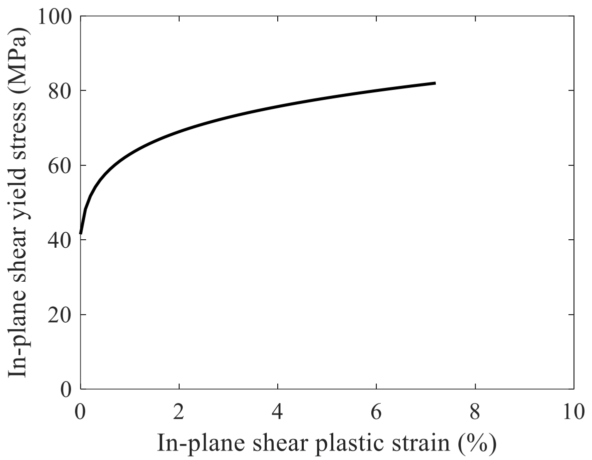
Constitutive Model Development: Implementation

- The model was derived numerically by the radial return mapping algorithm.
- Written in Fortran and Implemented in LS-DYNA as a user-defined material model (*MAT_43).

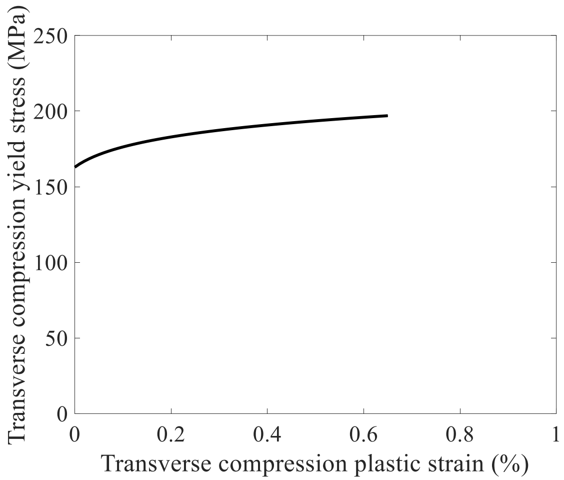


Calibration: Inputs for UD-NCF Carbon Fiber/Epoxy Composite

Parameter type	Parameter	Values
Elastic parameters	E_{11} [7]	120 GPa
	E_{22} [7]	8.6 GPa
	υ_{12} [11]	0.37
	G_{12} [7]	3.4 GPa
	G_{23}	3.2 GPa
Plastic potential parameter	$\beta_2\,(\frac{\varepsilon_{12p}}{\varepsilon_{23p}})$ [7]	1
Strain rate parameters	$\dot{\varepsilon}_{ref}$	0.000003 ms^{-1}
	$C_{E_{22}}$ [7]	0.04
	$C_{G_{12/23}}$ [7]	0.02
	C_{T_Y} [7]	0.10
	C_{S_Y} [7]	0.11
Others	Density [11]	0.00125 $g.mm^{-3}$



