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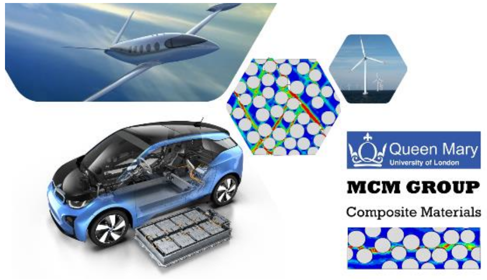
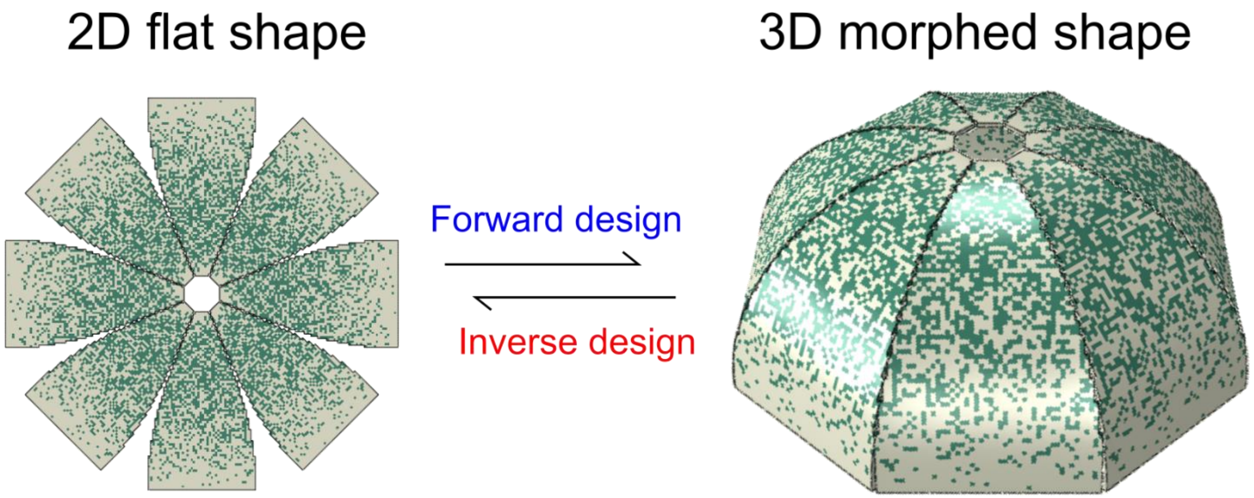


Inverse design of shape-morphing structures based on functionally graded composites

International Conference on Composite Materials 2023

Hirak Kansara
PhD Student in Mechanical Engineering
Queen Mary University of London

With excellent contributions from:
Mingchao Liu (Birmingham), **Yinfeng He** (Nottingham), **Wei Tan** (QMUL)



Outline

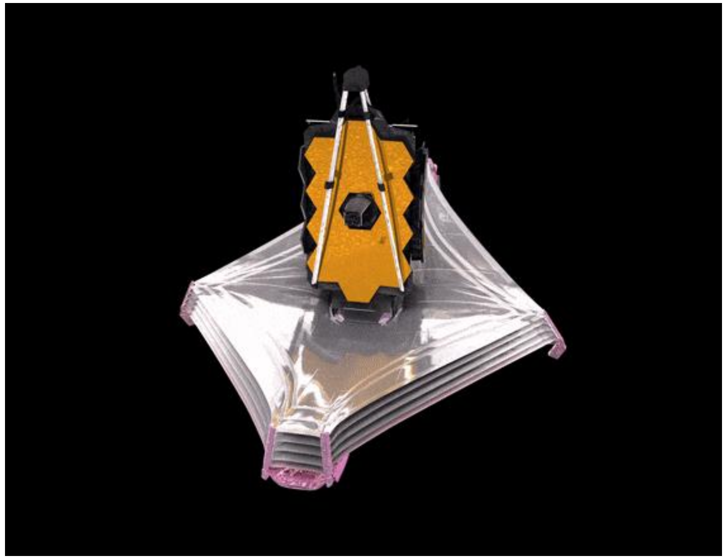
1. Motivation

2. Methods

3. Results

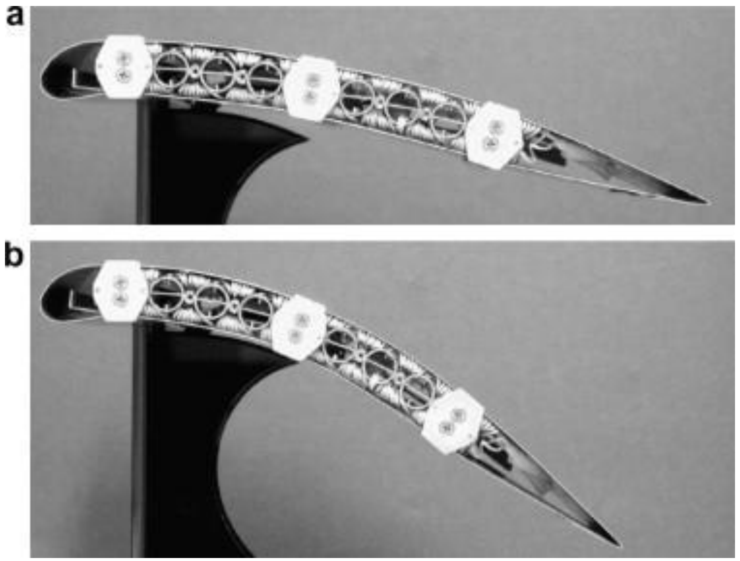
Morphing structures

JWST's sunshield



NASA's Goddard Space Flight Center

Aircraft wing



Sofla, A.Y.N., et al. *Materials & Design*, 2010.

Soft robots

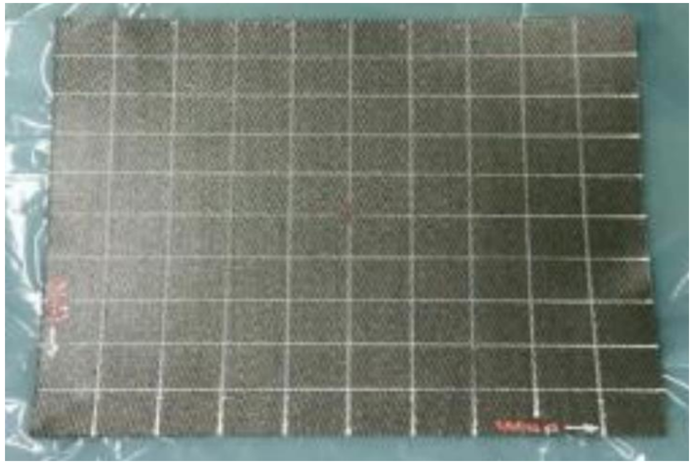
Advanced Robotics at Queen Mary

- Morphing structures have the ability to change their shape as **deployable structures,** improve **aerodynamic** performance, or achieve specific **manipulation/motion.**

Some candidate morphing structures:

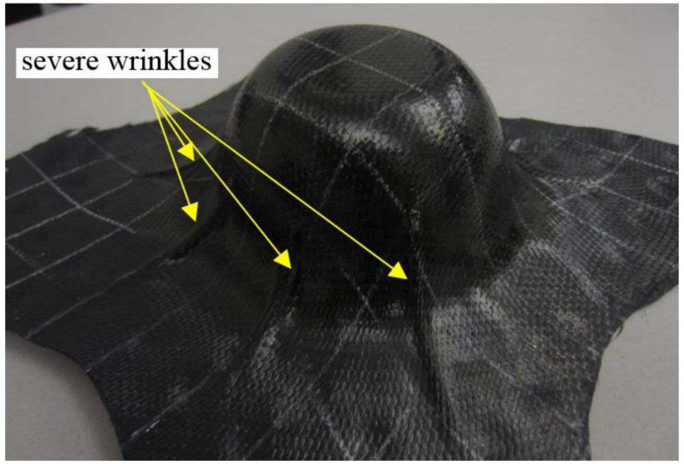
Shape-morphing structures that transform from **flat 2D sheets** to **3D shapes** are desired due to simplicity in fabrication and transportation (via stacking).

2D flat fabric



Deformation

3D shape after draping



[Gupta et al. 2019]

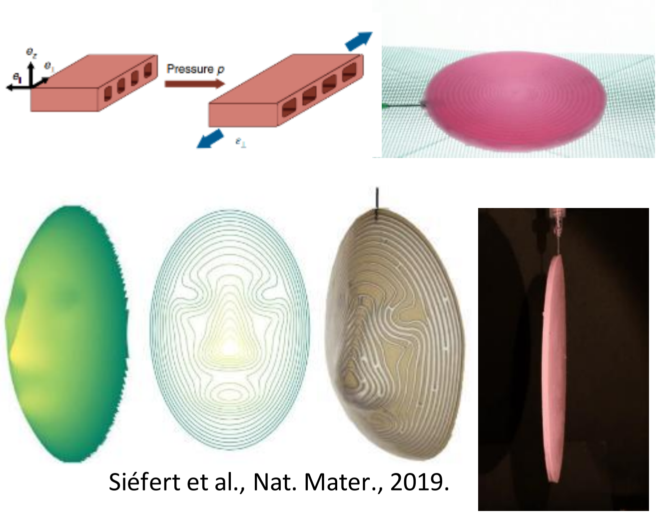
$\kappa = 0$ $\kappa = 0$

- **Isometries cannot alter the Gaussian curvature (κ).**

Methods to achieve morphing from flat sheet:

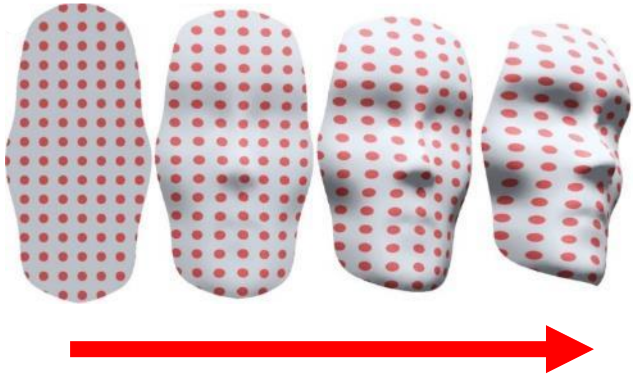
Locally expansion or actuation

Pneumatic expansion



Siéfert et al., Nat. Mater., 2019.

Thermal expansion



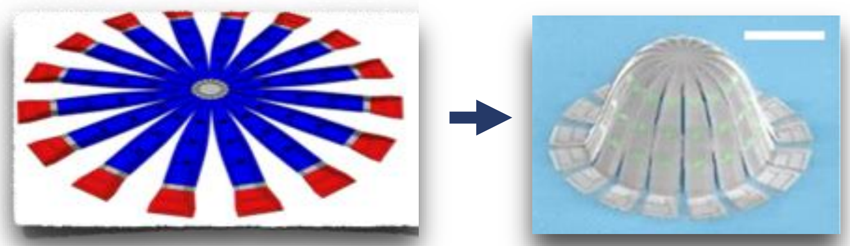
Flat sheet is heated

Aharoni, Hillel, et al. 2018

But induces excessive strain!

Specific cuts

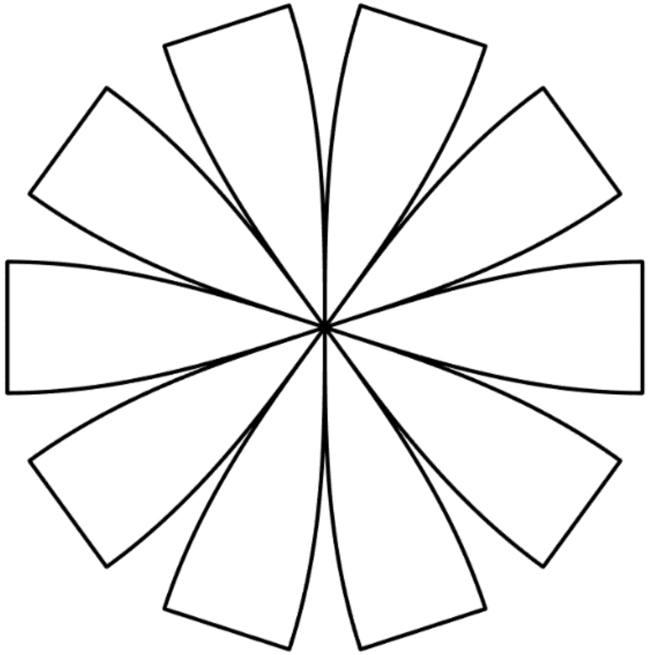
Kirigami



Lee et al., Nat. Com., 2018.

Axisymmetric-type structures can be formed by making repeated cut patterns around a central hub.

Inverse design problem:



Initial 2D pattern

Trial-and-Error

Forward design

Inverse design

Analytical or
Data-driven
approaches

Final 3D shape

➢ **How to determine a 2D pattern that deforms into a desired 3D shape?**

Outline

1. Motivation

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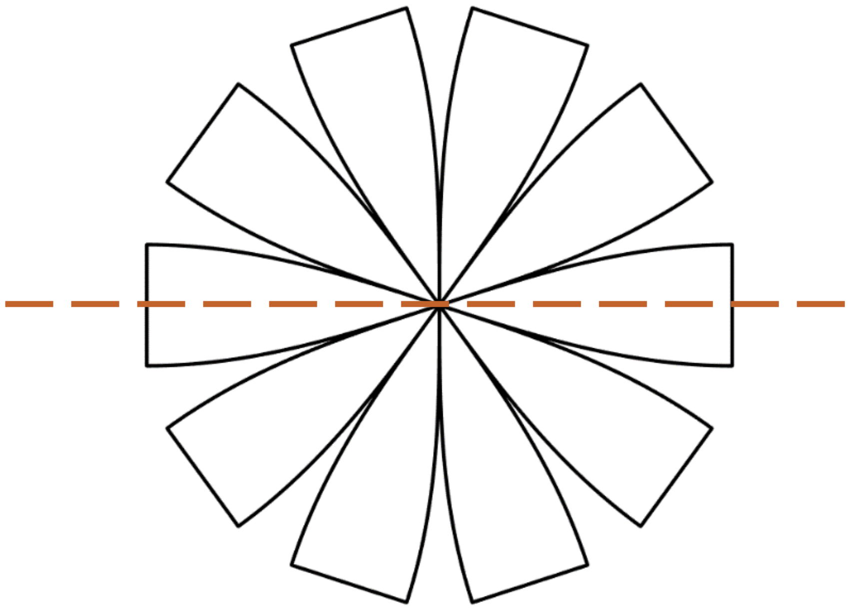
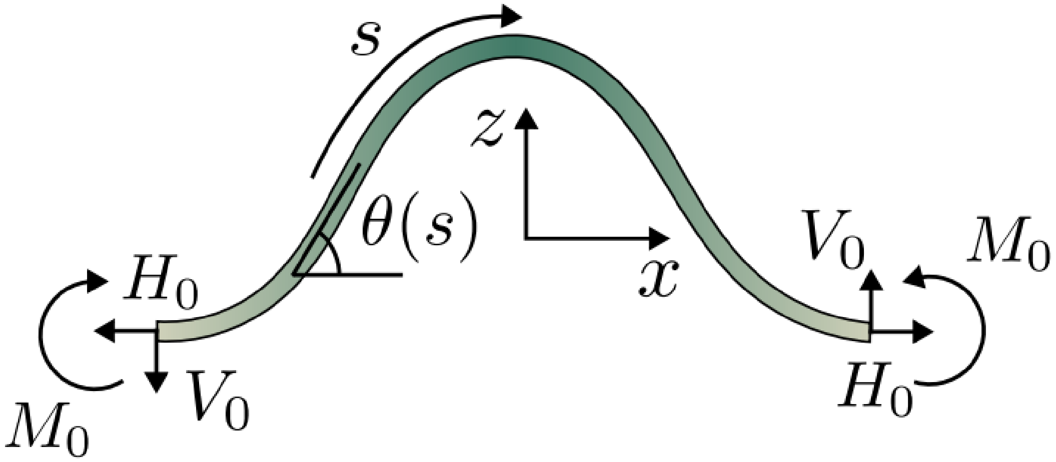
Start from the beam problem:

