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# A SEMI-ANALYTICAL METHOD FOR MEASURING THE STRAIN ENERGY RELEASE RATES OF ELLIPTICAL CRACKS- A NOVEL FINITE FRACTURE MECHANICS-BASED FAILURE CRITERION APPLICATION

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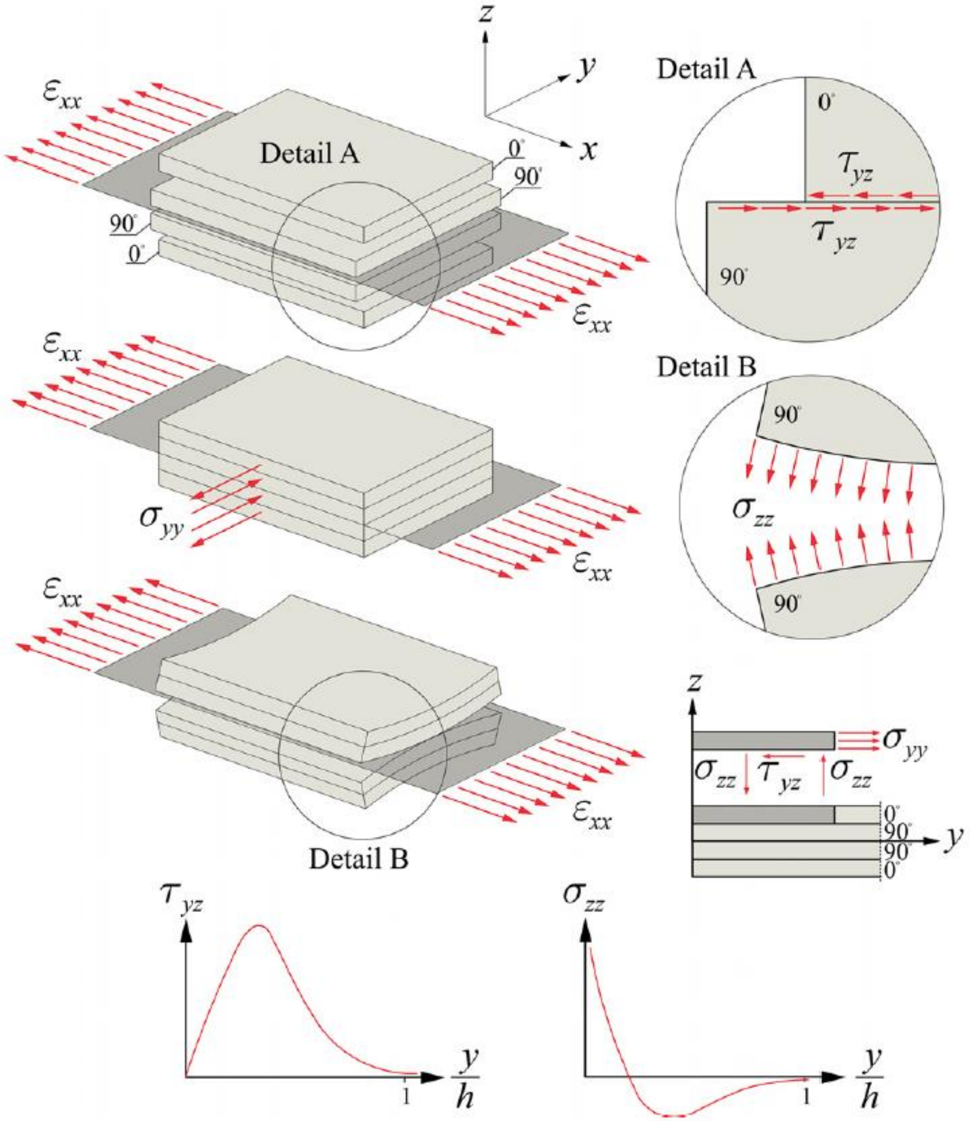
# BACKGROUND OVERVIEW & MOTIVATION

- Background (Free-edge effect):

Potential singular localized three-dimensional stress fields arise in the vicinity of interfaces between dissimilar laminate layers.

- Motivation:

1. Although free-edge effect has been under scientific investigation for more than 5 decades, it remains an open problem to predict the stress concentrations and associated delamination accurately.
2. Numerous closed-form, semi-analytical, and numerical methods have been proposed, however, each have documented either limitations or are computationally expensive.




Free-edge effect [1]



[1] Mittelstedt C, Becker W, Kappel A, Kharghani N. Free-Edge Effects in Composite Laminates-A Review of Recent Developments 2005-2020. Appl Mech Rev 2022;74:1–18

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# AIMS & OBJECTIVES

- To develop a method to obtain interfacial fracture parameters in a symmetric/hybrid stacked laminates.

- Utilising the above parameters to propose a failure criterion based on Finite Fracture Mechanics (FFM).

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# INTERFACIAL STRAIN ENERGY RELEASE RATE

- Preliminary dimensional analysis for a general two-ply bi-material system.

$$\mathcal{G} = \mathcal{G}\left(\begin{matrix}(\sigma_\infty),(a,b,location),(\phi),(h,H,W,L),\\(E_1^s,E_2^s,G_{12}^s,\upsilon_{12}^s,\upsilon_{23}^s),(E_1^m,E_2^m,G_{12}^m,\upsilon_{12}^m,\upsilon_{23}^m)\end{matrix}\right)$$

$$\mathcal{G}_i(\omega,\mu,\eta,\alpha,\beta,\phi) = \frac{{\sigma_\infty}^2 h}{E'}\psi_i^2(\omega,\mu,\eta,\alpha,\beta,\phi) \quad i \in \{x,y,z\}$$

