

Micromechanics of intra-laminar hybrid lamina with hollow fibres: a RVE model

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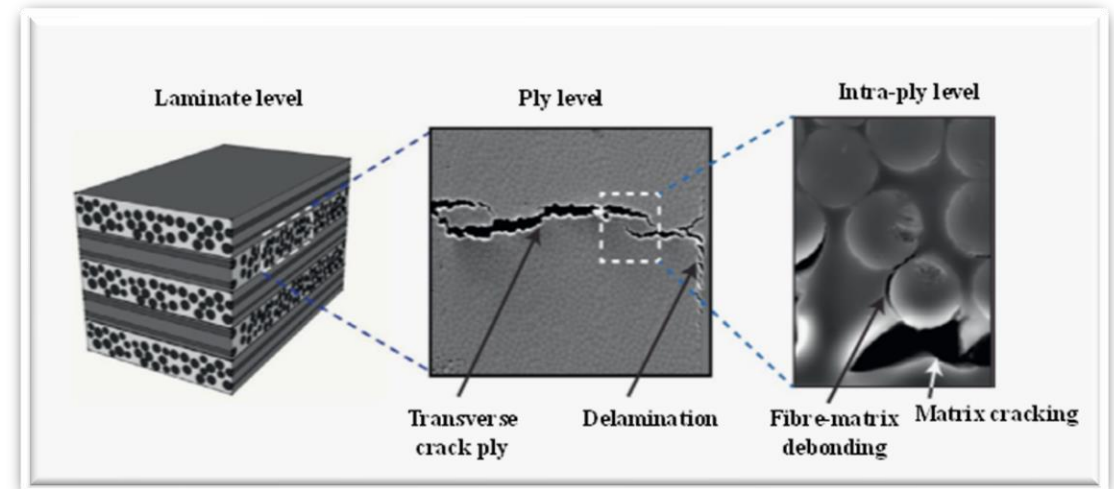
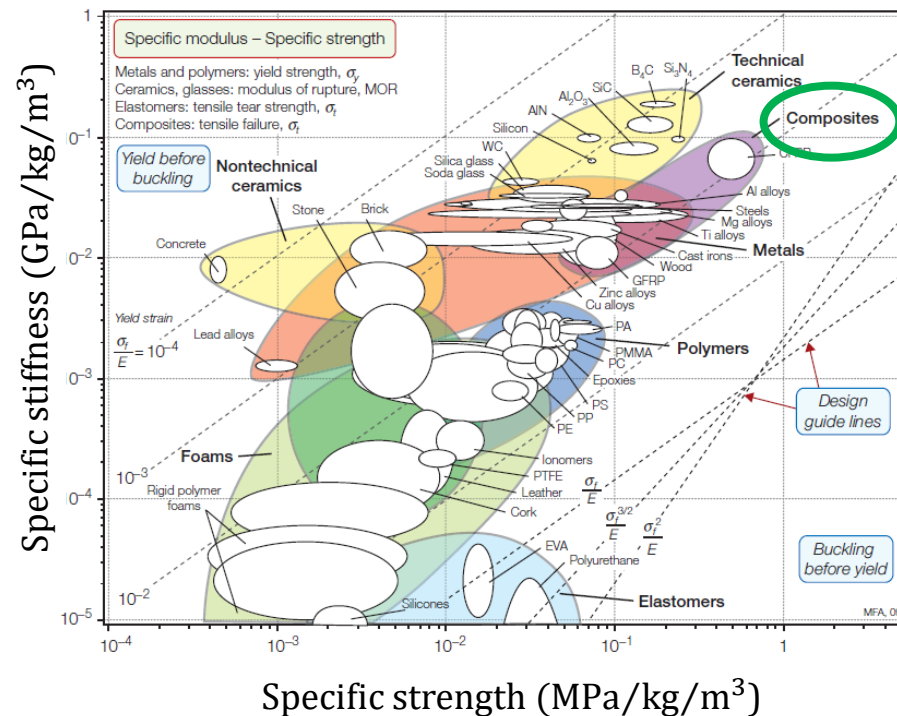
Thermosetting composites advantages and disadvantages

Advantages:
Specific strength
Specific stiffness

Disadvantages:
Poor damage tolerance
Low toughness



Barely visible external damage can lead to significant delamination



How to balance these properties?