We can simplify the axisymmetric problem into 1D beam problem:

Top view



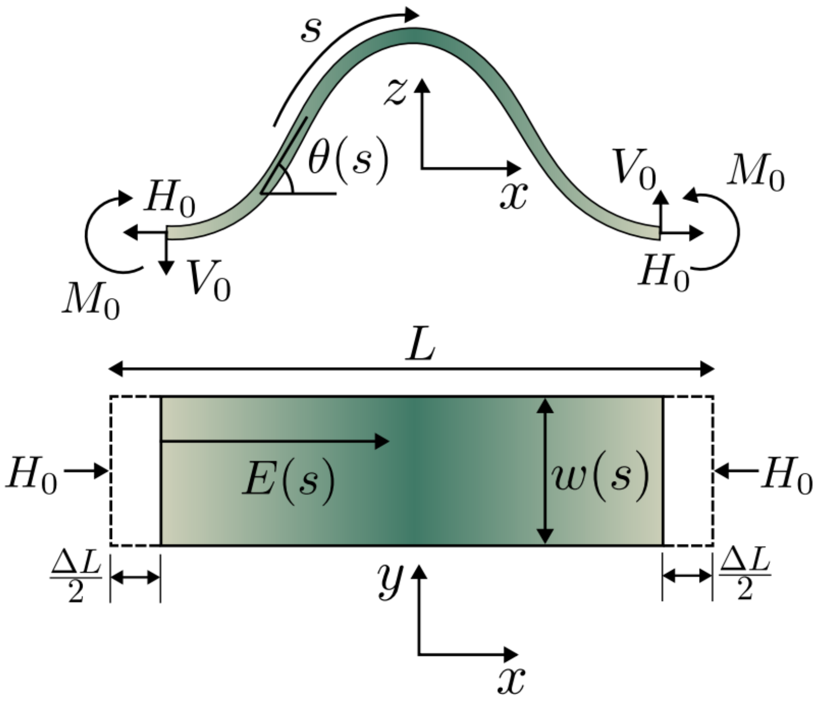
Side view



- **The load from the transverse and out-of-plane direction will introduce beam buckling.**

Tapered beam equation

Tapered beam equation relates the shape of an elastic strip (LHS) under mechanical loading to the curvature of a 3D shape (RHS) using non-linear beam theory.



Tapered Elastica equation:

$$\frac{d}{d\xi}\left[\hat{E}(\xi)\cdot\hat{I}(\xi)\cdot\frac{d\theta}{d\xi}\right] = -\hat{H}\frac{d\hat{z}}{d\xi} - \hat{V}\frac{d\hat{x}}{d\xi}$$

$$\hat{I}(\xi) = \frac{\hat{w}(\xi)\cdot\hat{t}(\xi)^3}{12}$$

Control:

- **Moment of inertia (width/thickness)**
- **Local Young's Modulus**

□ **Control the local bending stiffness to achieve a specific morphing shape.**

Previous work: Uniform thickness and tessellation

Without tessellation

Applying horizontal force and assuming uniform thickness.

$$\hat{w}(\xi) = \tilde{H}\frac{(\hat{z}_* - \hat{z})}{\theta_\xi(\xi)}$$

With tessellation

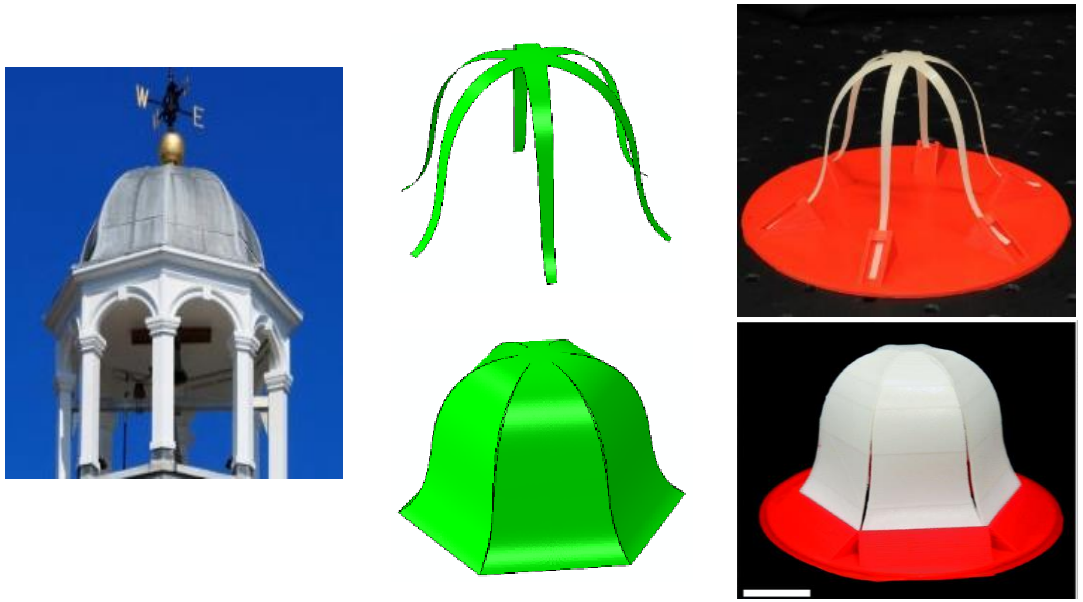
Following tessellation condition.

$$\hat{w}(\xi) = \frac{2L}{w_0}\cdot\hat{x}(\xi)\cdot\tan\left(\frac{\pi}{N}\right)$$

$$\hat{t}(\xi)^3 = \tilde{H}\frac{(\hat{z}_* - \hat{z})}{w(\xi)\cdot\theta_\xi}$$

Cupola of St Thomas church

Target FEM Experiment



Without tessellation

With tessellation

[Liu, M. et al. Soft Matter, 2020, **16**, 7739-7750]

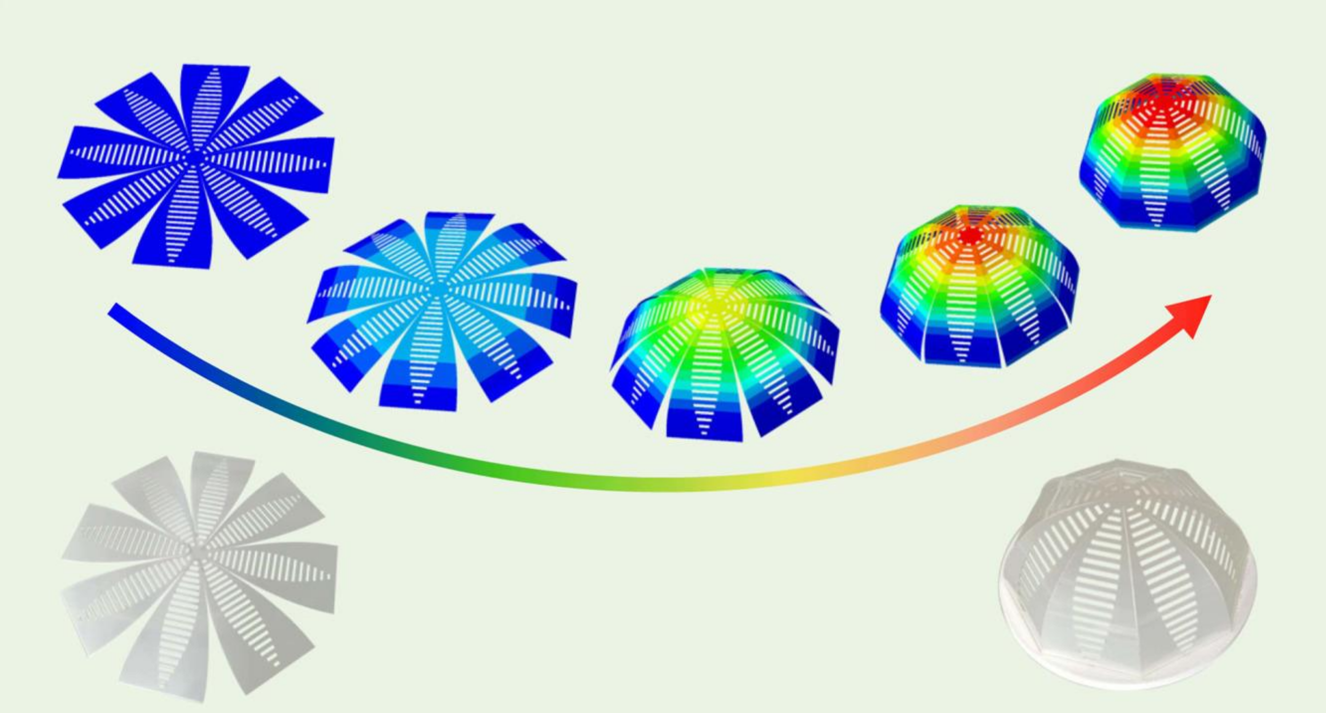
- **Structures with varying thickness are difficult to store and incompatible with brittle material.**

Previous work: Distributed local porosity

Assuming uniform thickness and following the tessellation condition.

Level of local porosity, ϕ, enables bending stiffness to be varied.

$$\phi(\xi) = 1 - \tilde{H}\,\frac{(\hat{z}_* - \hat{z})}{\hat{w}(\xi)\cdot\theta_\xi}$$

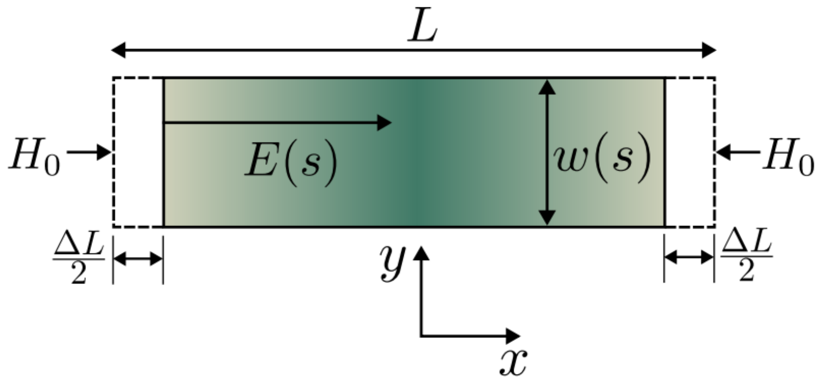


[Zhang, Y. et al. Ext Mech Letters, 56, 2022, 101857]

- **However, size of porosity may adversely affect load bearing capacity of morphing structures.**

Our work: Modulus-graded beam

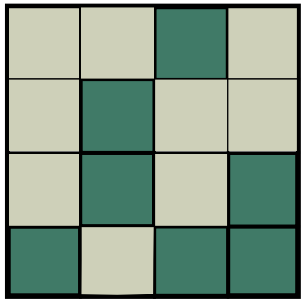
Assuming non-uniform modulus distribution with uniform thickness and tessellation.



$$\hat{E}(\xi) = \tilde{H}\frac{(\hat{z}_* - \hat{z})}{\hat{w}(\xi)\cdot\theta_\xi}$$

Graded beam equation

Modulus distribution is achieved by varying volume fraction of bi-phase composites. But…



Target

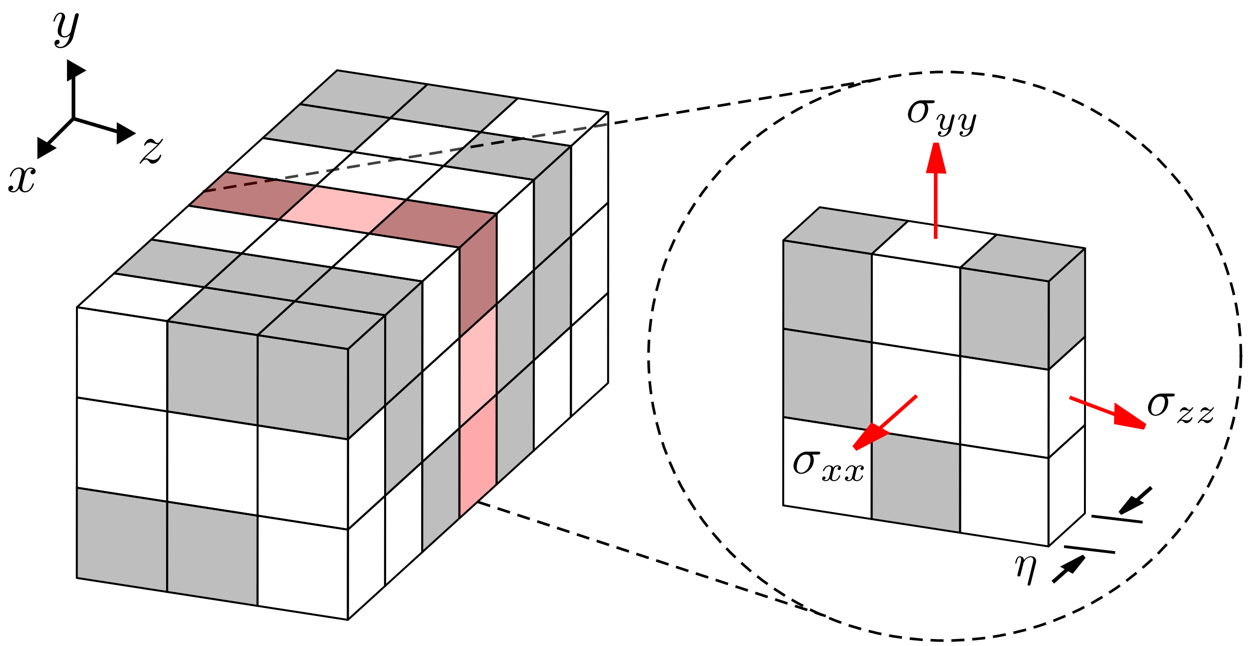
- ☐ How to determine the composition of each material?

AND

- ☐ How to fabricate the graded composite?