Hardening curve for the in-plane shear mode

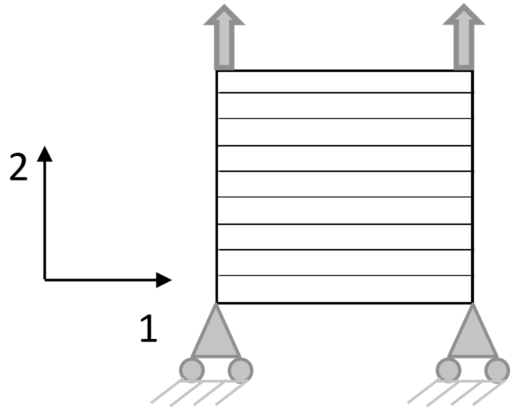


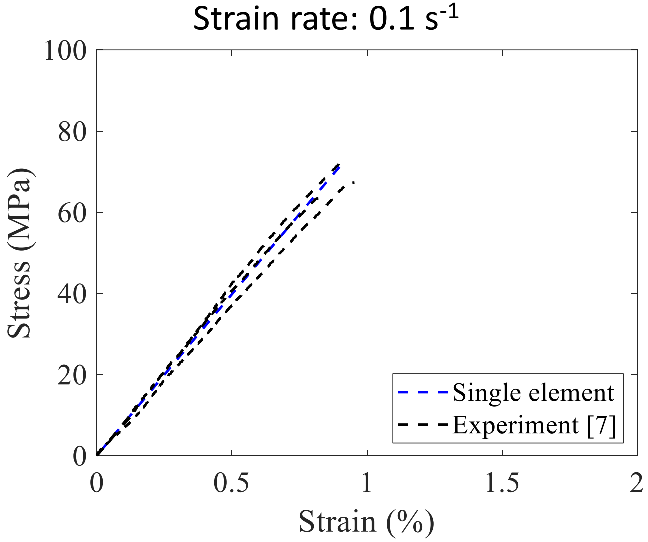
Hardening curve for transverse compression mode

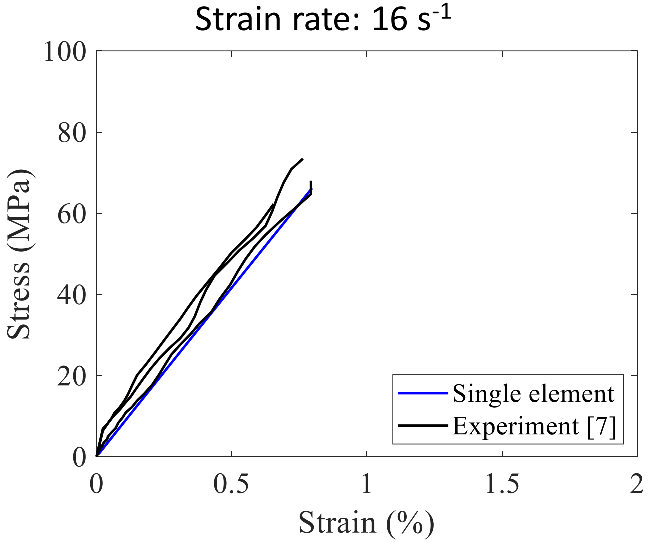
The hardening behavior for the transverse shear is scaled by a factor of 0.95 from the in-plane shear response.

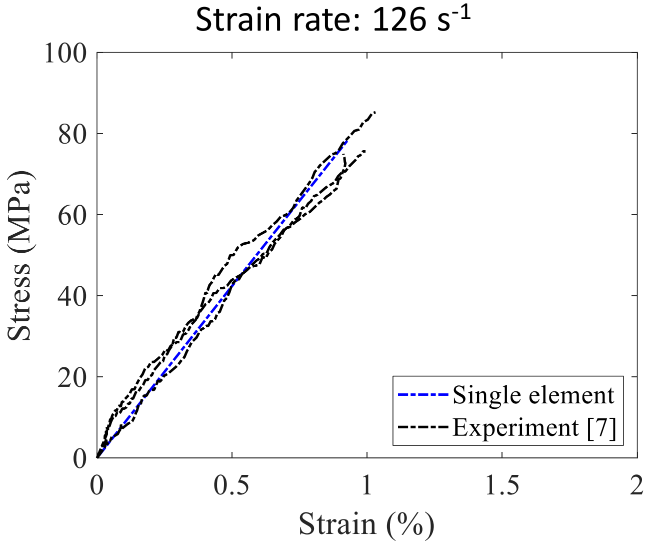
[7] Rouf, K., Worswick, M., & Montesano, J. (In-preparation). Effect of strain rate on the in-plane mechanical response of a unidirectional non-crimp fabric carbon fiber/snap-cure epoxy composite.
[11] Suratkar, A. P. (2022). Damage in non-crimp fabric carbon fiber reinforced epoxy composites under various mechanical loading conditions. PhD Dissertation, Western University.