Hybridization in composites: an overview

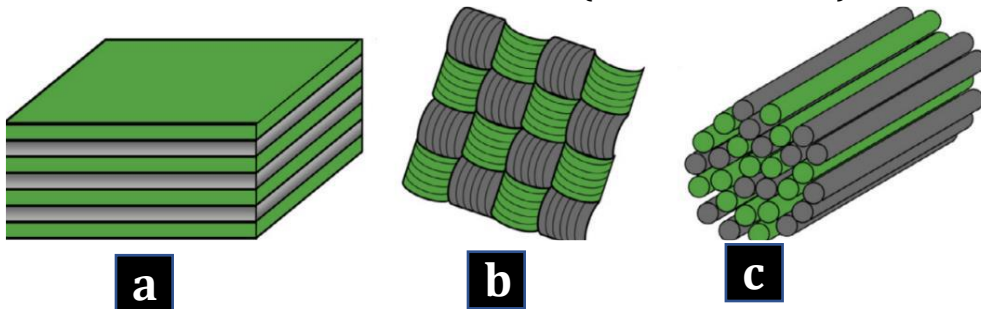
Hybrid Composites:

- Fibre hybridization (i.e. two or more fibre types)
- Matrix hybridization (i.e. two or more matrix types)

Hybridisation scale:

- Inter-layer **a**
- Intra-layer **b**
- Intra-yarn **c**

(Swolfs et al., 2014)



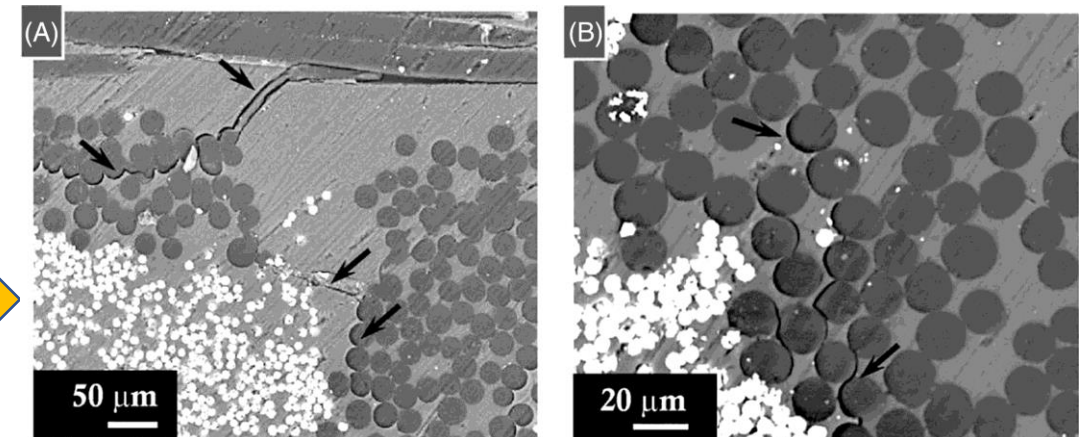
Balance in-plane and out-of-plane properties