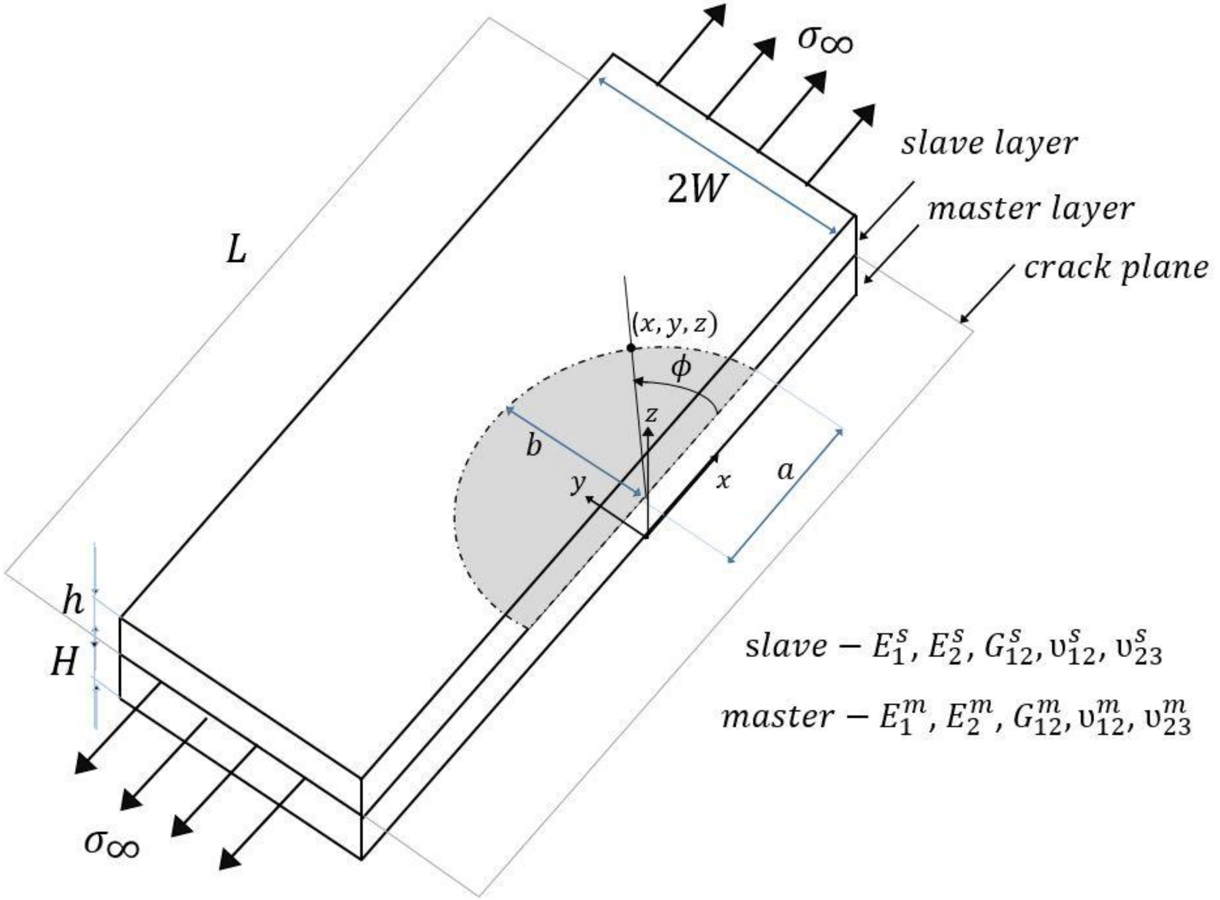
Where $\psi_i$ is non-dimensional correction factor,

$\omega = \mathfrak{I}_m/\mathfrak{I}^*$, $\mu = \mathfrak{I}_s/\mathfrak{I}^*$ ← normalised material parameters (*traces* [2])

$\mathfrak{I}^*$ ← reference value (average 3D *trace* of CFRP considered)

$E' = \mathfrak{I}_m.\omega/\mu$ ← equivalent modulus

$$C = \begin{bmatrix} C_{xx} & C_{xy} & C_{xy} & 0 & 0 & 0 \\ C_{yx} & C_{yy} & C_{yz} & 0 & 0 & 0 \\ C_{yx} & C_{zy} & C_{yy} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2C_{ss} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2C_{ss} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2C_{zz} \end{bmatrix}$$




$\alpha = a/H$, $\beta = b/H$ ← normalised crack semi-axes

$\eta = H/h$ ← normalised geometric parameter

$\phi$ ← polar angle along the crack front

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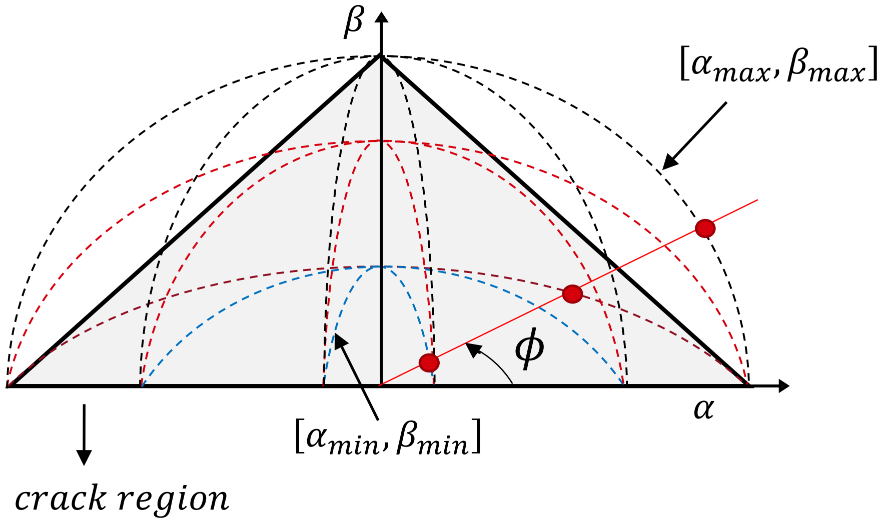
[2] Tsai SW, Melo JDD. An invariant-based theory of composites. Compos Sci Technol 2014;100:237–43. https://doi.org/10.1016/j.compscitech.2014.06.017.