Verification – Transverse Tension: UD-NCF Composite



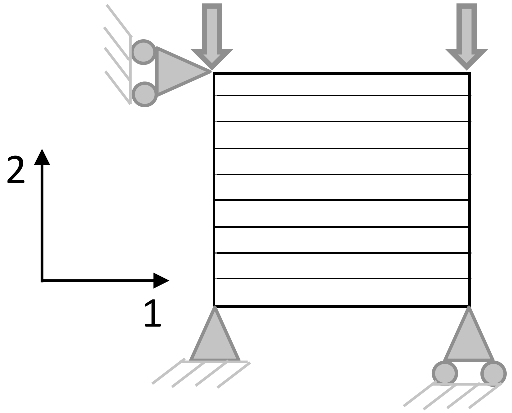


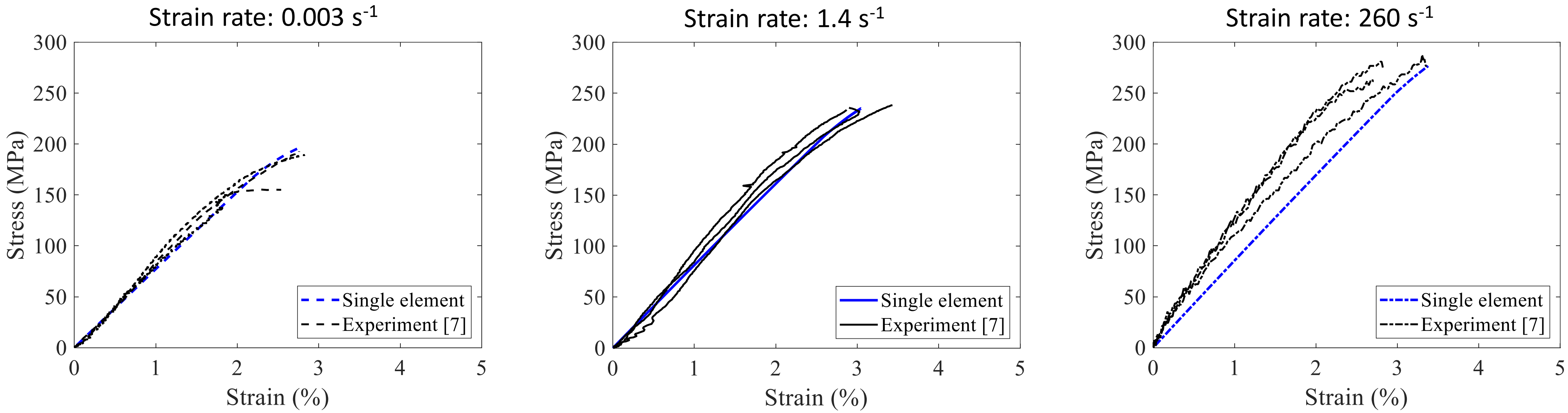




[7] Rouf, K., Worswick, M., & Montesano, J. (In-preparation). Effect of strain rate on the in-plane mechanical response of a unidirectional non-crimp fabric carbon fiber/snap-cure epoxy composite.

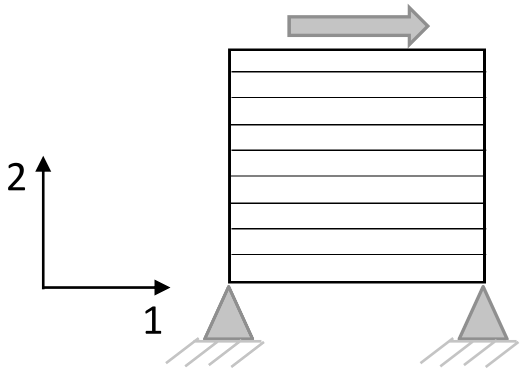
Verification – Transverse Compression: UD-NCF Composite