Cost-effective solution to improve damage tolerance



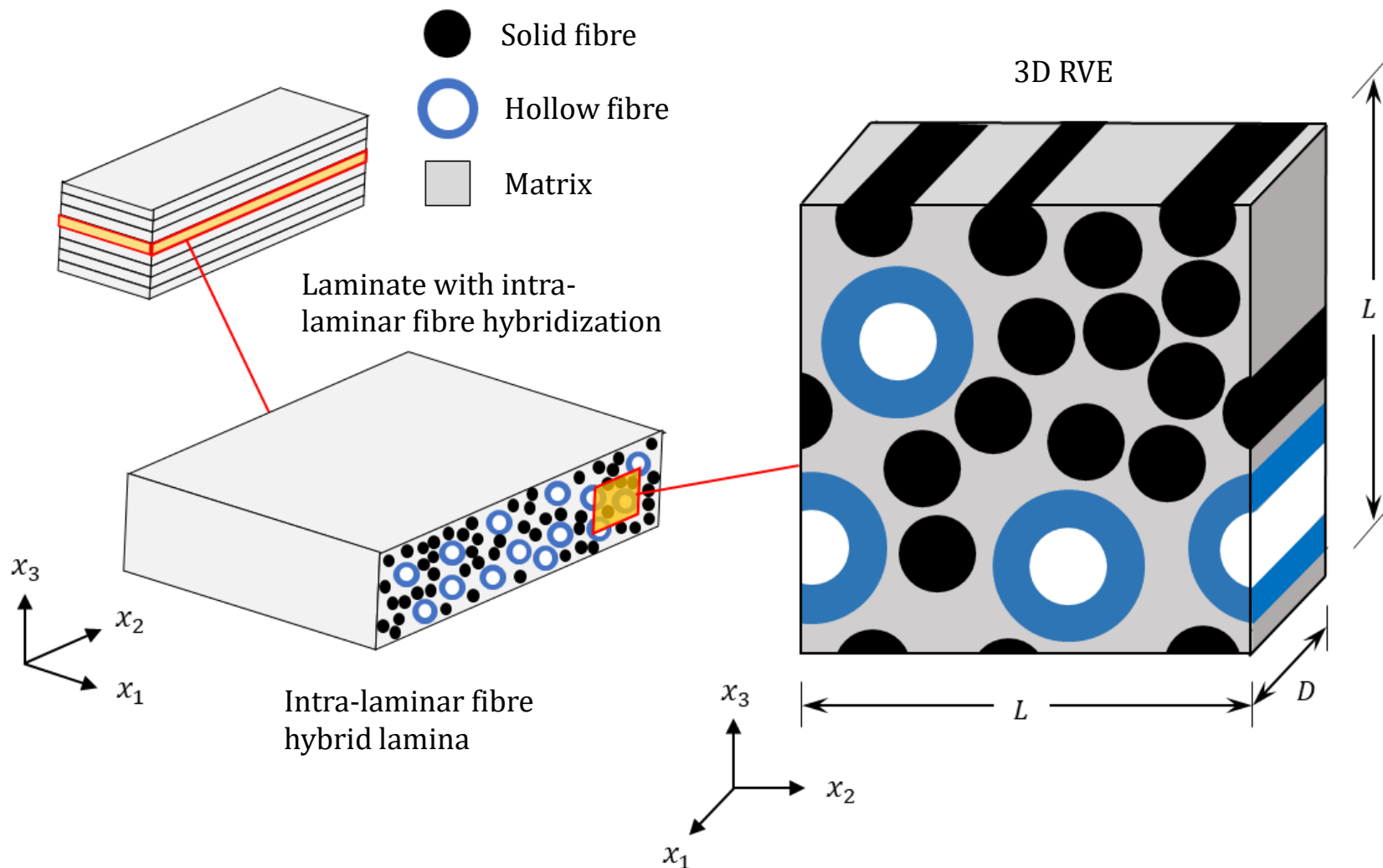
S-glass/PP/epoxy

(Dalfi et al., 2019)



Conductive failure modes to improve damage tolerance

Solid/Hollow fibres hybridization: an opportunity

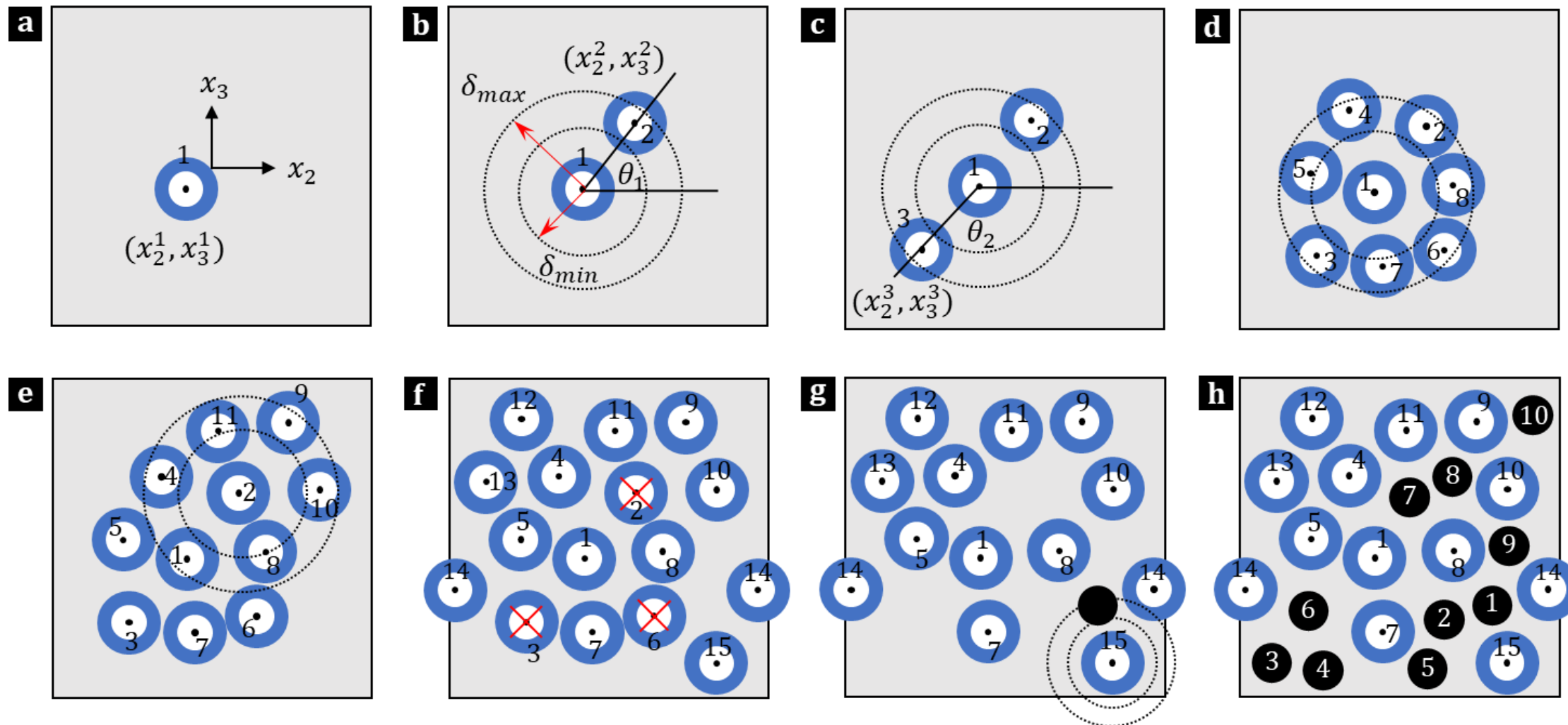


✓ Hollow fibre can be used as a channel to inject resin and repair the damaged structure