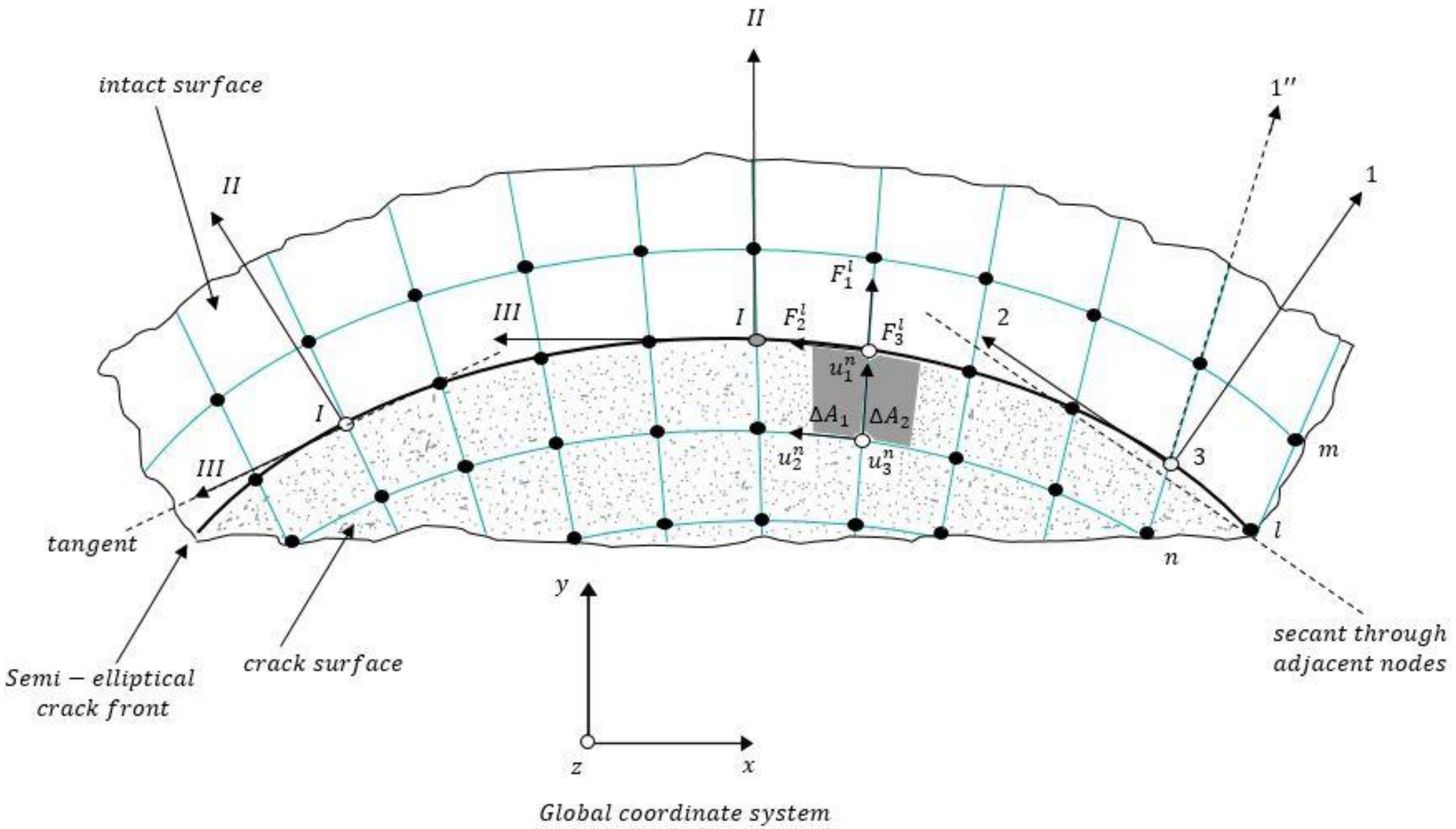
# MODE-MIXITY ALONG CRACK FRONT

- For a given material $(\omega, \mu)$ and thickness $\eta$ parameter, mode-mixity ratios are dependant on polar angle $\phi$ & crack profile $(\alpha, \beta)$:

$$M_{II}(\phi, \alpha, \beta) = \frac{\mathcal{G}_{II}}{\mathcal{G}_{I} + \mathcal{G}_{II} + \mathcal{G}_{III}}$$

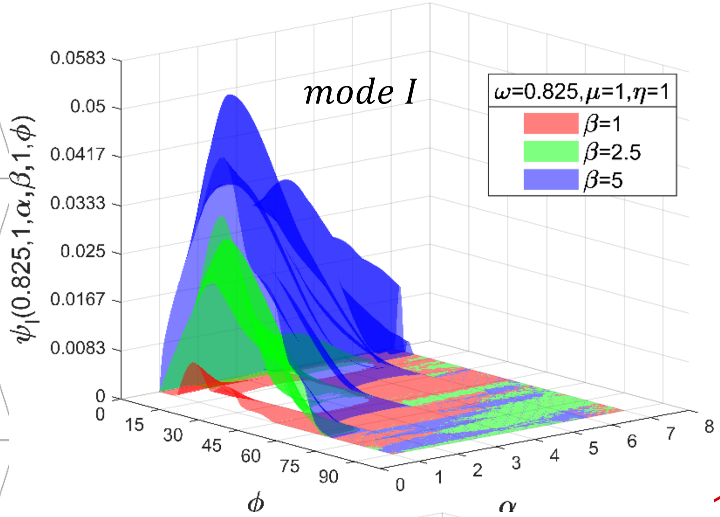
$$M_{III}(\phi, \alpha, \beta) = \frac{\mathcal{G}_{III}}{\mathcal{G}_{I} + \mathcal{G}_{II} + \mathcal{G}_{III}}$$

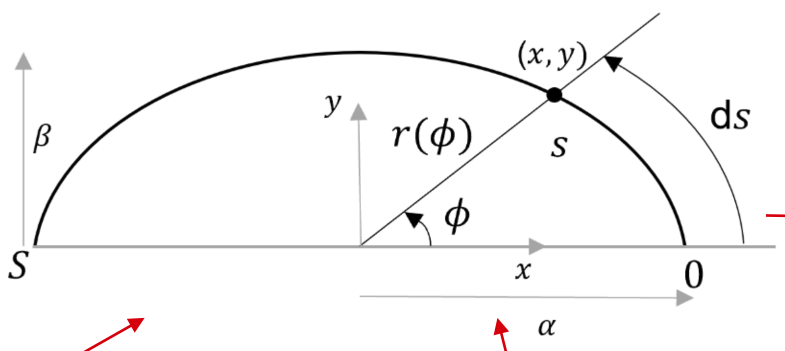


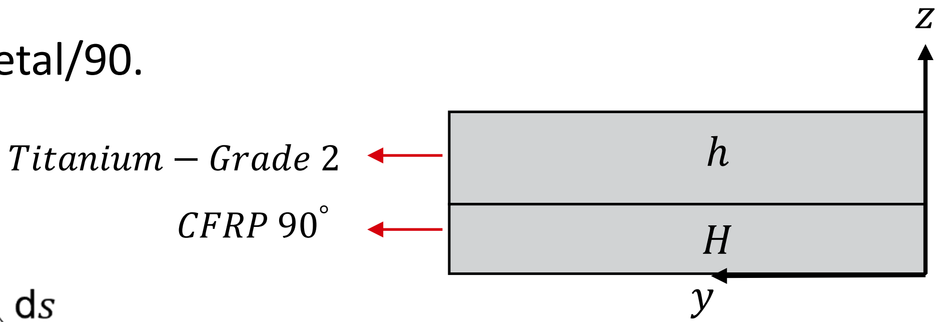



# Influence of crack parameters $\alpha$ and $\beta$ on norm. ERR

- ERR distribution is computed using 3D-VCCT (Abaqus in-built) for metal/90.