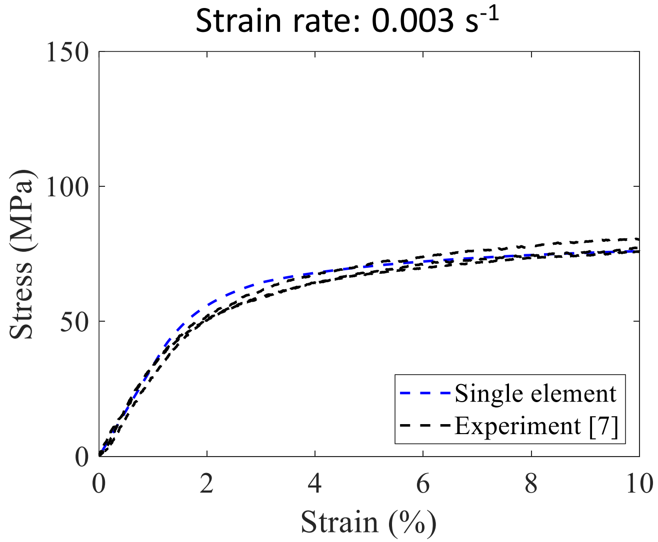


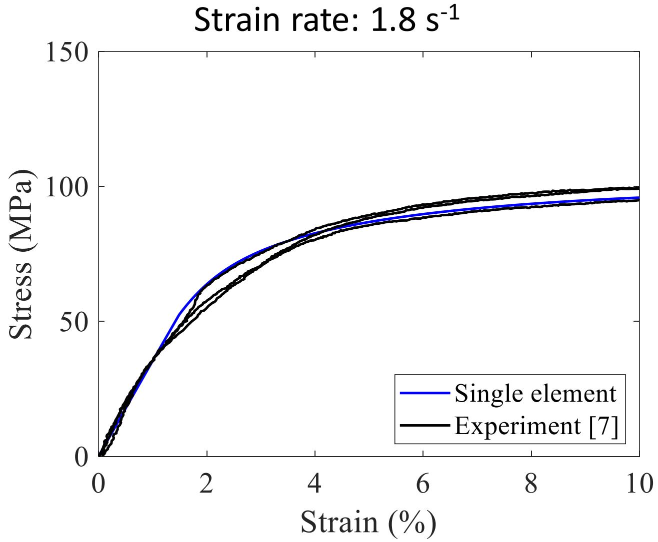


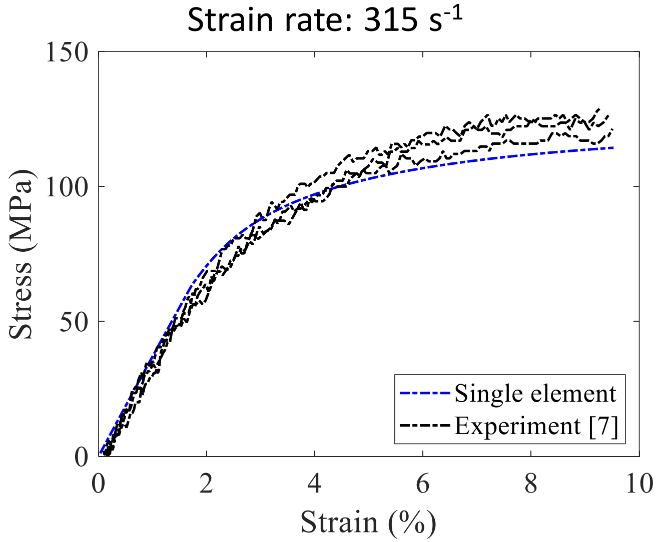
[7] Rouf, K., Worswick, M., & Montesano, J. (In-preparation). Effect of strain rate on the in-plane mechanical response of a unidirectional non-crimp fabric carbon fiber/snap-cure epoxy composite.