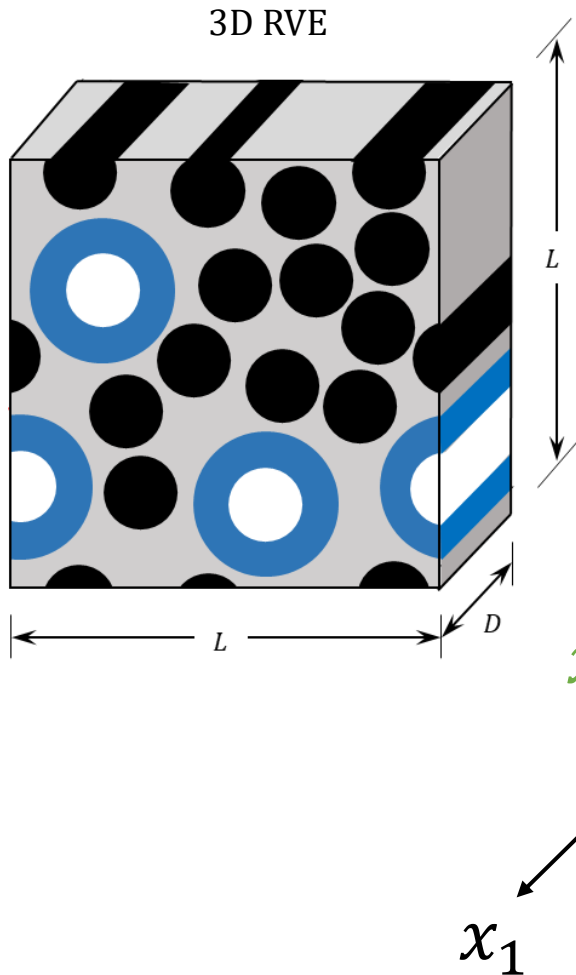
✓ Alter the specific elastic properties and micro stress fields compared to the non-hollow hybrid RVE