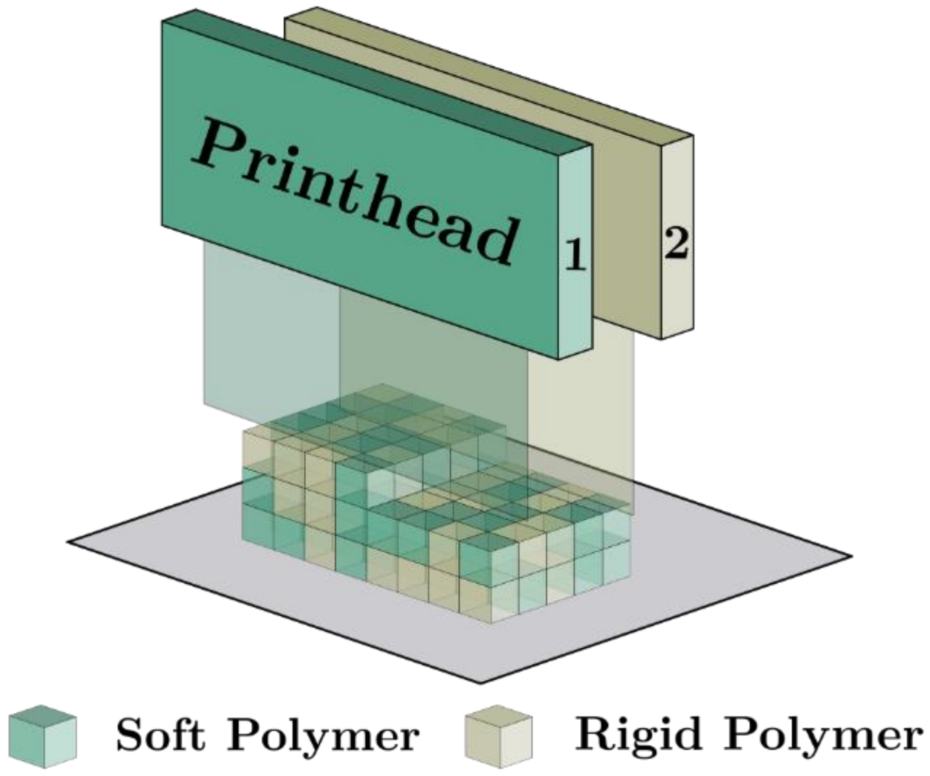
Voxelated graded composites

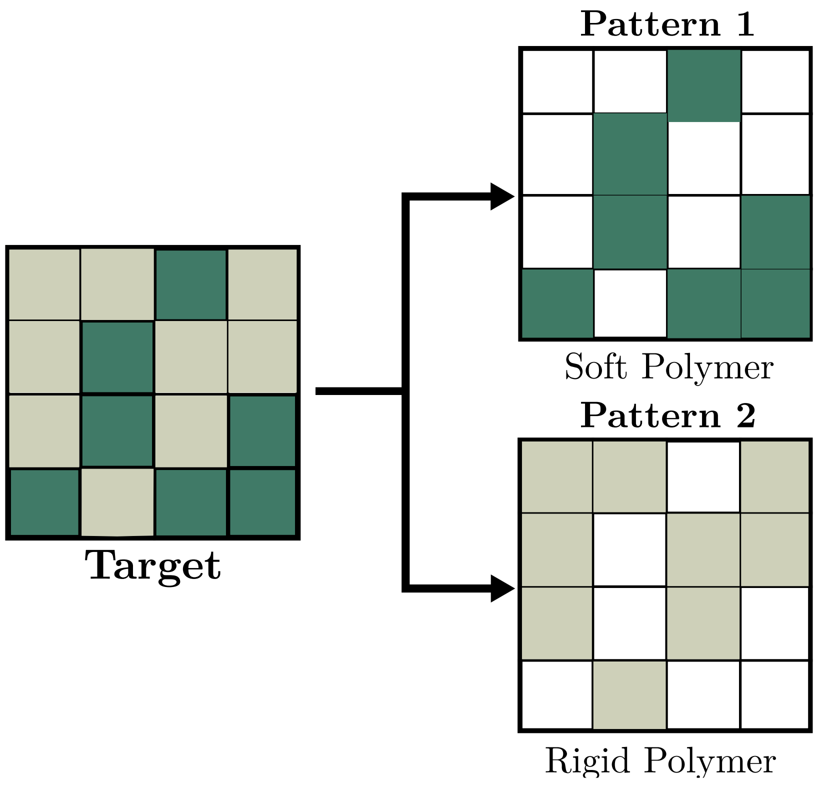
One strategy involves discretising the geometry using voxels.

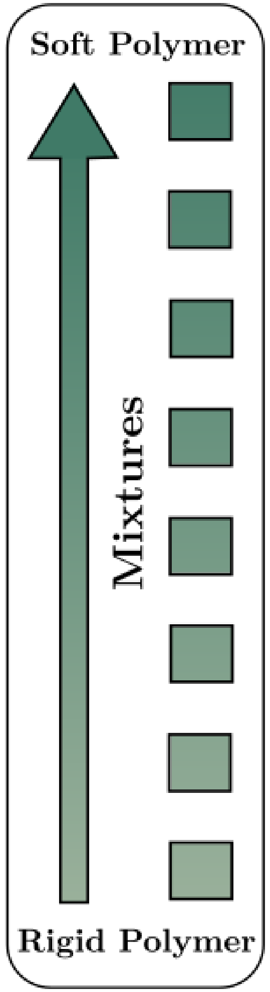


- ☐ **The modulus of each slice can be designed to achieve a certain modulus.**

Additive Manufacturing of Graded Composites

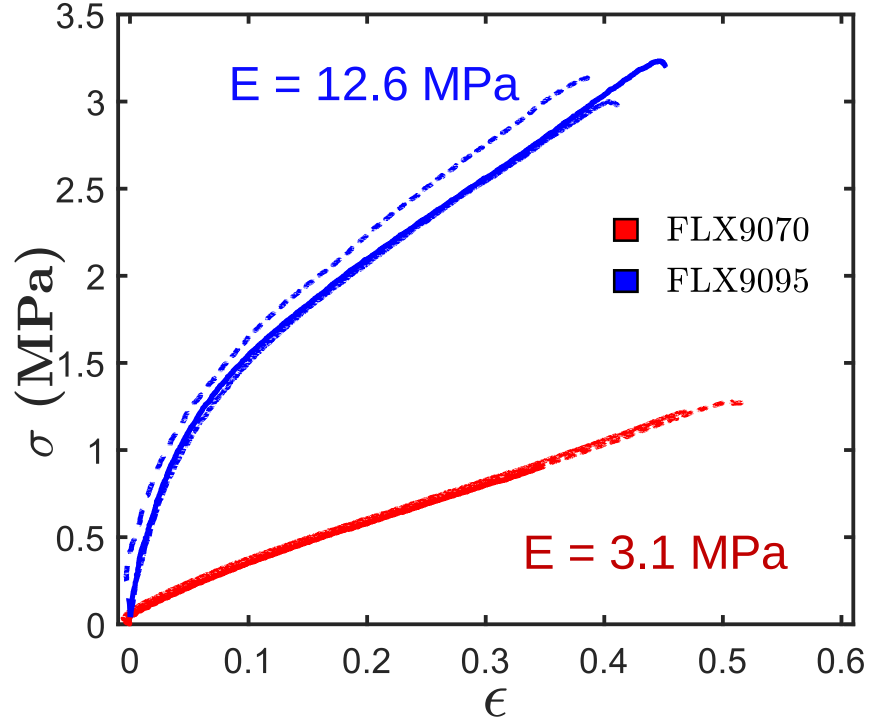






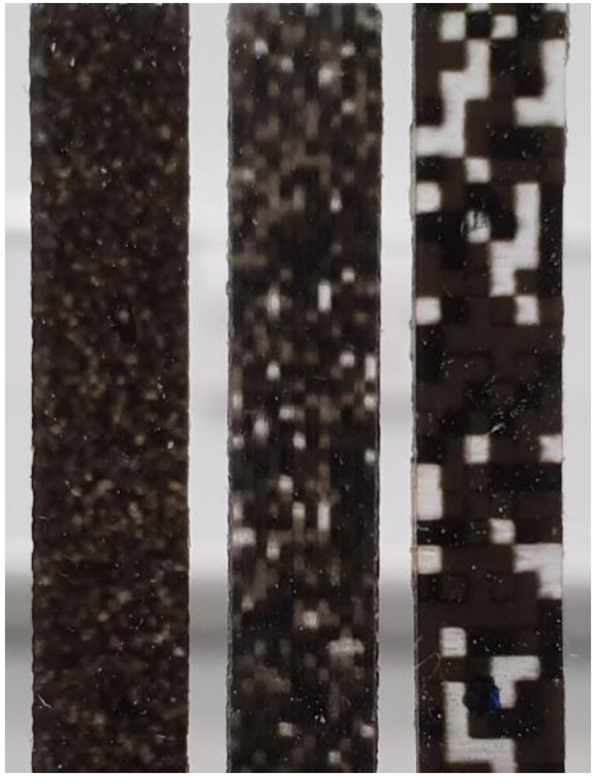
- **Dual Inkjet printing** with UV curing process.
- The two compatible polymers have good **interface bonding**.

Additive Manufacturing of Graded Composites



- ☐ **Two compatible elastomers are used to manufacture graded composite (FLX9070 is soft and FLX9095 is rigid).**

Increase in voxel size

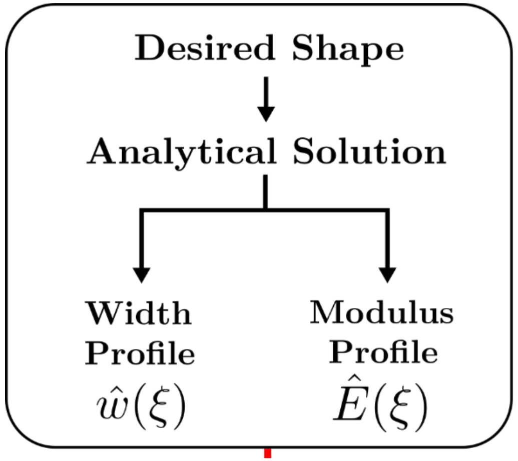


0.25 mm 0.5 mm 1 mm

50% composition

Design principle

Design strategy employed to inverse design desired 3D shape.



Outline

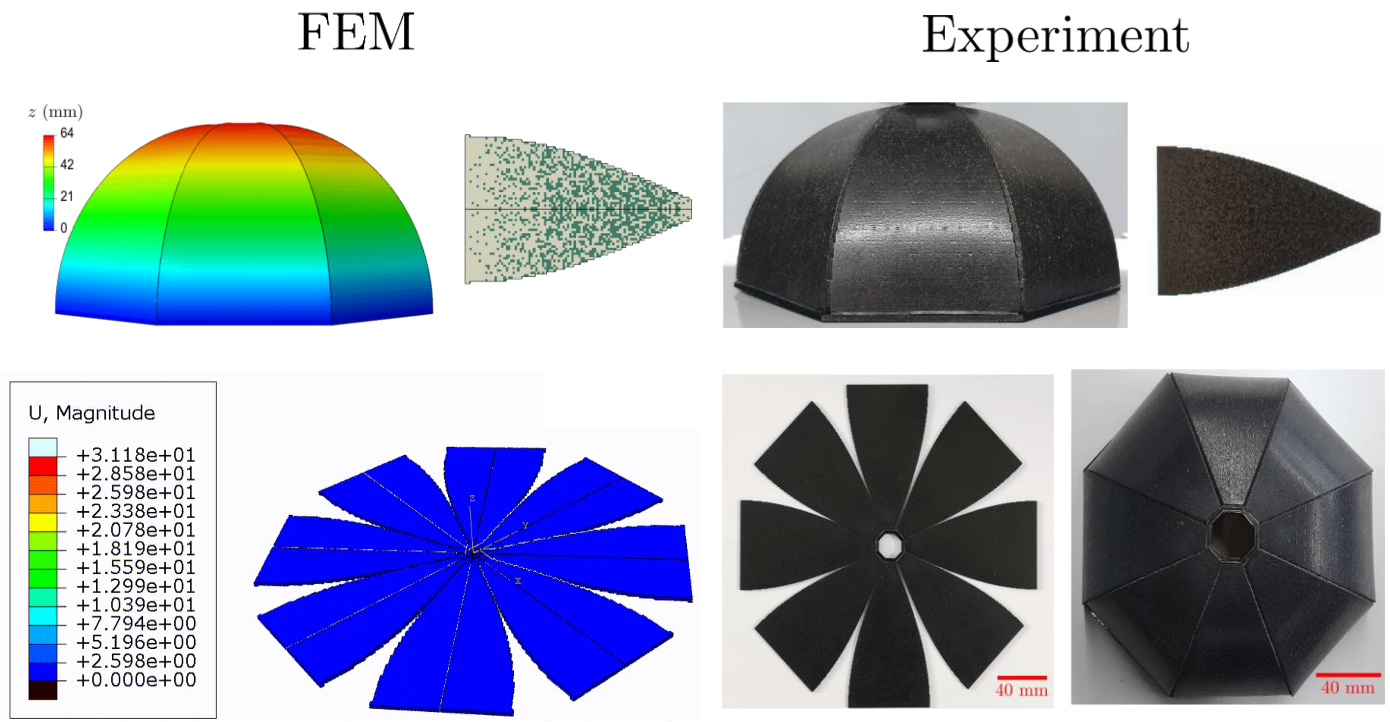
1. Motivation

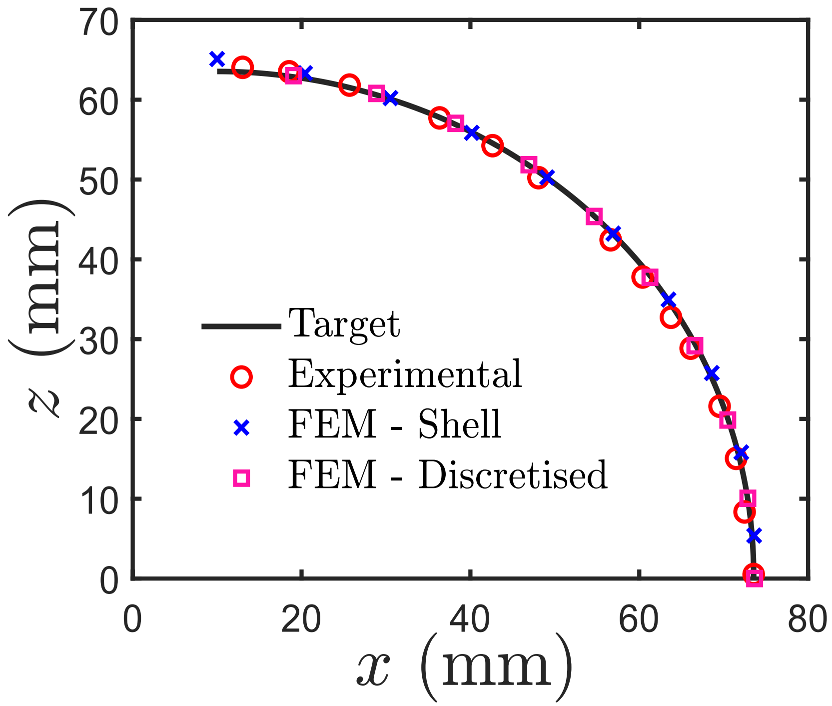
2. Methodologies

3. Results

Validation (FEM vs Experiment)

Target: 3D tessellating hemisphere



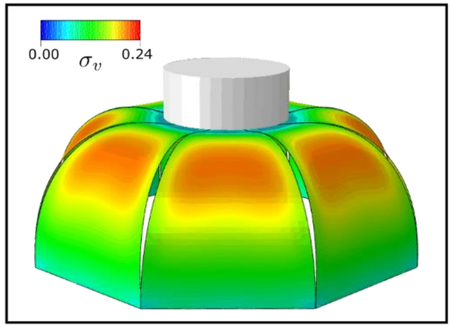


☐ **A good agreement between FEM predictions, analytical solution and experiment.**

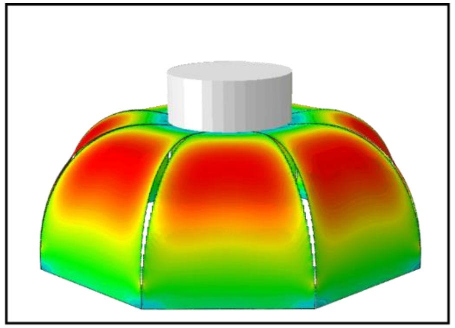
Load-bearing capacity

It is essential to evaluate their load-bearing capacities, before engineering application.