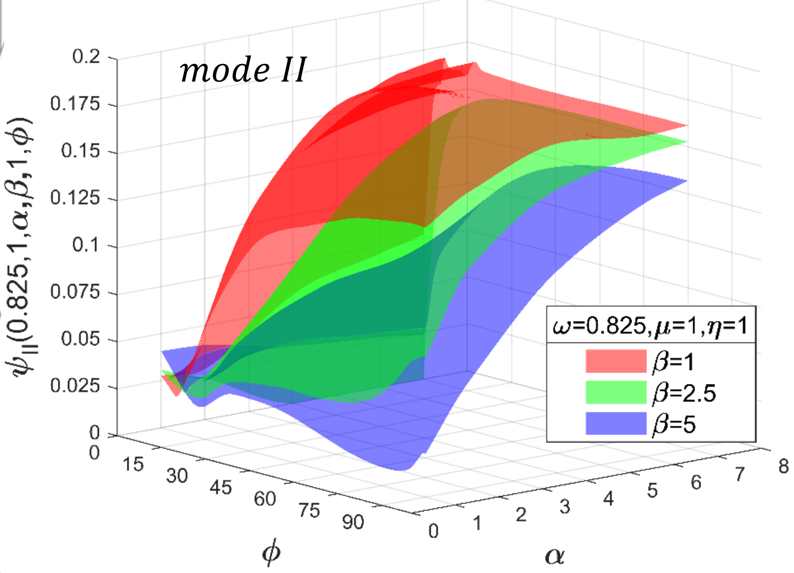


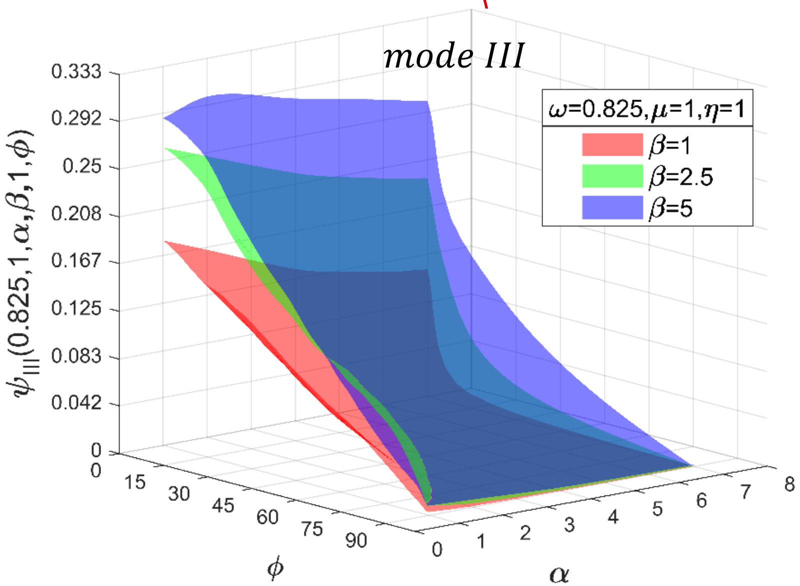



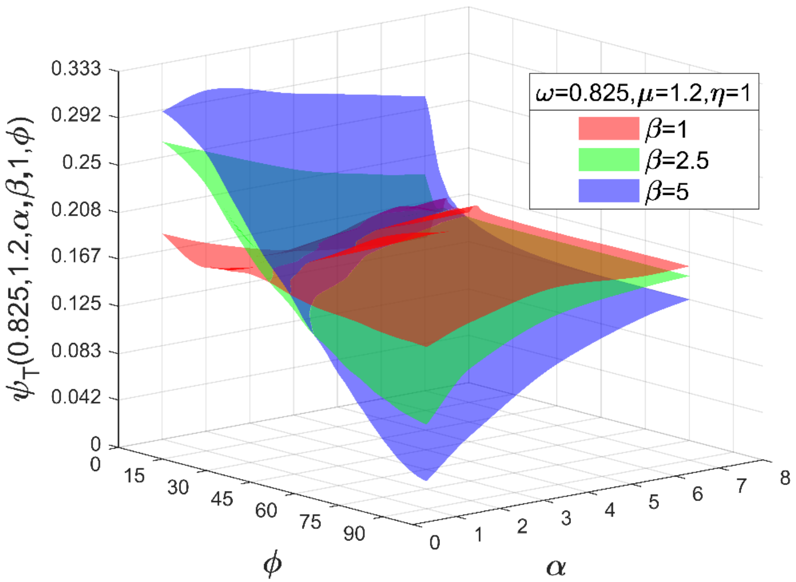




Dependence of norm. ERRs on $\alpha$ and $\beta$ is due to not only crack profile but ply orientation also.

$$\psi_T^2 = \psi_1^2 + \psi_2^2 + \psi_3^2$$



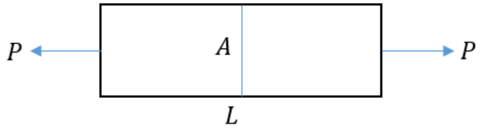



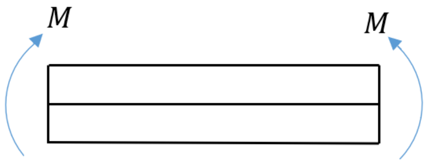



# FRACTURE CRITERION SPECTRUM

## Strength of materials

No stress singularity

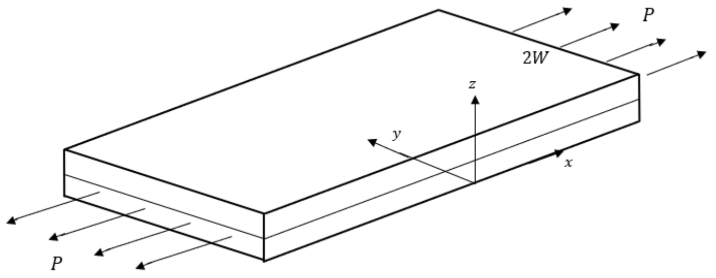


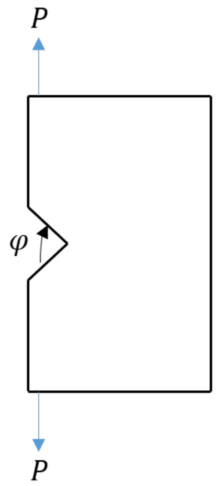




- When stresses are greater than the strength

$$f(\sigma_{ij}(x_i),\sigma_c) \geq 1$$

## Weak stress singularity and absence of a crack (flaw)



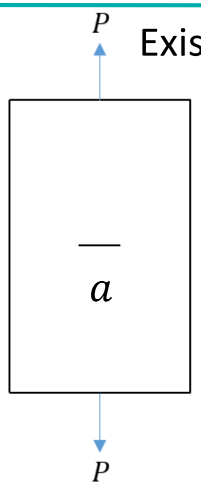

- Strength of materials and energy based fracture mechanics prevail paradoxical results due to singular behaviour of stresses and absence of physical crack.