Modified RSE algorithm

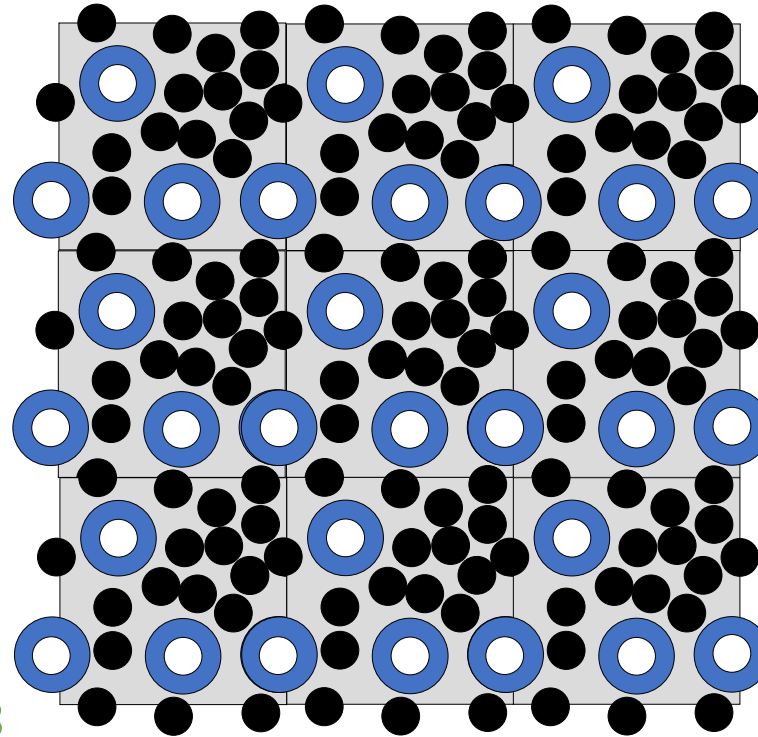
Matrix Hollow fibre Solid fibre



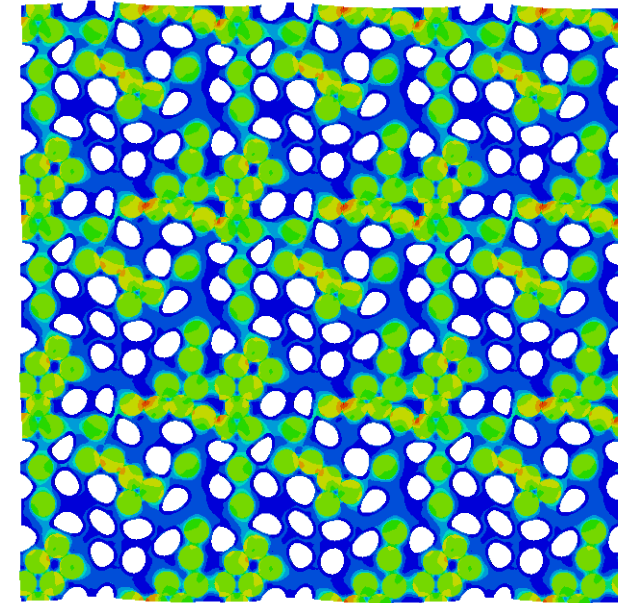
Periodic Boundary Conditions



Periodic microstructure

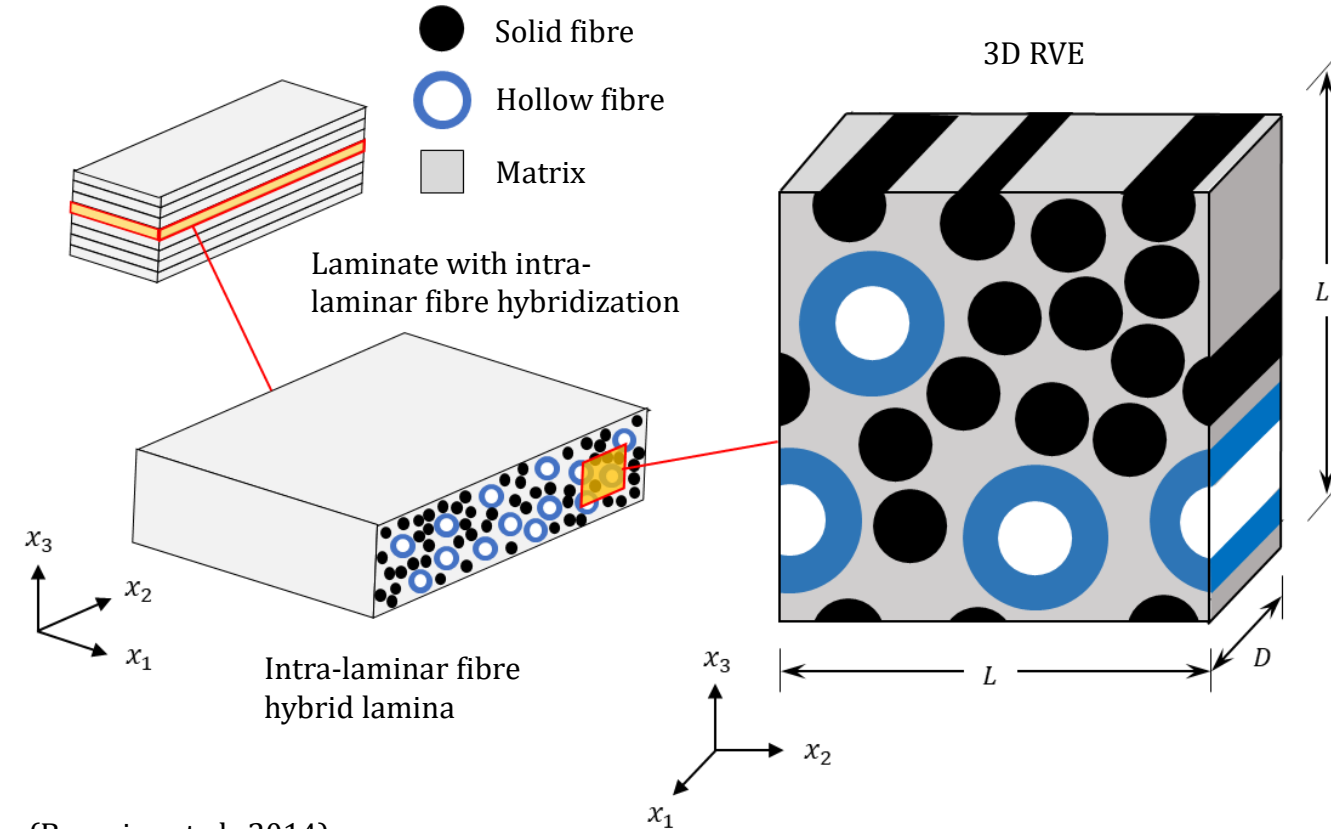


Periodic displacement



$$\begin{aligned}\vec{u}(0, x_2, x_3) - \vec{u}(D, x_2, x_3) &= \vec{U}_1 \\ \vec{u}(x_1, 0, x_3) - \vec{u}(x_1, L, x_3) &= \vec{U}_2 \\ \vec{u}(x_1, x_2, 0) - \vec{u}(x_1, x_2, L) &= \vec{U}_3\end{aligned}$$