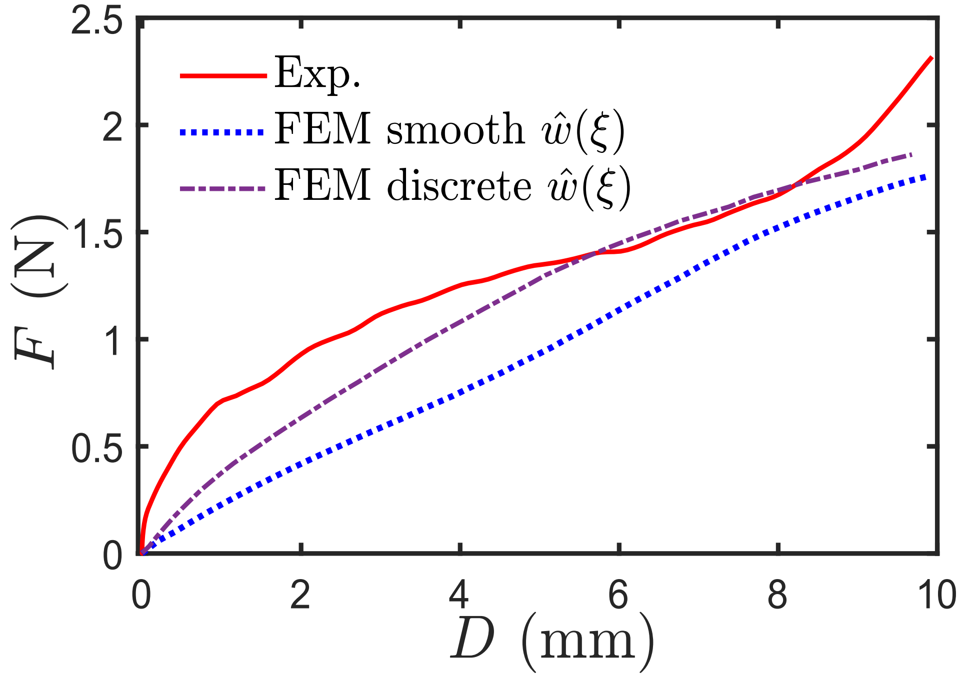
Experiment



Smooth $w(\xi)$



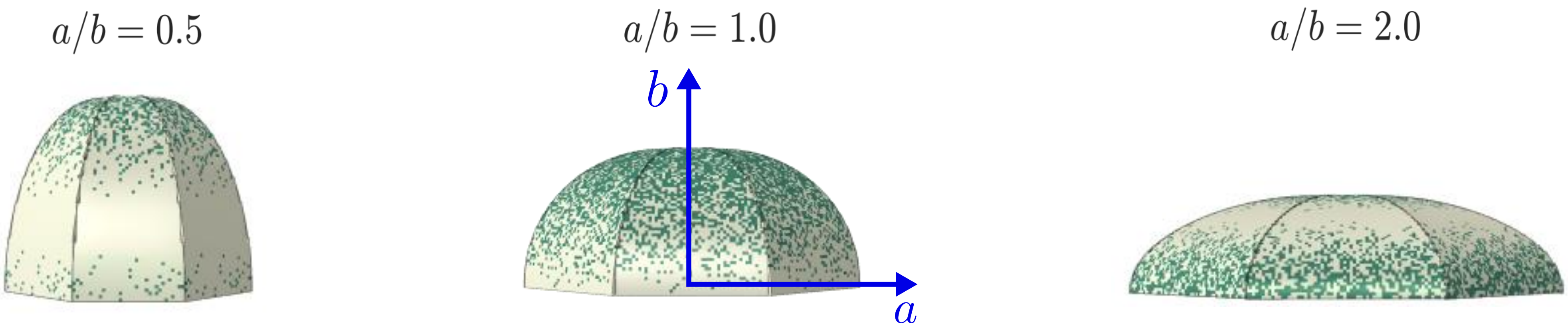
Discrete $w(\xi)$

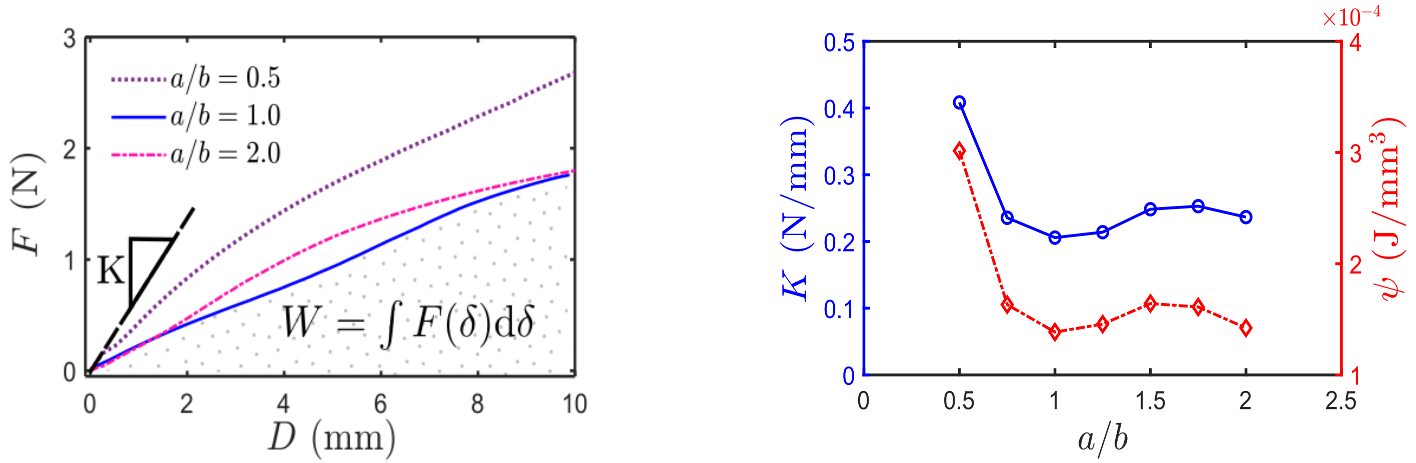


- **Indentation tests to determine the stiffness and the energy absorption capacity.**

Load-bearing capacity

3D tessellating hemiellispsoids with varying aspect ratios.





- Indentation tests to determine the stiffness (K) and the energy absorption capacity (ψ).

More cases:

A nose cone structure

$\kappa > 0$

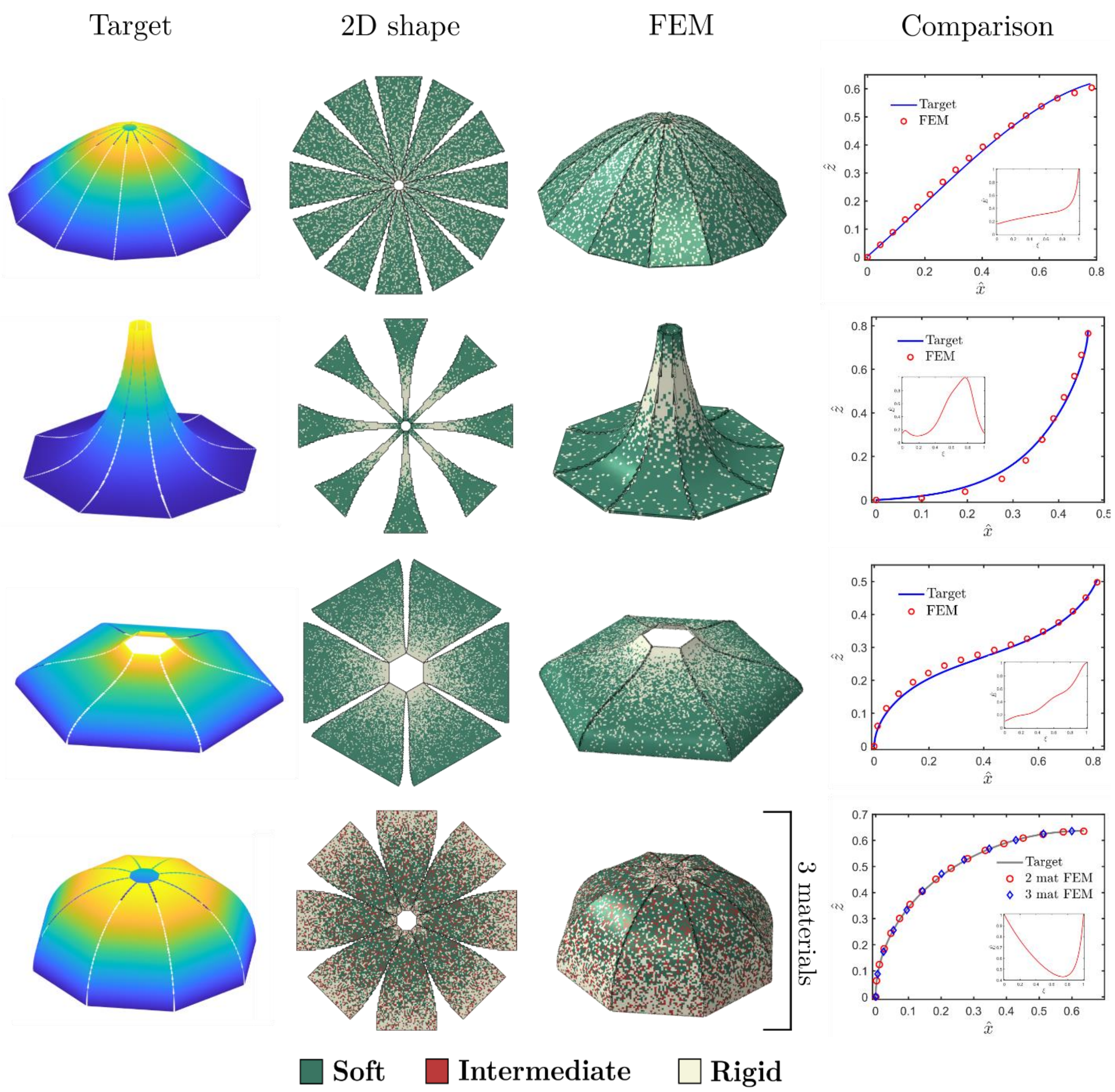
A trumpet structure

$\kappa < 0$

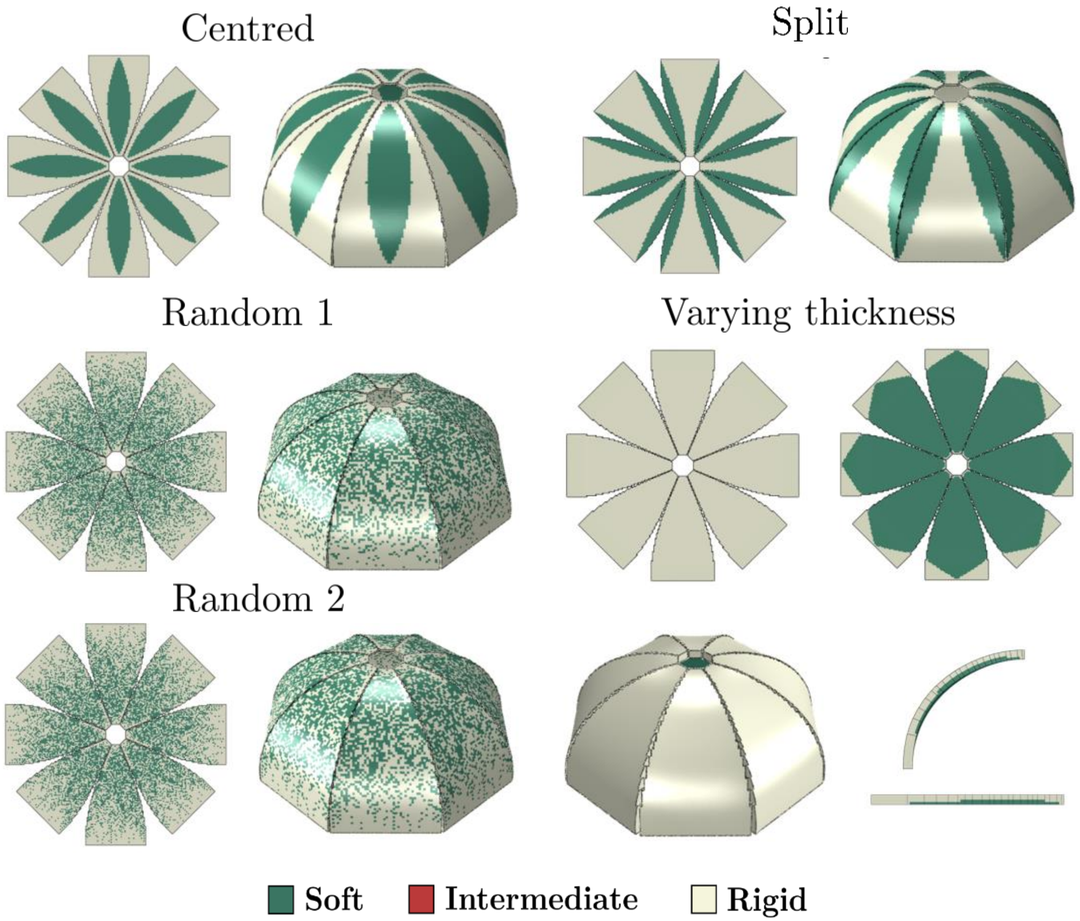
A roof structure

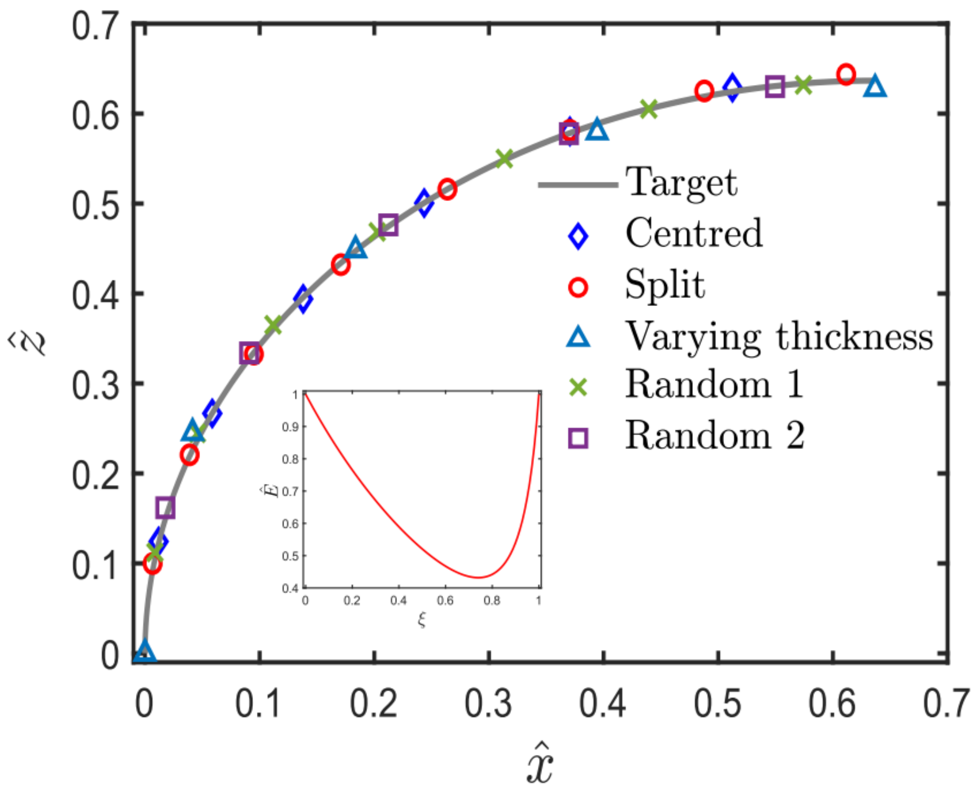
Varying κ

A hemisphere formed using three-phase materials



Aggregate patterns of materials



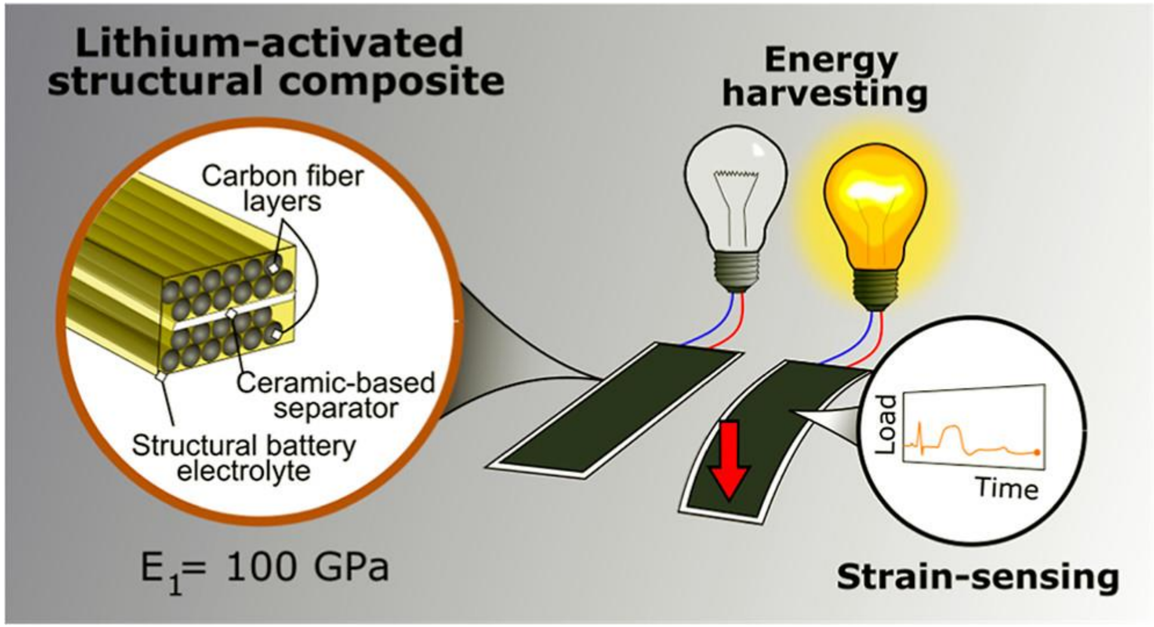


- The same morphing structure using of various **aggregate patterns** in either width or thickness direction.
- This is to avoid generating too many **interfaces** if the bonding is not very well.

Why use composites instead of a single material for morphing?

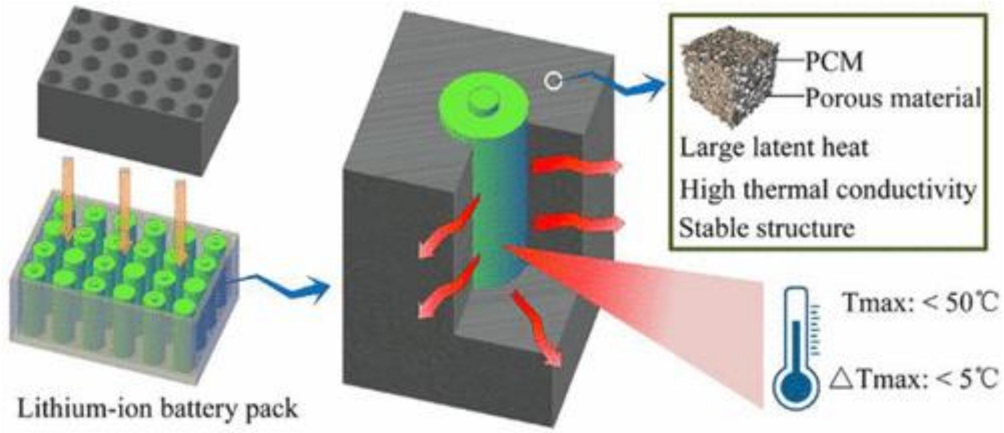
- Composites can be designed to blend the distinct advantages of two different materials.

Sensing and energy storage



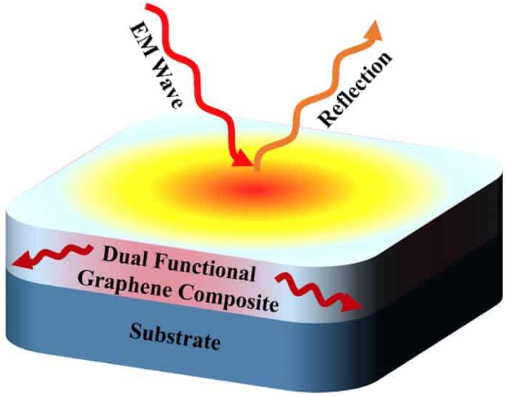
[ACS Appl. Mater. Interfaces 2022, 14, 29, 33871−33880]

Heat management



[Energy Fuels 2022, 36, 8, 4153–4173]

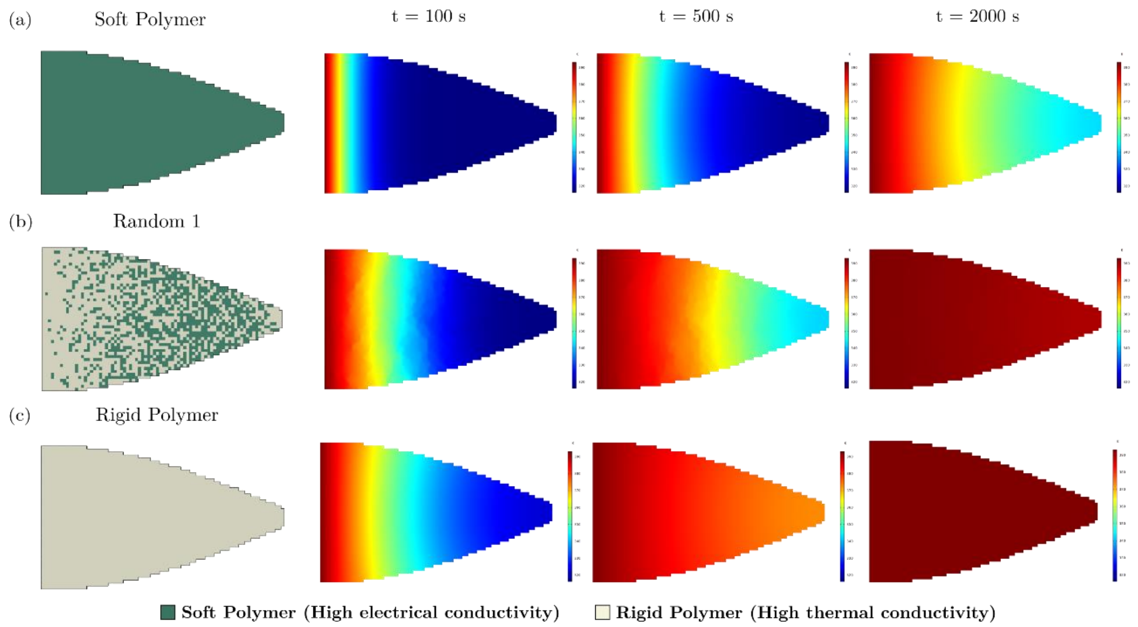
Electromagnetic shielding interface



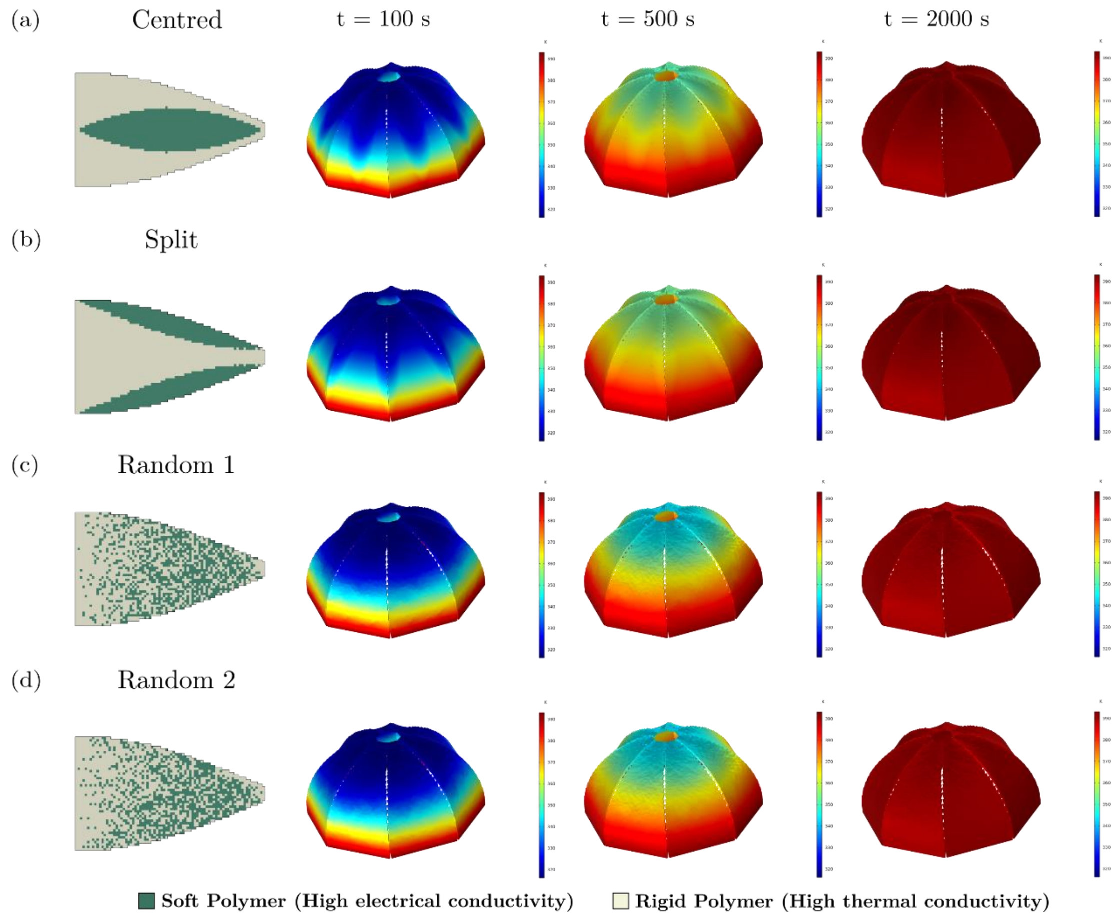
https://physicsworld.com/a/cool-graphene-composites-block-em-radiation/

Multifunction – Hemisphere (Heat transfer)

Before morphing



After morphing



- Composite material that exhibits intermediate **thermal conductivity**.

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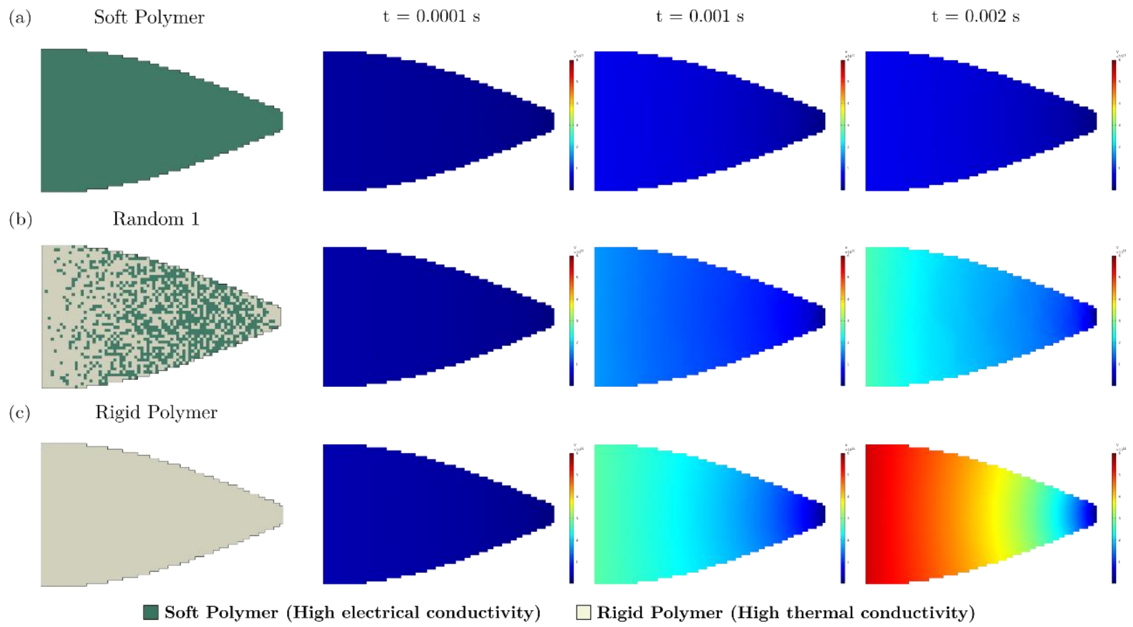
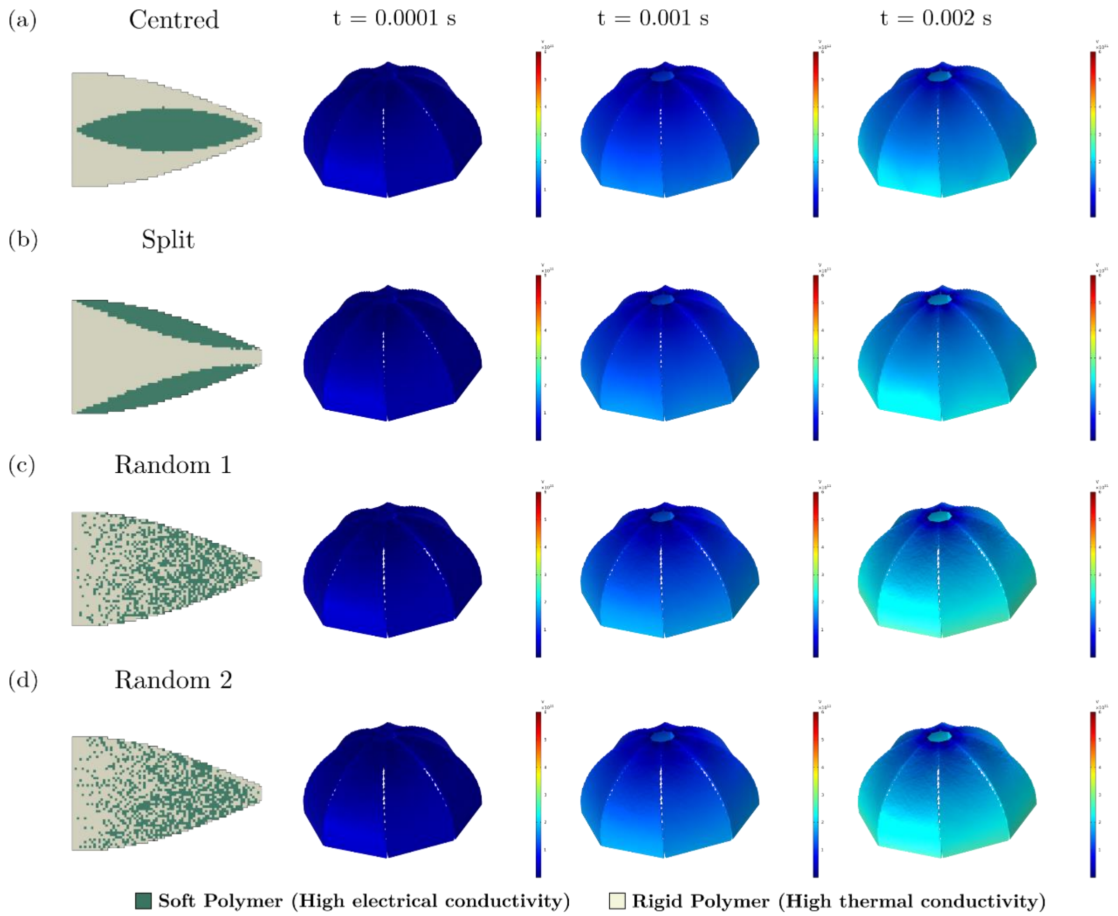
Multifunction – Hemisphere (Electric potential)