Verification – In-plane Shear: UD-NCF Composite



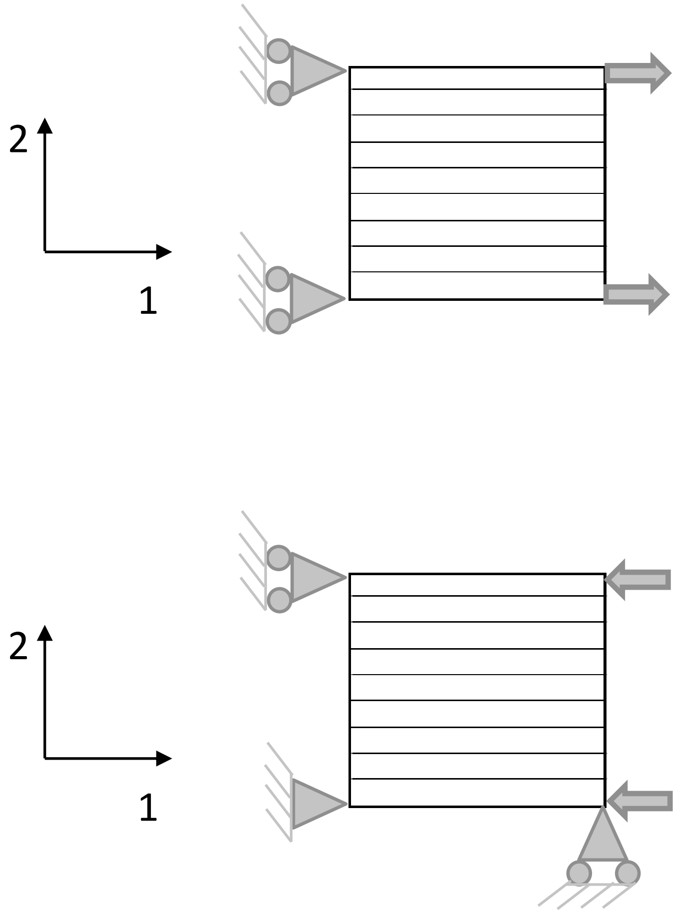


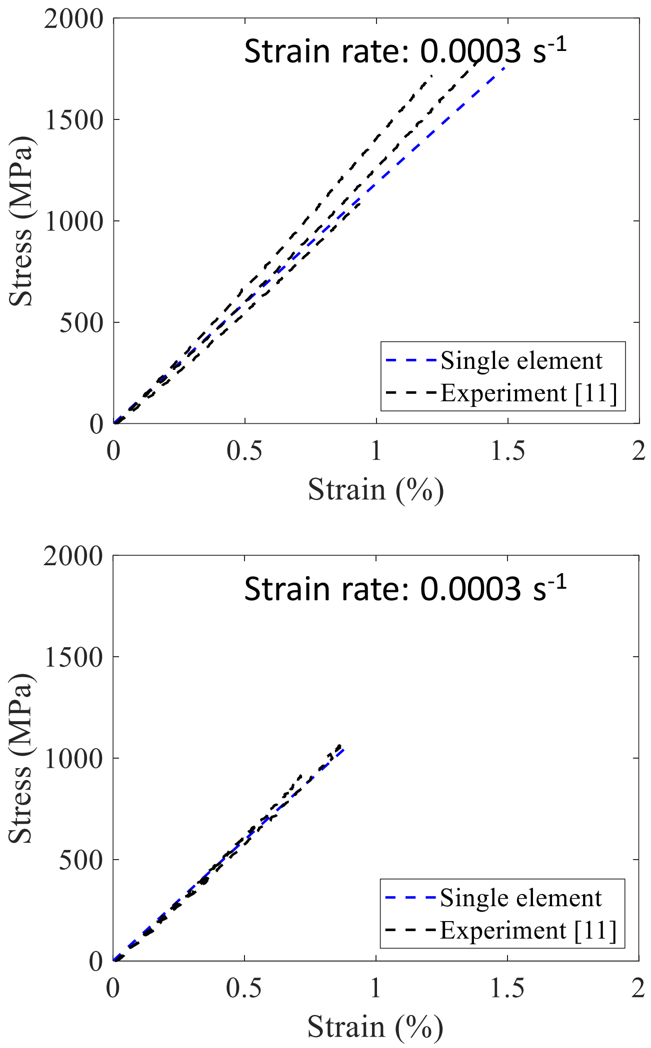




[7] Rouf, K., Worswick, M., & Montesano, J. (In-preparation). Effect of strain rate on the in-plane mechanical response of a unidirectional non-crimp fabric carbon fiber/snap-cure epoxy composite.