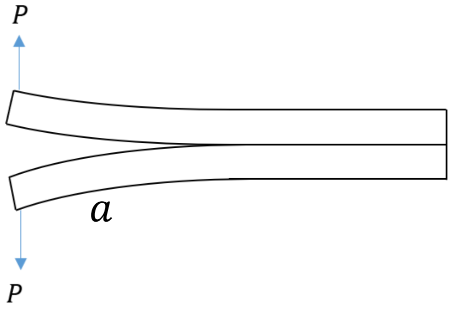
**Finite Fracture Mechanics** (FFM)[3]: Both conditions are necessary and together as one they become sufficient for a crack onset.

$$f(\sigma_{ij}(x_i),\sigma_c) \geq 1 \quad \wedge \quad h(\mathcal{G}_g(\Delta a),\mathcal{G}_c) \geq 1$$

## Fracture mechanics

Existence of a crack (flaw)




- When energy release rate is greater than the fracture toughness

$$h(\mathcal{G}_g(\Delta a),\mathcal{G}_c) \geq 1$$

[3] Leguillon D. Strength or toughness? A criterion for crack onset at a notch. Eur J Mech A/Solids 2002;21:61–72.

# INCREMENTAL ENERGY RELEASE RATE (IERR)

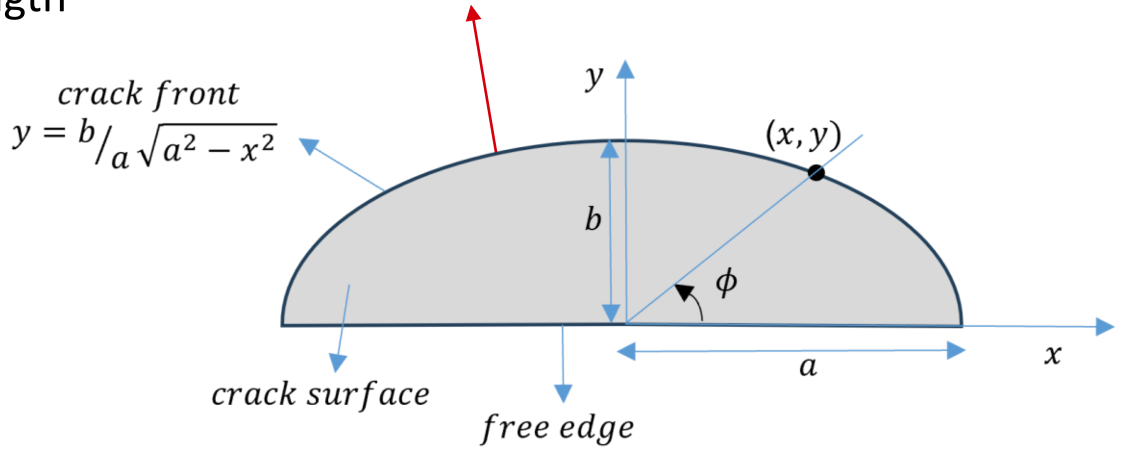
- Grifft h's ERR: differential form

$$\mathcal{G} = -\frac{d\pi}{da}$$ where a is crack length

- Incremental ERR: incremental form

$$\bar{\mathcal{G}} = -\frac{\Delta\pi}{\Delta a}$$

Semi-elliptical crack front




- Therefore, $\bar{\mathcal{G}}_i$ can be calculated from integral average of $\mathcal{G}_i$

$$\bar{\mathcal{G}}_i = \frac{1}{\Delta a}\int_0^{\Delta a} \mathcal{G}_i \, da = \frac{{\sigma_\infty}^2 h}{E'} \frac{1}{A} \iint_0^A {\psi_i}^2 da = \frac{{\sigma_\infty}^2 h}{E'} \Lambda_i$$

$\Lambda \longrightarrow$ norm. IERR

$\bar{\mathcal{G}}_i$ is IERR for mode $i$, gives energy rate required to nucleate a crack of area $A$(area of semi-ellipse).

# INTERLAMINAR STRESSES COMPUTATION

- Interlaminar stresses computed using Abaqus for a general two-ply bi-material system :