Volume average homogenization



$$\hat{\sigma}_{ij} = \frac{1}{V} \int_V \sigma_{ij} dV$$

$$\hat{\epsilon}_{ij} = \frac{1}{V} \int_V \epsilon_{ij} dV$$

$$V = L * L * D$$

Lamina constants

$$\begin{aligned} \hat{E}_{11} \\ \hat{E}_{22} &= \hat{E}_{33} \\ \hat{G}_{12} &= \hat{G}_{13} \\ \hat{G}_{23} \\ \hat{\nu}_{12} &= \hat{\nu}_{13} \\ \hat{\nu}_{23} \end{aligned}$$

- ✓ $\sigma_{ij}, \epsilon_{ij}$ are the micro stresses and micro strains
- ✓ $\hat{\sigma}_{ij}, \hat{\epsilon}_{ij}$ are the macro stresses and macro strains

Assumptions:

- ✓ Linear elastic behaviour of the constituent materials
- ✓ Isotropic or transversely isotropic fibres
- ✓ Isotropic matrix
- ✓ No micro voids
- ✓ Perfect fibre-matrix interface bonding
- ✓ No yielding or failure

(Banerjee et al., 2014)

	E_{11} [GPa]	$E_{22} = E_{33}$ [GPa]	$G_{12} = G_{13}$ [GPa]	G_{23} [GPa]	$\nu_{12} = \nu_{13}$	ν_{23}	ρ [g/cm ³]
E-glass	72.4	72.4	30.2	30.2	0.2	0.2	2.54
Carbon	263	19	27.6	7.04	0.2	0.35	1.78
Epoxy	3.5	3.5	1.29	1.29	0.35	0.35	1.29

Validation of the RVE model

(Banerjee et al., 2014)

Rule of mixture

$$\hat{E}_{11} = E_{11c} V_{fc} + E_{11g} V_{fg} + E_m V_m$$

$$\hat{\nu}_{12} = \nu_{12c} V_{fc} + \nu_{12g} V_{fg} + \nu_m V_m$$

Modified Halpin-Tsai

$$\frac{\hat{E}}{E_m} = \frac{1 + \xi(\eta_c V_{fc} + \eta_g V_{fg})}{1 - (\eta_c V_{fc} + \eta_g V_{fg})}$$

$$\frac{\hat{G}}{G_m} = \frac{1 + \xi(\eta_c V_{fc} + \eta_g V_{fg})}{1 - (\eta_c V_{fc} + \eta_g V_{fg})}$$

ξ (Fibre packing, material combination)
 η (Loading, ξ)

$$\begin{aligned}\hat{E} &= \hat{E}_{22} = \hat{E}_{33} = \hat{E}_T \\ \hat{G} &= \hat{G}_{12} = \hat{G}_{13} = \hat{G}_{LT} \\ \hat{G} &= \hat{G}_{23} = \hat{G}_T\end{aligned}$$

Transverse Isotropy

$$\hat{\nu}_{23} = \hat{\nu}_T = \frac{\hat{E}_T}{2\hat{G}_T} - 1$$

Variations are due to the packing system and element strategy used in the reference-> ξ chosen

The comparison of the homogenised lamina properties of **carbon/epoxy** lamina (with $V_{fc} \approx 0.60$) using the RVE and analytical models

	$\hat{E}_{11}$ [GPa]	$\hat{E}_{22} = \hat{E}_{33}$ [GPa]	$\hat{G}_{12} = \hat{G}_{13}$ [GPa]	$\hat{G}_{23}$ [GPa]	$\hat{\nu}_{12} = \hat{\nu}_{13}$	$\hat{\nu}_{23}$
RVE	158.63 $\pm$ 0.00	8.95 $\pm$ 0.03	5.18 $\pm$ 0.05	3.15 $\pm$ 0.01	0.25 $\pm$ 0.00	0.44 $\pm$ 0.00
Analytical	159.20	8.61	4.41	3.06	0.26	0.41
Variation (%)	0.36	3.95	17.46	2.94	3.85	8.14

The comparison of the homogenised lamina properties of **E-glass/epoxy** lamina (with $V_{fg} \approx 0.60$) using the RVE and analytical models

	$\hat{E}_{11}$ [GPa]	$\hat{E}_{22} = \hat{E}_{33}$ [GPa]	$\hat{G}_{12} = \hat{G}_{13}$ [GPa]	$\hat{G}_{23}$ [GPa]	$\hat{\nu}_{12} = \hat{\nu}_{13}$	$\hat{\nu}_{23}$
RVE	44.82 $\pm$ 0.00	13.98 $\pm$ 0.31	5.53 $\pm$ 0.11	5.13 $\pm$ 0.04	0.25 $\pm$ 0.01	0.38 $\pm$ 0.01
Analytical	44.84	12.21	4.47	4.32	0.25	0.40
Variation (%)	0.04	14.50	23.71	18.75	0.00	8.03