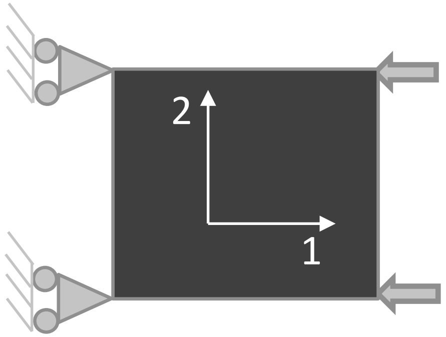
Verification – Longitudinal Tension & Compression: UD-NCF Composite



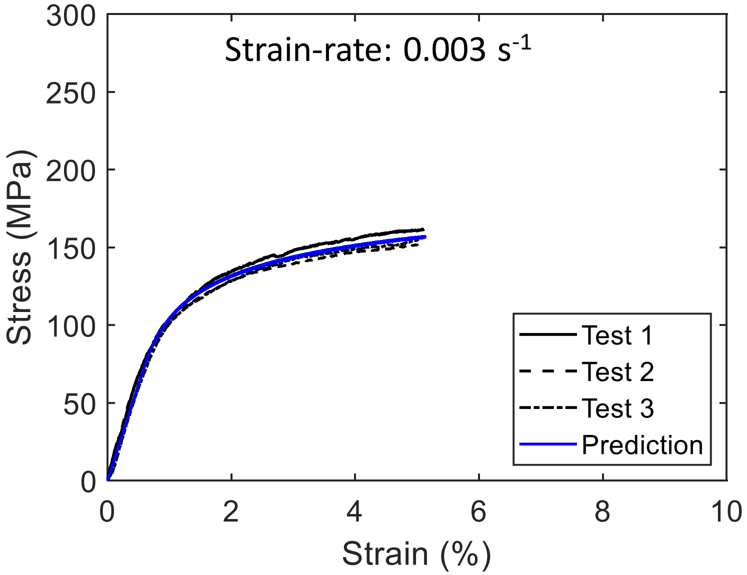


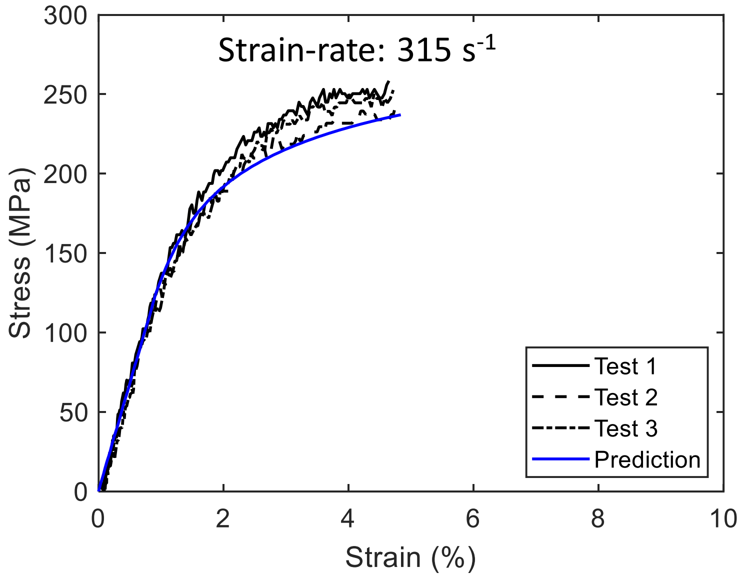
[11] Suratkar, A. P. (2022). Damage in non-crimp fabric carbon fiber reinforced epoxy composites under various mechanical loading conditions. PhD Dissertation, Western University.