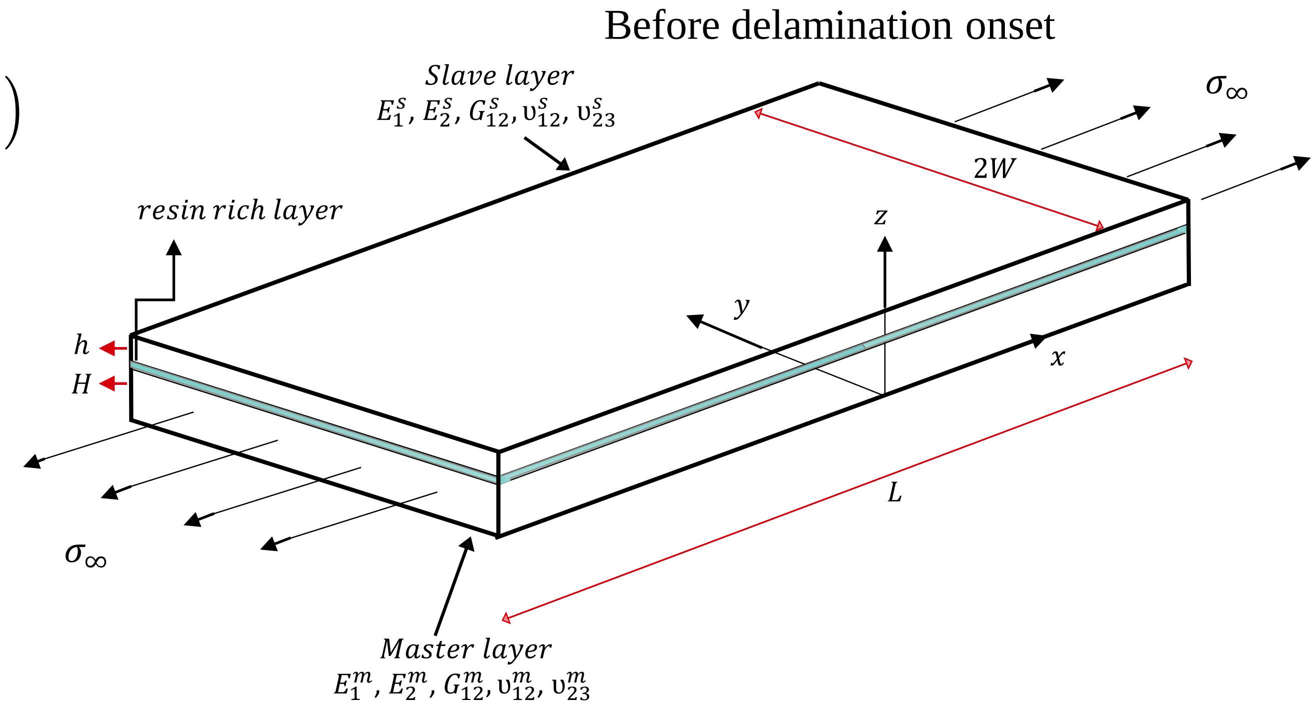
Dimensional analysis

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}\left(\begin{matrix}(\sigma_\infty), (y), (H, h), \\ (E_1^s, E_2^s, G_{12}^s, \upsilon_{12}^s, \upsilon_{23}^s), (E_1^m, E_2^m, G_{12}^m, \upsilon_{12}^m, \upsilon_{23}^m)\end{matrix}\right)$$

$$\sigma_{iz}(\omega, \mu, \eta, y') = \sigma_\infty\, \chi_{iz}(\omega, \mu, \eta, y') \quad i \in \{x, y, z\}$$

where $\chi_{iz}$ is non-dimensional stress functions.

$$y' = y/t$$ ← normalised geometrical parameter

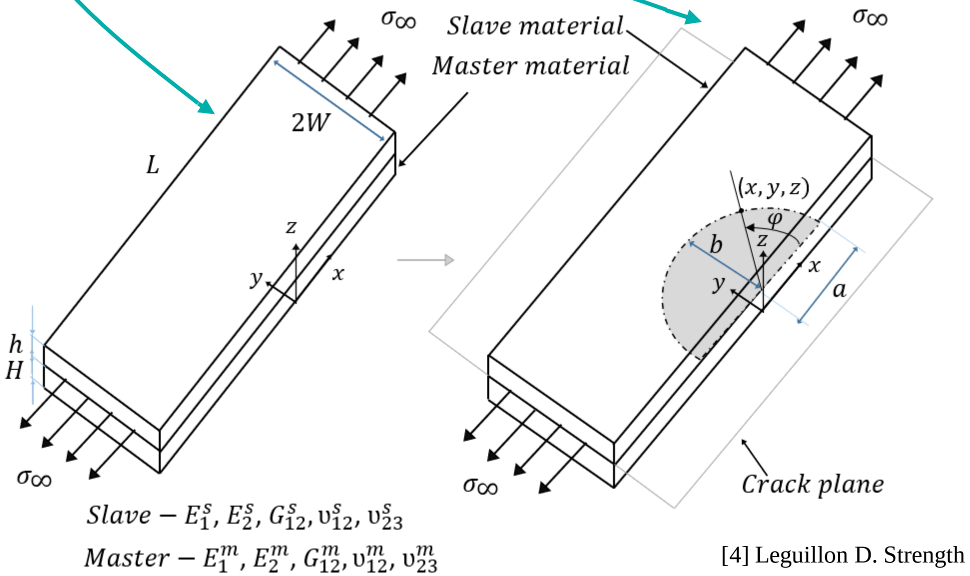



- Resin layer modelling: 2% of $H$ as thickness (revealed parametric study on angle-ply laminate in comparison with literature).

# FINITE FRACTURE MECHANICS (FFM) CRITERION

- FFM coupled criterion [4]: $f(\sigma_{ij}(x_i), \sigma_c) \geq 1 \quad \wedge \quad h(\mathcal{G}_g(\Delta a), \mathcal{G}_c) \geq 1$

$$\begin{cases} \dfrac{2\sigma_\infty}{\pi ab}\displaystyle\int_{-a}^{a}\int_{0}^{b/a\sqrt{a^2-x^2}} \sqrt{\left(\frac{\langle \chi_{zz}\rangle}{X_z}\right)^2 + \left(\frac{\chi_{zy}}{S_y}\right)^2 + \left(\frac{\chi_{zx}}{S_x}\right)^2}\, dy\, dx \geq 1 & \longrightarrow \textit{Stress criterion} \\ \dfrac{2{\sigma_\infty}^2 h}{\pi ab\, E'} \geq \dfrac{\mathcal{G}_c}{\int_{-a}^{a}\int_{0}^{b/a\sqrt{a^2-x^2}} \left({\psi_I}^2 + {\psi_{II}}^2 + {\psi_{III}}^2\right) dy\, dx} & \longrightarrow \textit{Energy criterion} \end{cases}$$




| Identify | Description |
| --- | --- |
| 2 non-linear integral system of equations | 3 variables: $\sigma_\infty, a, b$ |
| Interfacial intrinsic parameters (strength & fracture toughness) | $X_z, S_y, S_x;$ $\mathcal{G}_c,$ |
| Numerically calculate non-dimensional functions | $\chi_{zz}, \chi_{zy}, \chi_{zx};$ $\psi_I, \psi_{II}, \psi_{III}$ |

[4] Leguillon D. Strength or toughness? A criterion for crack onset at a notch. Eur J Mech A/Solids 2002;21:61–72.