The comparison of the homogenised lamina properties of **carbon/solid-E-glass/epoxy lamina** (with $V_{fc} \approx 0.15$ and $V_{fg} \approx 0.45$) using the RVE and analytical models

	$\hat{E}_{11}$ [GPa]	$\hat{E}_{22} = \hat{E}_{33}$ [GPa]	$\hat{G}_{12} = \hat{G}_{13}$ [GPa]	$\hat{G}_{23}$ [GPa]	$\hat{\nu}_{12} = \hat{\nu}_{13}$	$\hat{\nu}_{23}$
RVE	74.36 $\pm$ 0.00	12.34 $\pm$ 0.16	5.33 $\pm$ 0.08	4.40 $\pm$ 0.02	0.25 $\pm$ 0.00	0.40 $\pm$ 0.00
Analytical	73.43	11.27	4.45	4.31	0.26	0.31
Variation (%)	1.27	9.49	19.78	2.09	3.85	30.11

Specific elastic properties

The comparison of the **homogenised properties** of carbon/epoxy (L-1), E-glass/epoxy (L-2), carbon/solid-E-glass/epoxy (L-3) and carbon/hollow-E-glass/epoxy (L-4) laminae.

Lamina	$\hat{E}_{11}$ [GPa]	$\hat{E}_{22} = \hat{E}_{33}$ [GPa]	$\hat{G}_{12} = \hat{G}_{13}$ [GPa]	$\hat{G}_{23}$ [GPa]	$\hat{\nu}_{12} = \hat{\nu}_{13}$	$\hat{\nu}_{23}$	$\hat{\rho}$ [g/cm ³]
L-1	158.63 ± 0.00	8.95 ± 0.03	5.18 ± 0.05	3.15 ± 0.01	0.25 ± 0.00	0.44 ± 0.00	1.58
L-2	44.82 ± 0.00	13.98 ± 0.31	5.53 ± 0.11	5.13 ± 0.04	0.25 ± 0.01	0.38 ± 0.01	2.04
L-3	74.36 ± 0.00	12.34 ± 0.16	5.33 ± 0.08	4.40 ± 0.02	0.25 ± 0.00	0.40 ± 0.00	1.93
L-4	67.81 ± 0.00	10.60 ± 0.04	4.98 ± 0.04	3.67 ± 0.00	0.25 ± 0.00	0.44 ± 0.00	1.70

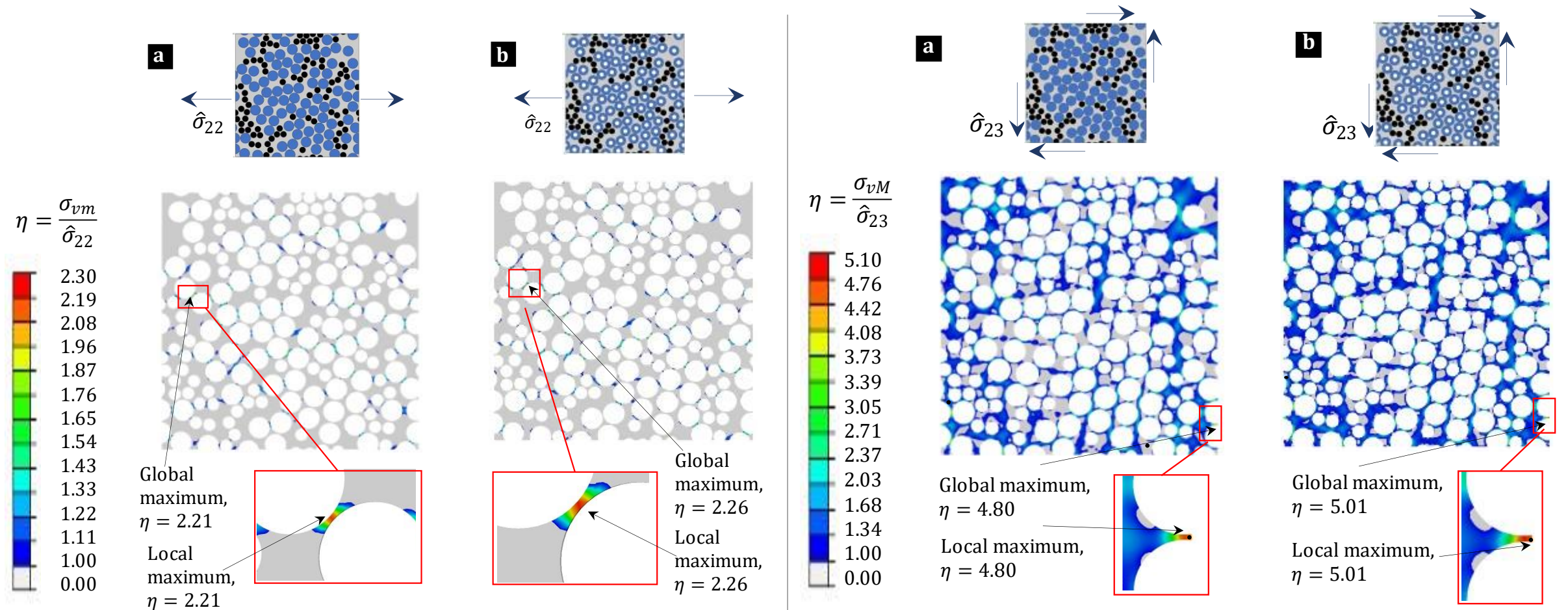
$V_{fC} \approx 0.15$
 $V_{fE} = V_{fH} \approx 0.45$
 $V_{fH-net} \approx 0.36$
 20% hollowness