Before morphing

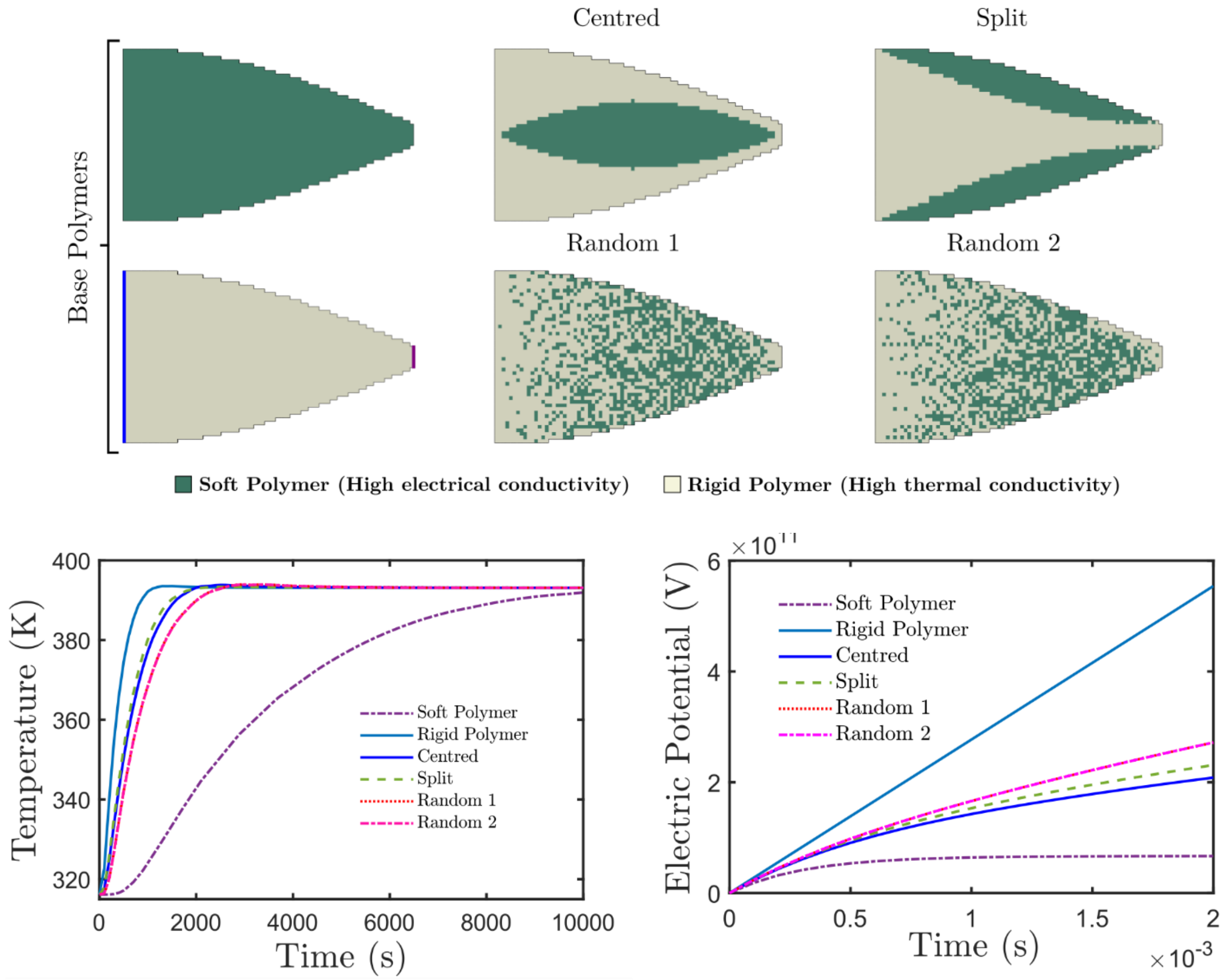


After morphing



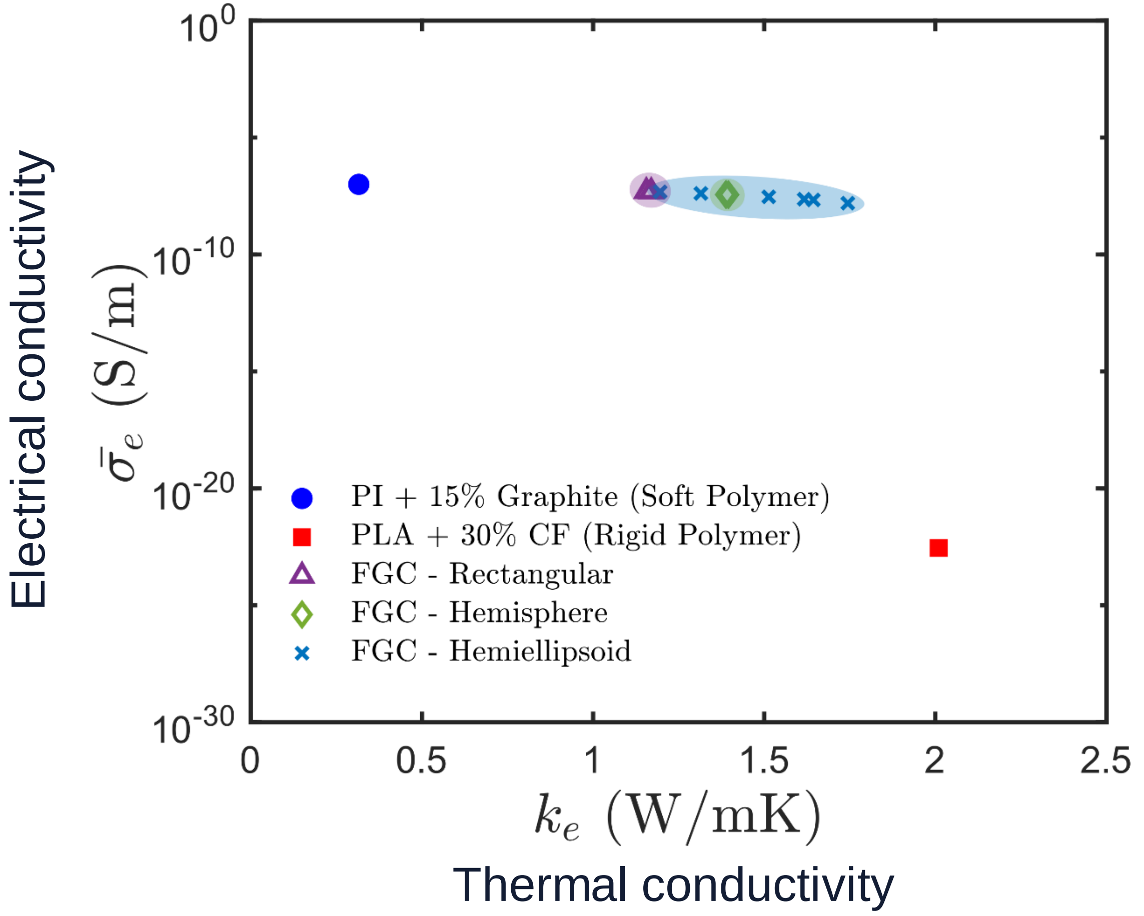
- Composite material that exhibits intermediate **electrical conductivity**.

Multifunction – Hemisphere



- The temperature and electrical potential profiles vary with different compositional patterns.

Composite blending



- Composite material that exhibits favourable properties for both electrical and thermal conductivities.

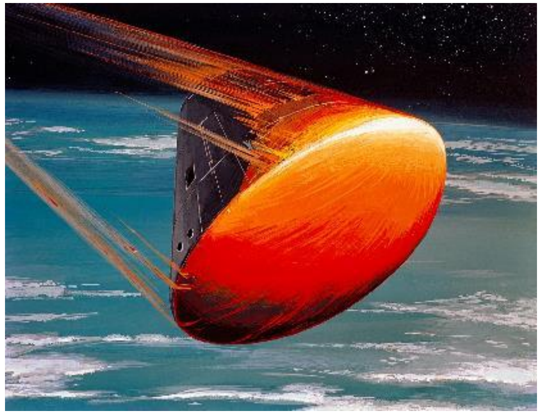
Potential multifunctional applications：

Spherical Radar Radome

Dome Theatre

O2 arena

Nosecone



Heat shield

Conclusion:

- The modulus profile of tapered spokes needed to achieve a desired 3D deformation was inversely designed using the **Tapered Beam equation**.
- We have achieved the graded morphing composites based on **rule of mixtures**, **discretised voxels** and **additive manufacturing**.
- **Multifunctional morphing composites** blend the distinct advantages of two different materials and combined effective properties in a single structure.

Future work:

- Achieving **self-actuation** by embedding stimuli-responsive material within the print or as an addition to the printed structure.

Thank you for your attention!

References:

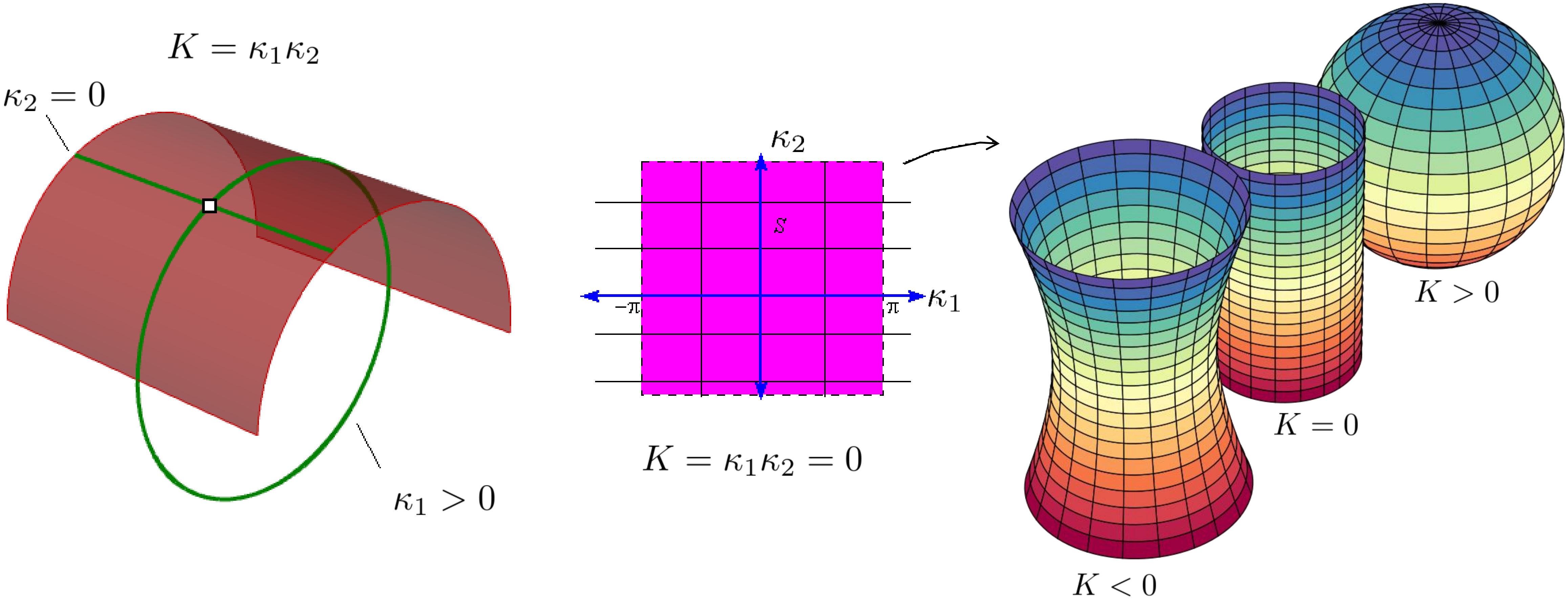
1. **Kansara, H.**, Liu, M.*, He, Y., Tan, W.* (2023). Inverse design and additive manufacturing of shape-morphing structures based on functionally graded composites. **J Mech. Phys. Solids**, Under Review.
2. Liu, M., Domino, L., Vella, D.* (2020). Tapered elasticæ as a route for axisymmetric morphing structures. **Soft Matter**, 16, 7739–7750.
3. Zhang, Y., Yang, J., Liu, M.*, Vella, D. (2022). Shape-morphing structures based on perforated kirigami. **Extre. Mech. Lett.**, 56, 101857.

Research sponsors:

Engineering and Physical Sciences Research Council

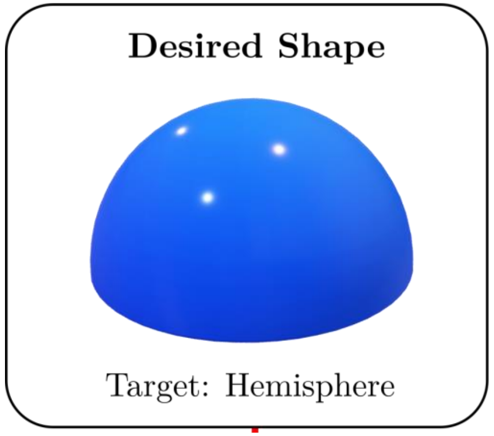
Open-source research code:

Gaussian curvature [Gauss, 1827]



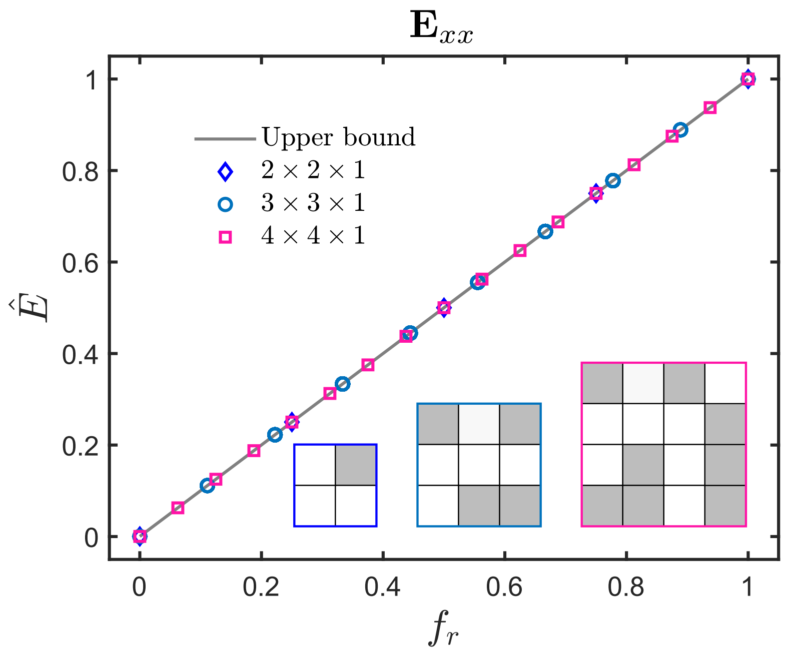
- **The Gaussian curvature of a surface is invariant under local isometry.**

Summary of the overall framework



Voxelated graded composites

Plotting the composite modulus against the volume fraction for longitudinal direction, $\mathbf{E}_{xx}$.