Validation: UD-NCF Composite

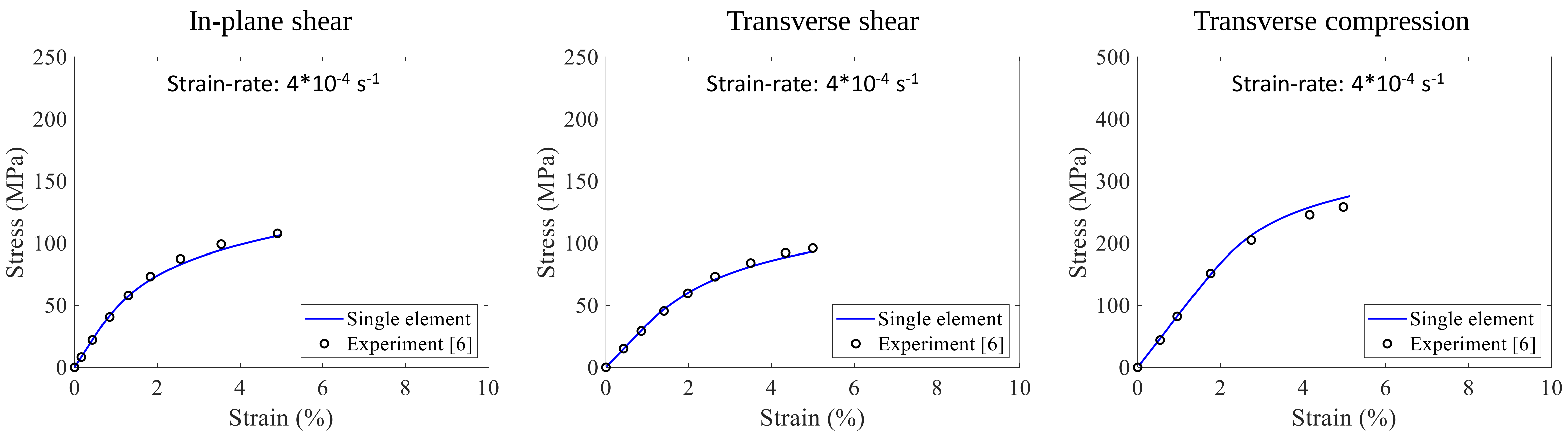


Stacking Sequence $[\pm45^o]_{2s}$



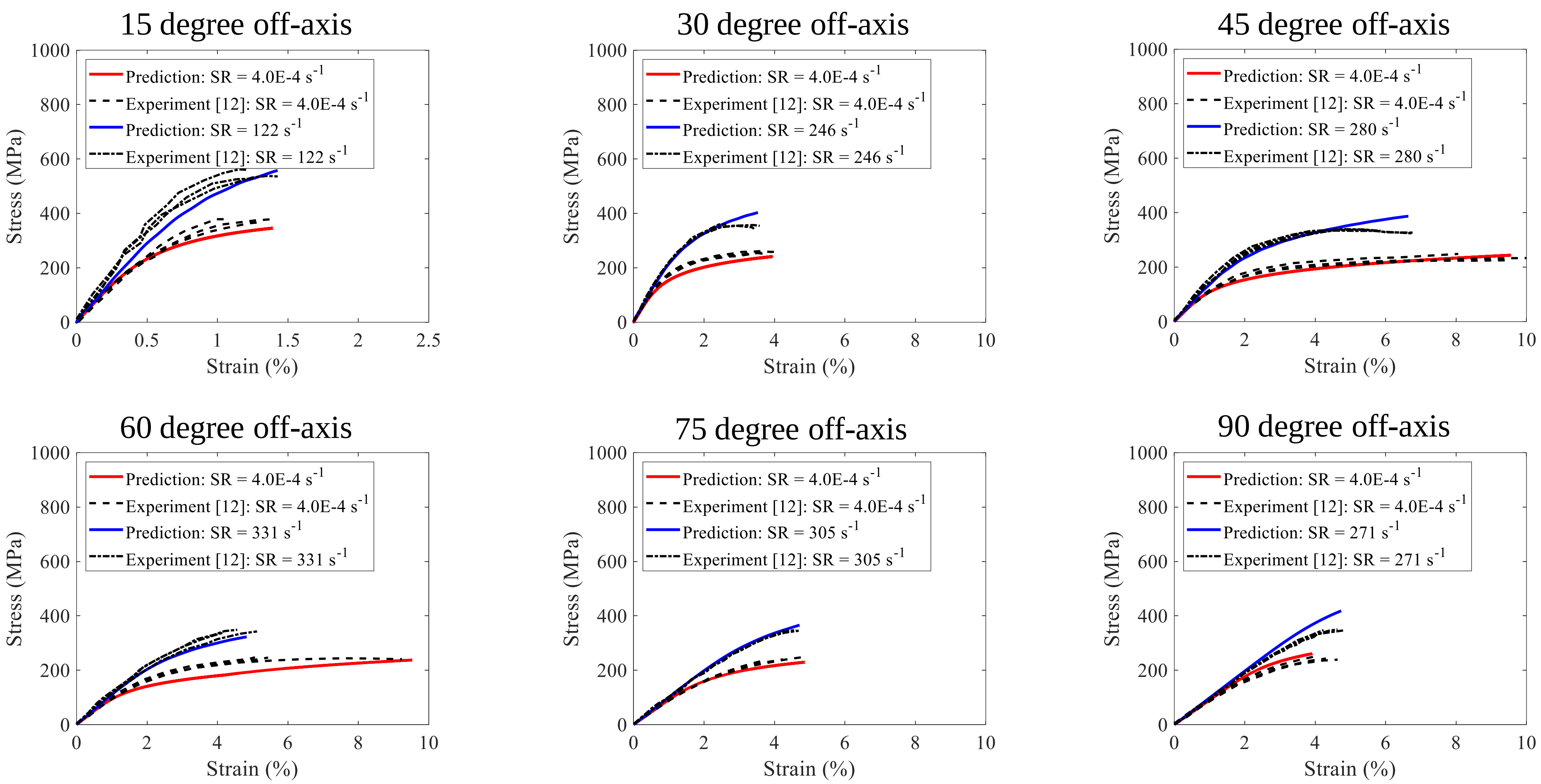


Verification: IM7-8552 UD Composite



[6] Vogler, M., Rolfes, R., & Camanho, P. P. (2013). Modeling the inelastic deformation and fracture of polymer composites–Part I: plasticity model. Mechanics of Materials, 59, 50-64.

Validation: IM7-8552 UD Composite



[12] Koerber, H., Kuhn, P., Ploeckl, M., Otero, F., Gerbaud, P. W., Rolfes, R., & Camanho, P. P. (2018). Experimental characterization and constitutive modeling of the non-linear stress–strain behavior of unidirectional carbon–epoxy under high strain rate loading. *Advanced Modeling and Simulation in Engineering Sciences*, *5*(1), 1-24.

Conclusions and Next Steps

- **Conclusions**
  - The developed material model accurately captured the strain rate-dependent elastic and inelastic response for a UD-NCF composite lamina and has been validated for an angle-ply laminate.
  - The material model also accurately captured the response of a UD lamina; however, for off-axis loading there are minor discrepancies at higher applied strains.

- **Next Steps**
  - Develop and implement rate-dependent failure initiation criteria in the material model.
  - Develop and implement a damage model for predicting the rate-dependent post-peak response.
  - Perform characterization experiments to capture the post-peak response at different strain rates.
  - Perform validation tests at the component level for different strain rates.

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Thanks for your attention