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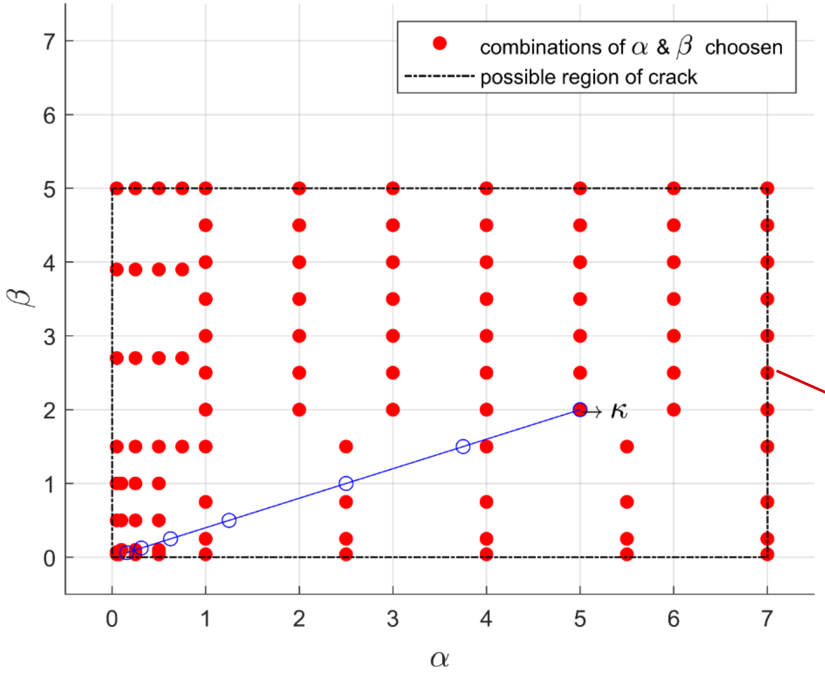
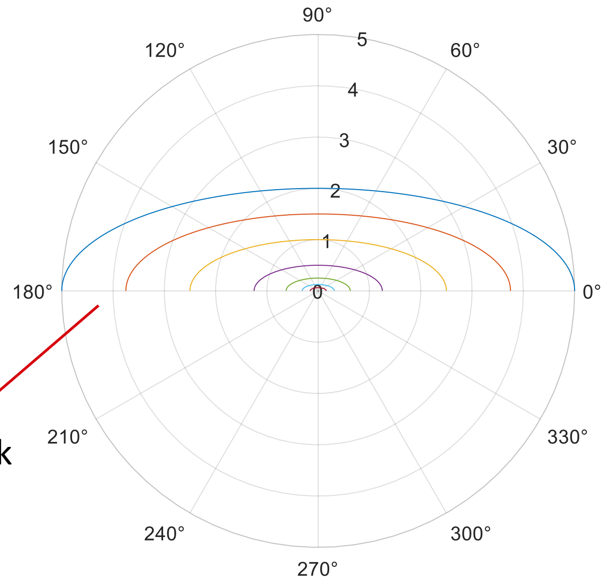
# HYPOTHESIS: HOMOTHETIC CRACK EXTENSION

- Homothetic crack extension: aspect ratio of crack constant




Homothetic Crack extension path

- 3D-interface semi-elliptical edge crack leads to optimization problem:

Obj function:

$$F_f = \min_{F,\Delta A} \begin{Bmatrix} F \mid f\left(\sigma_{ij}(x_i), S_x, S_y, X_z\right) \geq 1 \quad \forall \quad x_i \in \Omega_c \\ \wedge \quad h\left(\mathcal{G}_g(\Delta A), \mathcal{G}_{Ic}, \mathcal{G}_{IIc}, \mathcal{G}_{IIIc}\right) \geq 1 \end{Bmatrix}$$

Constraints:

$$\sigma_\infty\left(\sigma_{ij}(x_i), S_x, S_y, X_z\right) = \sigma_\infty\left(\mathcal{G}_g(\Delta A), \mathcal{G}_{Ic}, \mathcal{G}_{IIc}, \mathcal{G}_{IIIc}\right)$$

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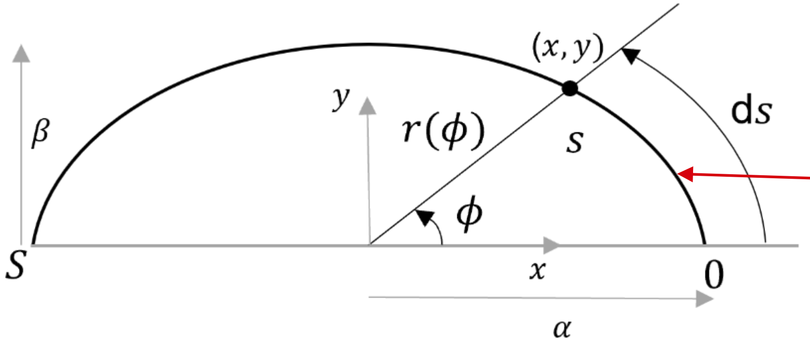
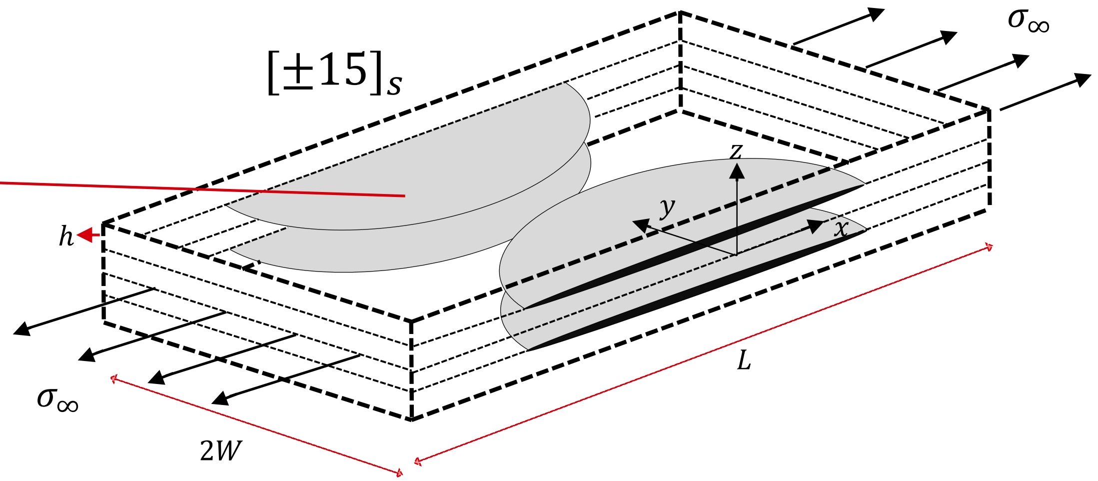
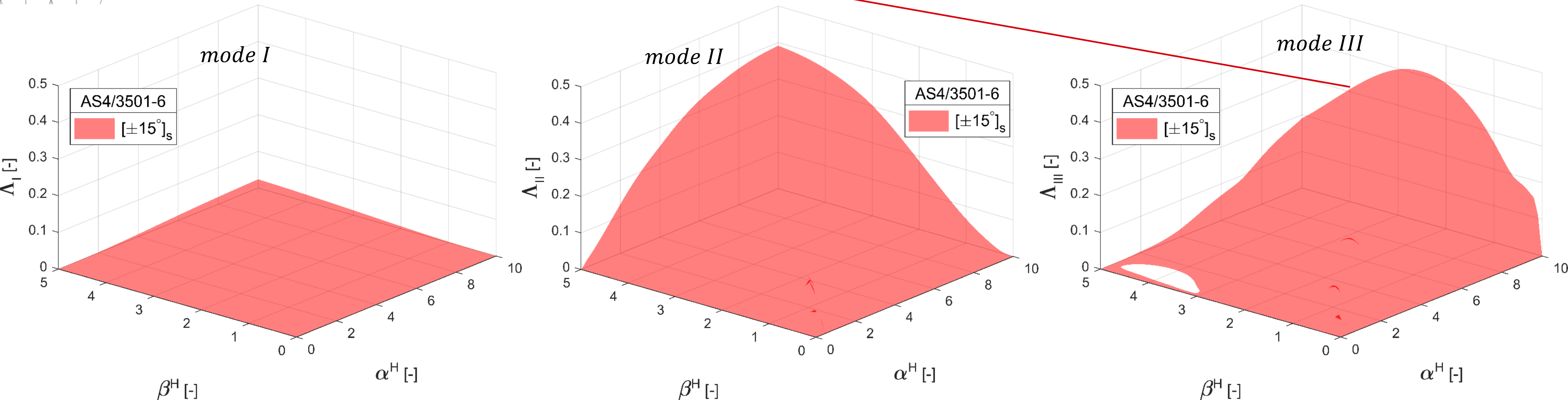
# NORM. IERR OF HERCULES AS1/3501-6