Interestingly, $\mathbf{E}_{xx}$ scales with the volume fraction due to affine deformation, whereby a global deformation is translated uniformly to microscale.

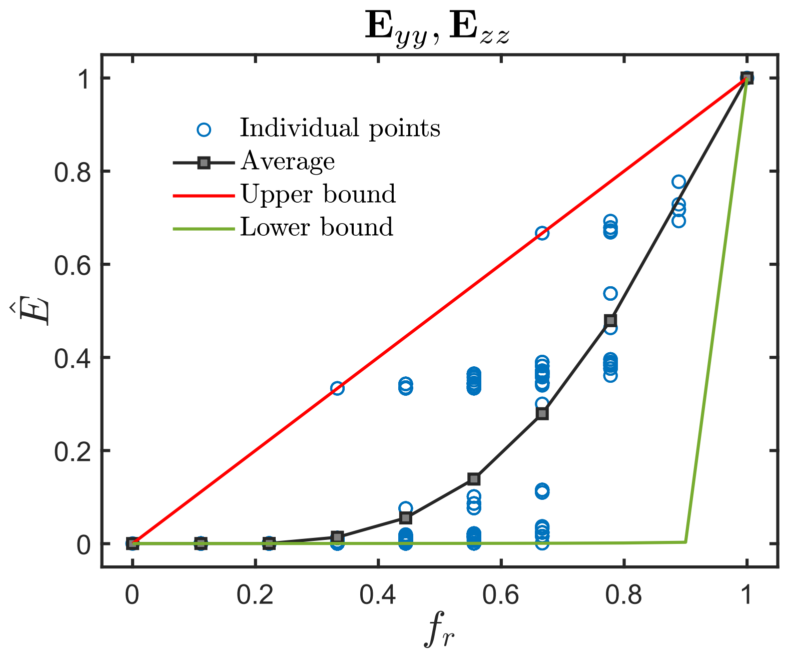
Results follow the Voigt (Upper) bound of rule of mixtures.

$$\mathbf{E}_{xx}(\xi) = f_r(\xi)E_r + f_s(\xi)E_s$$

Bending deformation is primarily dependent on longitudinal modulus, $\mathbf{E}_{xx}$, so we only need to match the modulus profile in that direction.

Voxelated graded composites

Plotting the composite modulus against the volume fraction for transverse, $\mathbf{E}_{yy}$, and out-of-plane directions, $\mathbf{E}_{zz}$.

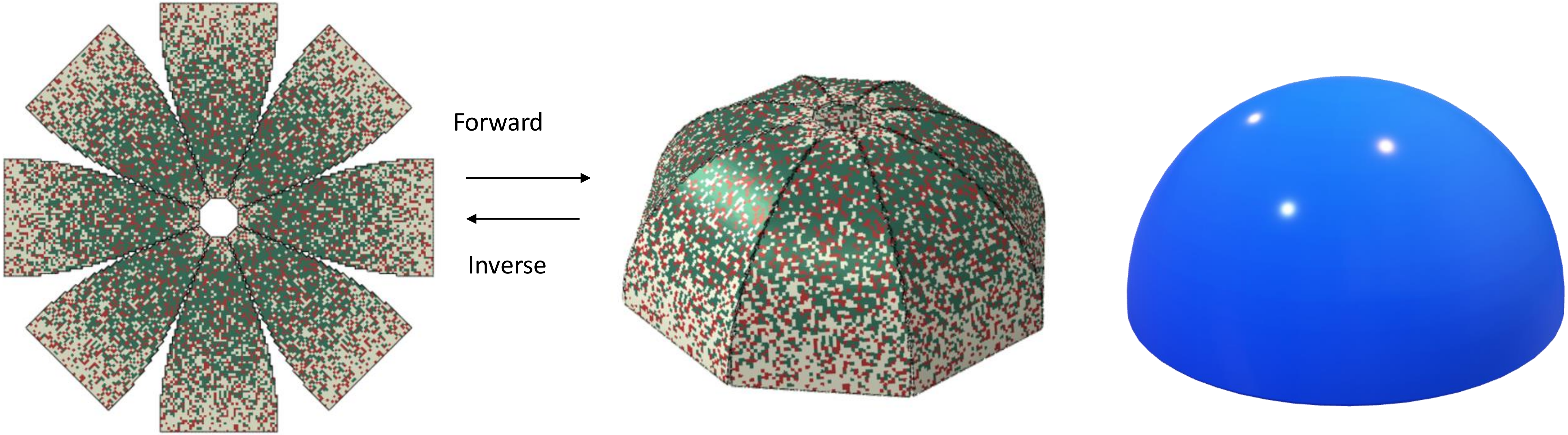


Voxels under constant stress undergo non-affine deformation, resulting from a mismatch of strain.

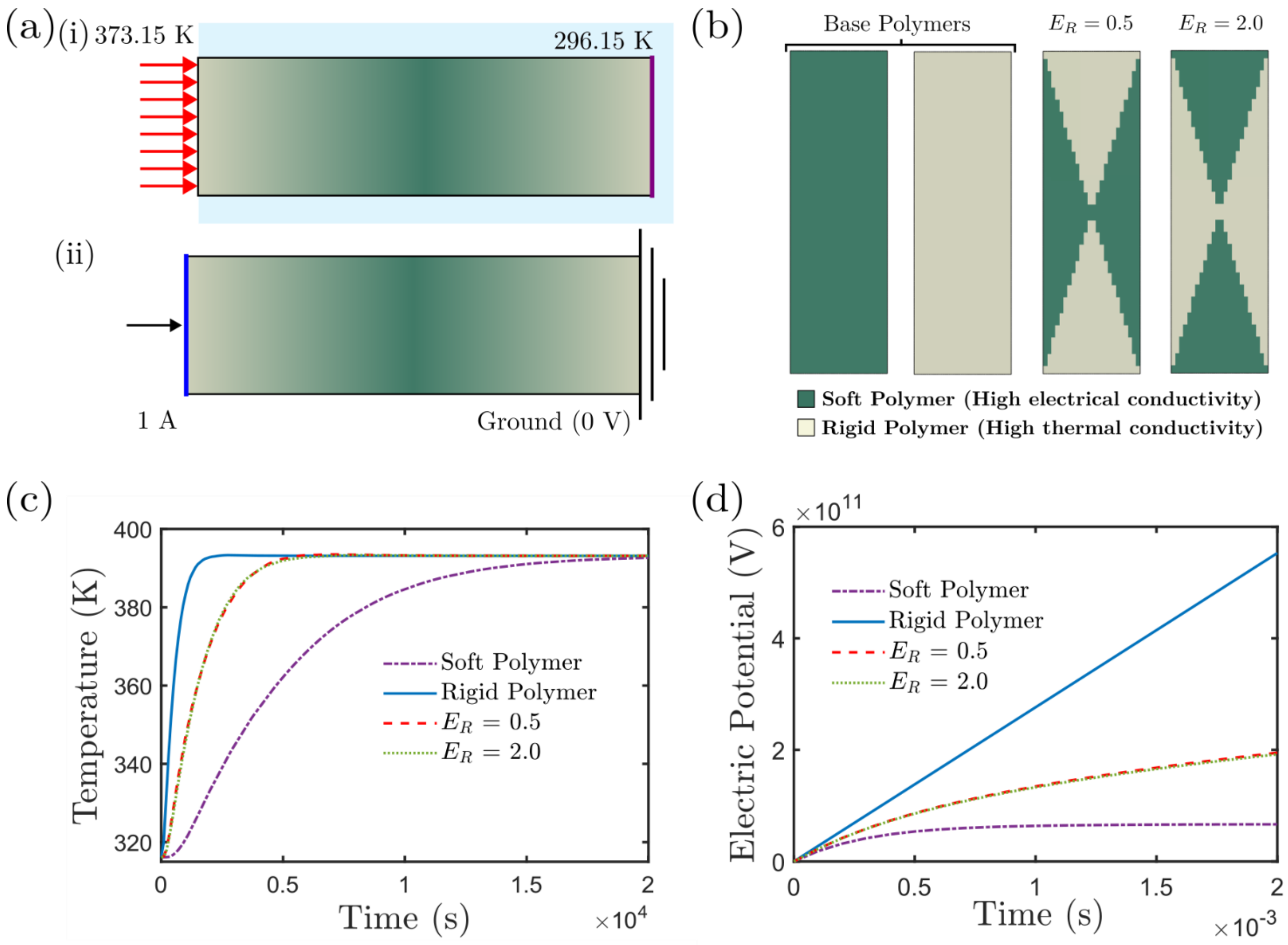
Points are bounded by the Voigt (Upper) and Reuss (Lower) bounds of rule of mixtures.

$$E_{yy} = E_{zz} = \left[\frac{f_r(\xi)}{E_r} + \frac{f_s(\xi)}{E_s}\right]^{-1}$$

Models such as Hashin--Shtrikman could also be used to capture micro-mechanics of bi-phase materials but at heightened complexity.


Forward
Inverse

Multifunctional – Rectangle shapes



	Soft Polymer (PI + 15% Graphite)	Rigid Polymer (PLA + 30% Carbon fibre)
Thermal conductivity (W/mK)	0.3155	2.01
Electrical conductivity (S/m)	1E-7	2.86E-23

Governing equations:

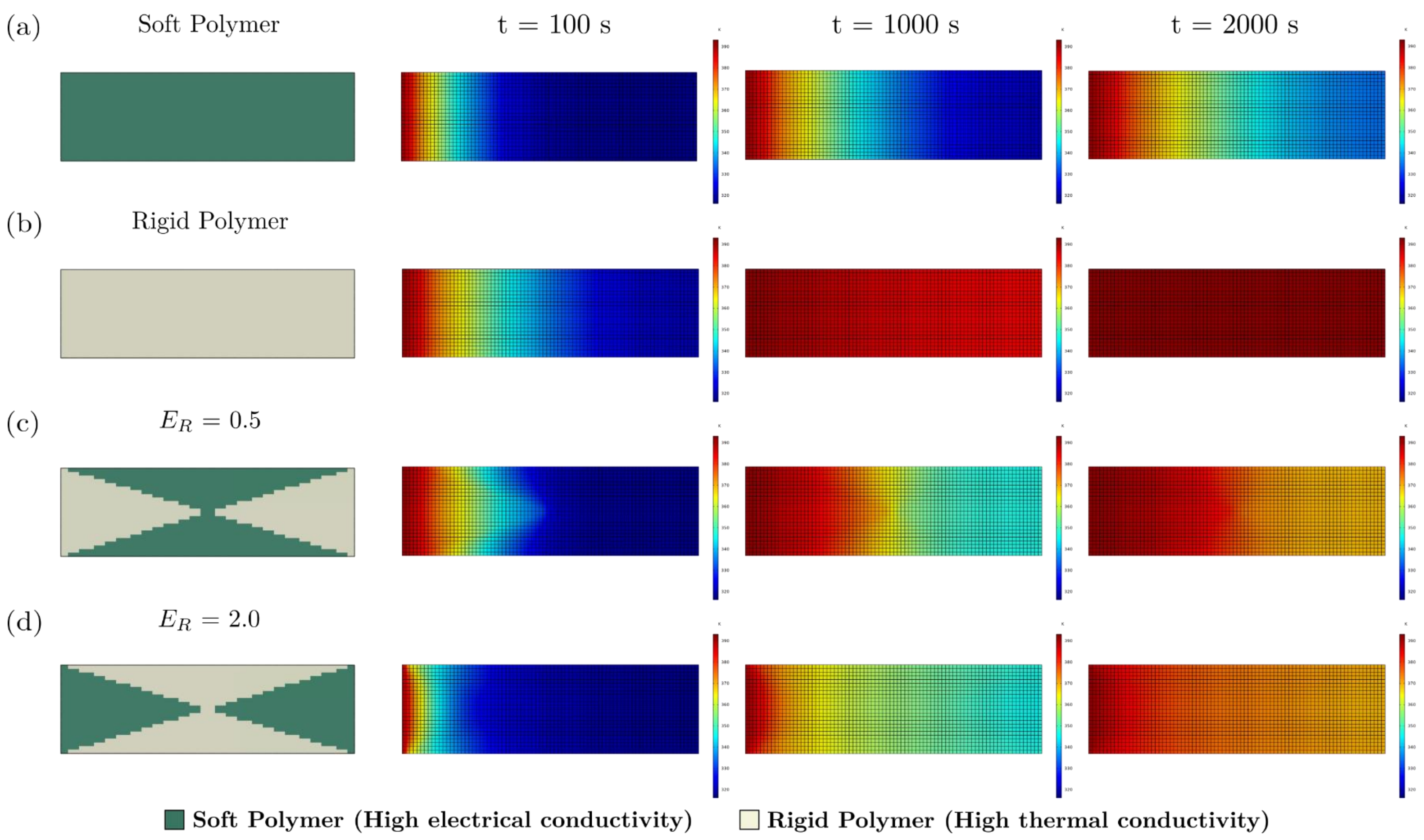
Transient heat transfer:

$$\nabla^2 T + \frac{\dot{q}}{k} = \frac{\rho C_p}{k}\frac{\partial T}{\partial t}$$

Electrical conductivity:

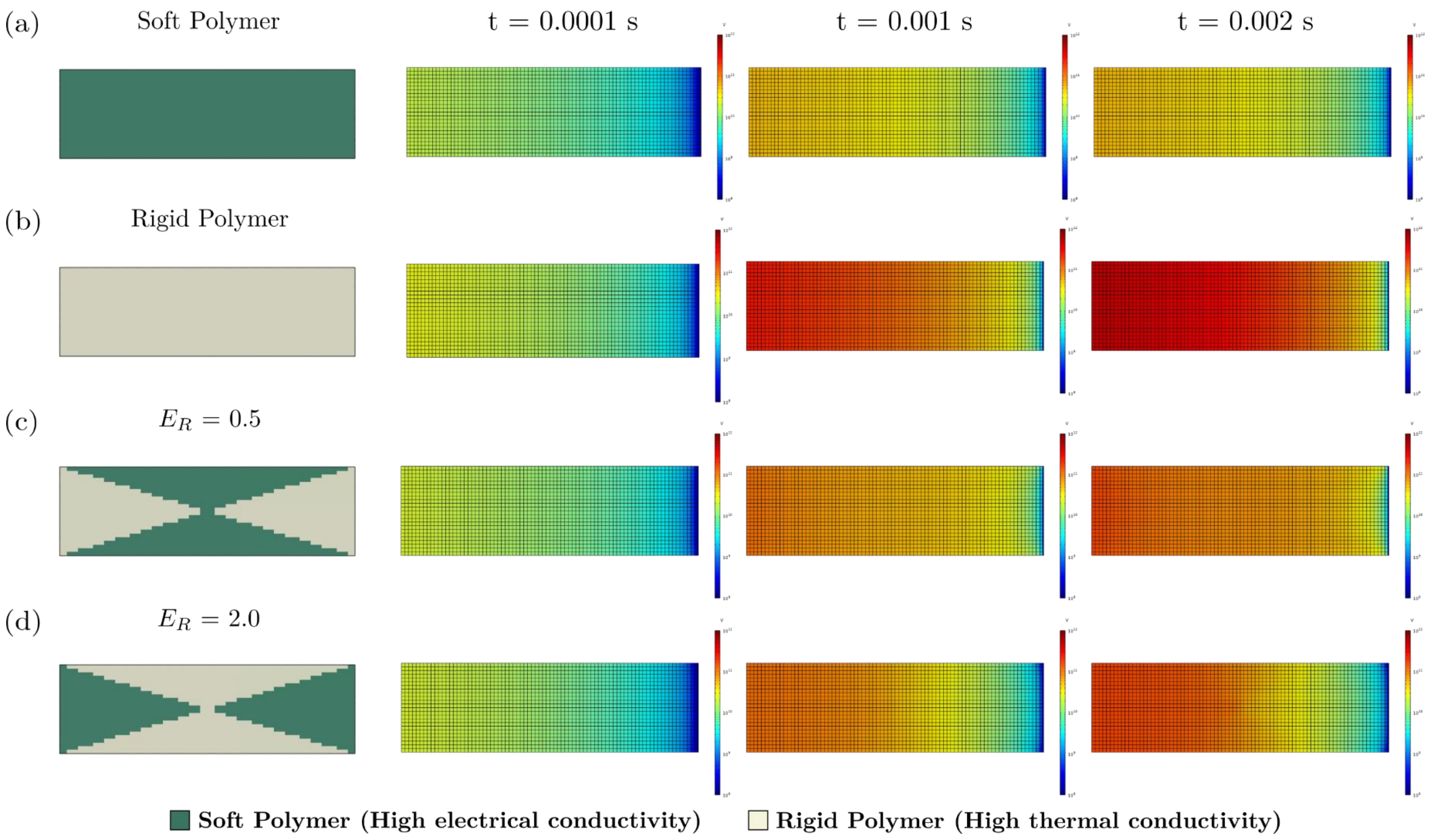
$$R \equiv \frac{V}{I} = \frac{1}{\sigma}\frac{L}{A}$$

Rectangle shape: heat transfer properties



- ➢ The temperature profiles vary with different compositional patterns.

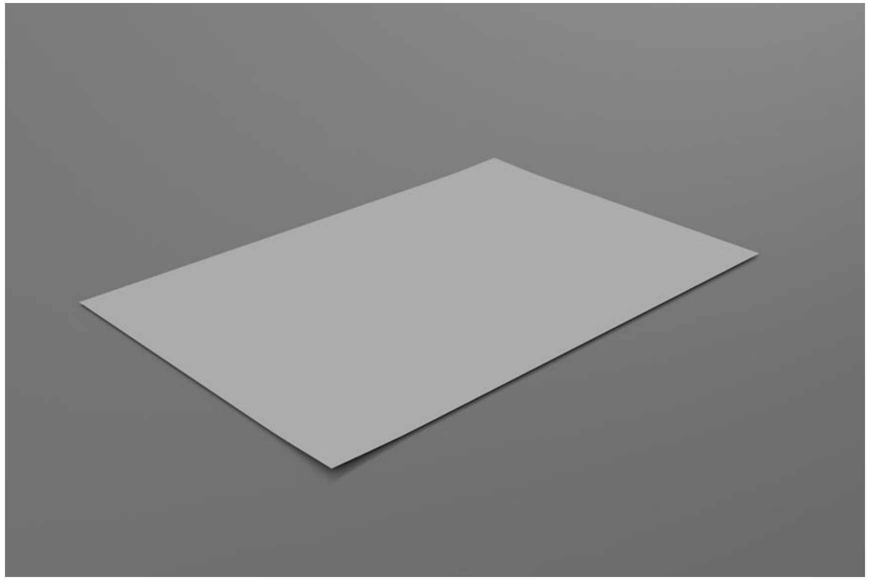
Rectangle shape: electrical conduction properties

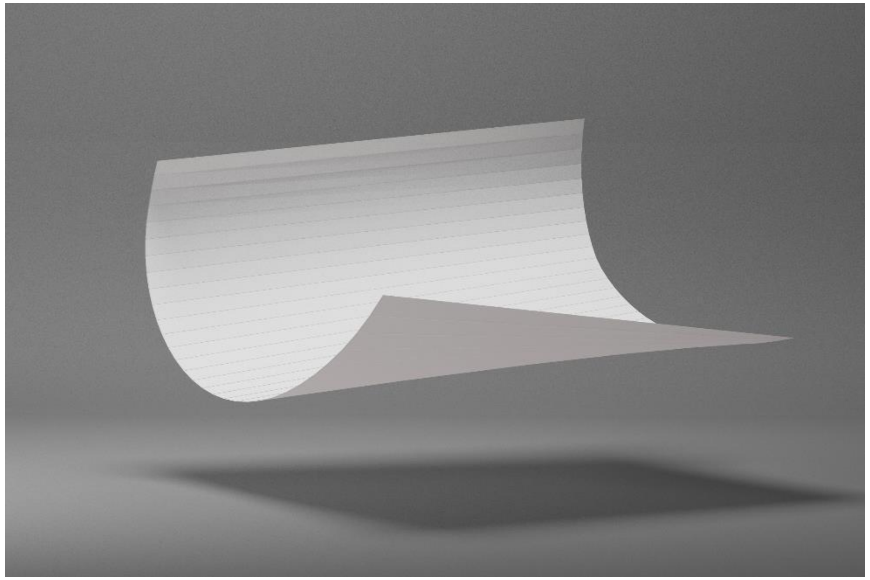


- The electrical potential profiles vary with different compositional patterns.

Some candidate morphing structures:

Shape-morphing structures that transform from flat 2D sheets to 3D are desired due to simplicity in fabrication and transportation (via stacking).



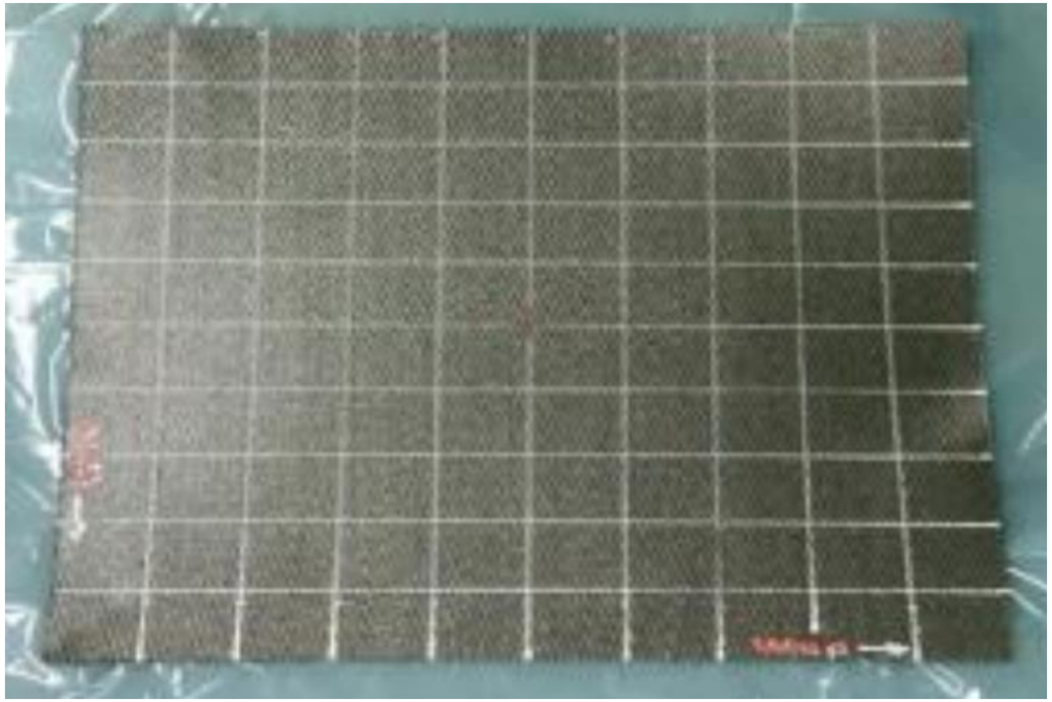


$$\kappa = 0 \quad \xrightarrow{\text{Loading}} \quad \kappa = 0$$

- ☐ **Keep the Gaussian curvature will not cause wrinkling.**

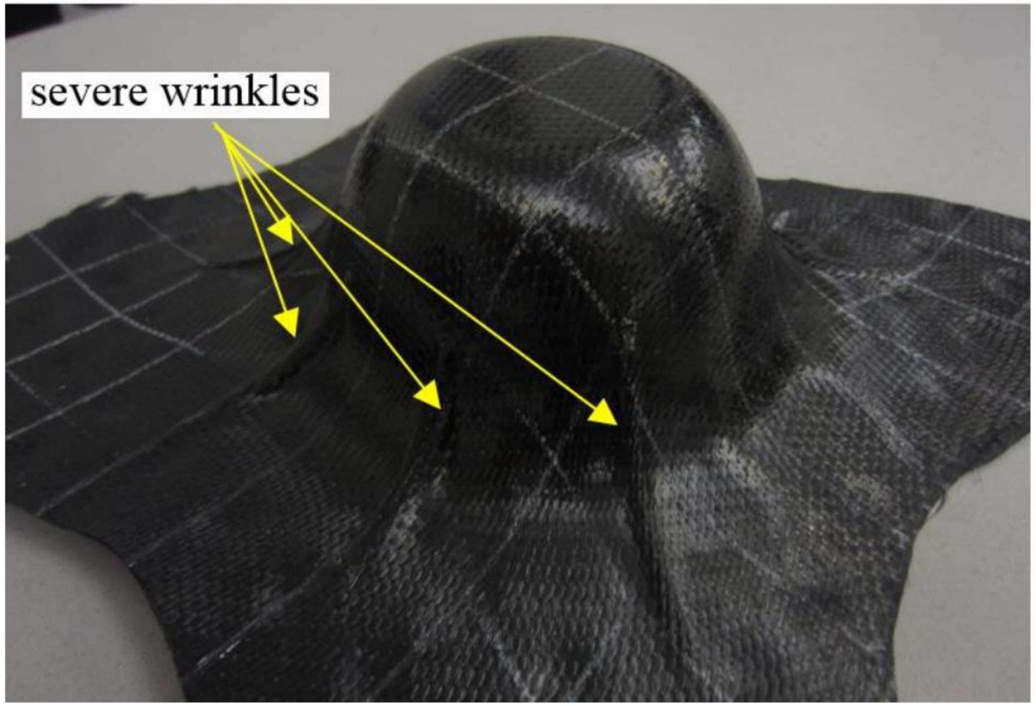
If you change Gaussian curvature:

2D flat fabric



$K = 0$

3D shape after draping



$K > 0$

- **Change the Gaussian curvature will cause severe wrinkling!**

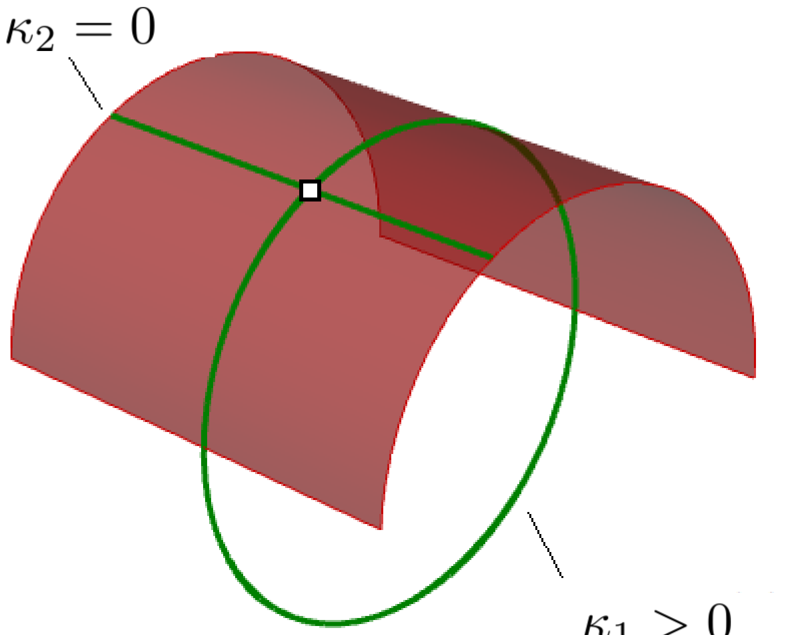
[Gupta et al. 2019]

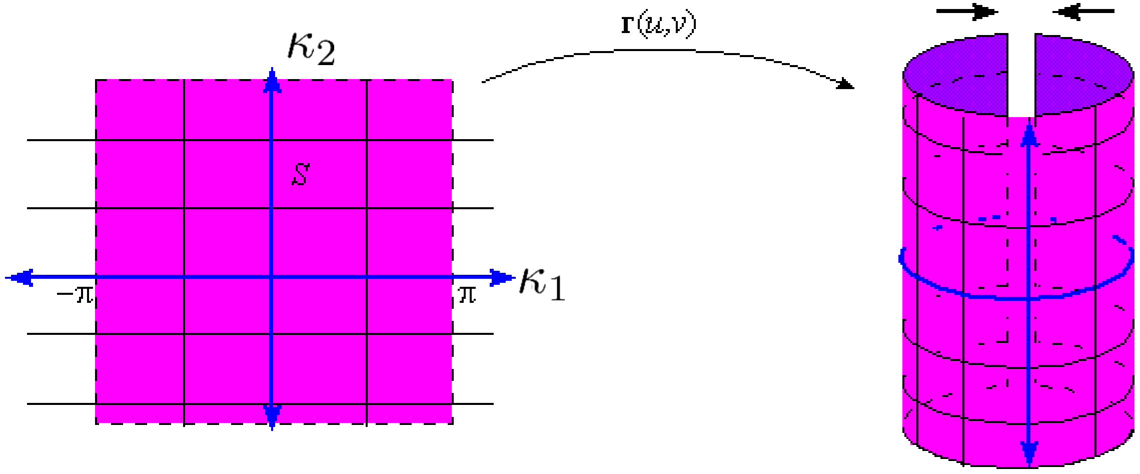
Some candidate morphing structures:

Shape-morphing structures that transform from **flat 2D sheets** to **3D shapes** are desired due to simplicity in fabrication and transportation (via stacking).

Gaussian curvature **[Gauss, 1827]**

$$K = \kappa_1 \kappa_2$$





$$K = \kappa_1 \kappa_2 = 0$$

$$K = \kappa_1 \kappa_2 = 0$$

- ☐ **The Gaussian curvature of a surface is invariant under local isometry.**