The comparison of the **specific homogenised properties** of carbon/epoxy (L-1), E-glass/epoxy (L-2), carbon/solid-E-glass/epoxy (L-3) and carbon/hollow-E-glass/epoxy (L-4) laminae

Lamina	$\hat{E}_{11}/\hat{\rho}$ [GPa. cm ³ /g]	$\hat{E}_{22}/\hat{\rho} = \hat{E}_{33}/\hat{\rho}$ [GPa. cm ³ /g]	$\hat{G}_{12}/\hat{\rho} = \hat{G}_{13}/\hat{\rho}$ [GPa. cm ³ /g]	$\hat{G}_{23}/\hat{\rho}$ [GPa. cm ³ /g]
L-1	100.15 ± 0.00	5.65 ± 0.02	3.27 ± 0.03	1.99 ± 0.01
L-2	21.97 ± 0.00	6.85 ± 0.15	2.71 ± 0.05	2.51 ± 0.02
L-3	38.61 ± 0.00	6.41 ± 0.08	2.77 ± 0.04	2.28 ± 0.01
L-4	39.95 ± 0.00	6.24 ± 0.02	2.93 ± 0.02	2.16 ± 0.00

- carbon/hollow-E-glass/epoxy density comparable to carbon/epoxy
- $\hat{E}_{11}/\hat{\rho}$ and $\hat{G}_{12}/\hat{\rho}$ are slightly higher in carbon/hollow-E-glass/epoxy than carbon/solid-E-glass/epoxy.
- The hollow fibre content lowers the specific transverse elastic properties ($\hat{E}_{22}/\hat{\rho}$, $\hat{E}_{33}/\hat{\rho}$ and $\hat{G}_{23}/\hat{\rho}$) compared to E-glass/epoxy lamina.
- $\hat{E}_{11}/\hat{\rho}$ and $\hat{G}_{12}/\hat{\rho} = \hat{G}_{13}/\hat{\rho}$ are increased compared to E-glass/epoxy

Matrix von Mises micro-stress fields



- Slightly higher maximum stress for 20% hollowness
- Stress redistribution and larger stress amplification region

Conclusions

- The effective density of carbon/hollow-E-glass/epoxy lamina is comparable to that of carbon/epoxy lamina.
- Higher longitudinal modulus ($\hat{E}_{11}$) is obtained for carbon/solid-E-glass/epoxy and carbon/hollow-E-glass/epoxy laminae when compared to that of solid-E-glass/epoxy lamina.
- An increase in the transverse Poisson's ratio ($\hat{\nu}_{23}$) is observed in carbon/hollow-E-glass/epoxy lamina because of the hollow fibre content compared to solid-E-glass/epoxy lamina.
- The hollow fibre content lowers the specific transverse elastic properties ($\hat{E}_{22}/\hat{\rho}$, $\hat{E}_{33}/\hat{\rho}$ and $\hat{G}_{23}/\hat{\rho}$), while having a negligible effect on the major Poisson's ratios ($\hat{\nu}_{12} = \hat{\nu}_{13}$) compared to solid-E-glass/epoxy lamina.
- The specific longitudinal elastic modulus ($\hat{E}_{11}/\hat{\rho}$), and the specific longitudinal shear modulus ($\hat{G}_{12}/\hat{\rho} = \hat{G}_{13}/\hat{\rho}$) are increased compared to solid-E-glass/epoxy.

Future work

- Investigate different carbon/E-glass/epoxy fibre volume fraction
 - Investigate different hollowness % for hollow E-glass fibres
 - Interfacial stresses analysis
 - Progress damage modelling
- } —————> Alter the micro stresses and specific elastic properties



Thank you for
your attention!

Questions?