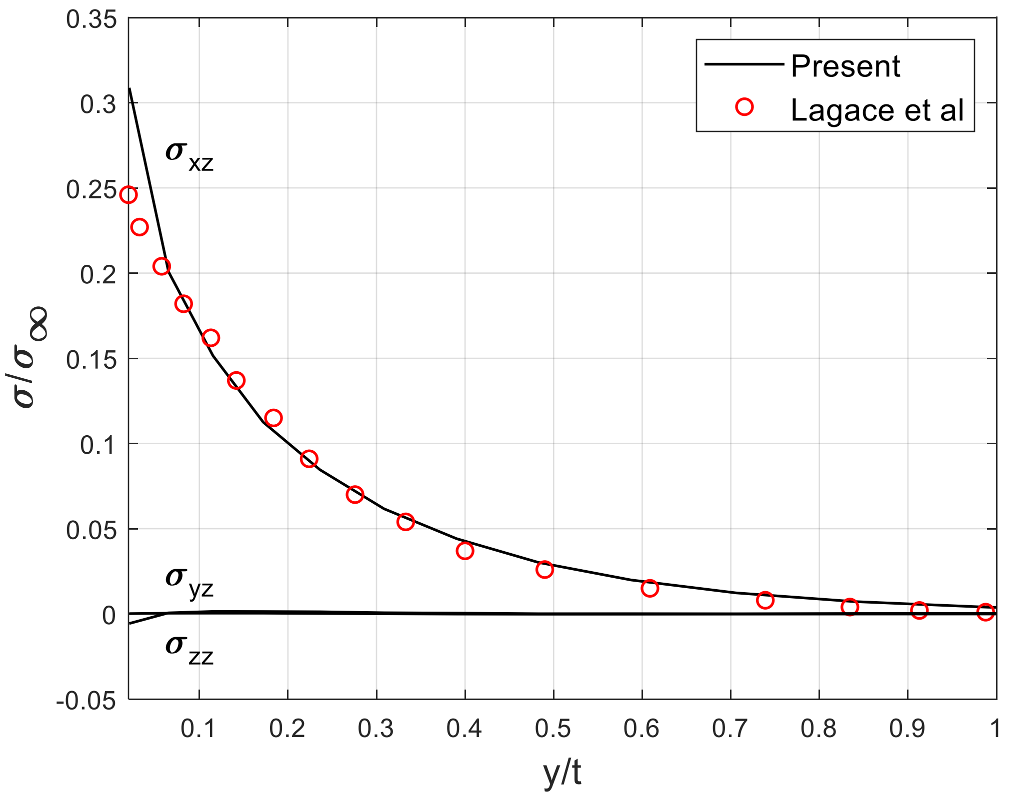



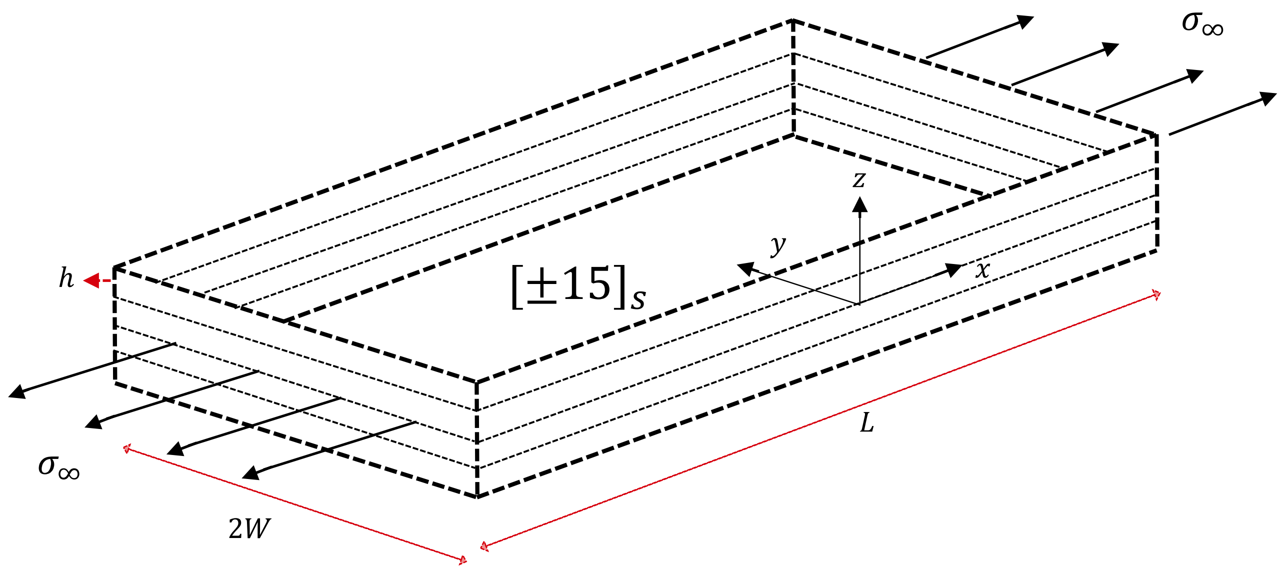

Dependence of ERRs on $\alpha$ and $\beta$ is due to the crack profile and ply orientation



# INTERLAMINAR STRESSES OF HERCULES AS1/3501-6

- Weak singularity is observed at the free edge in interlaminar shear stress.

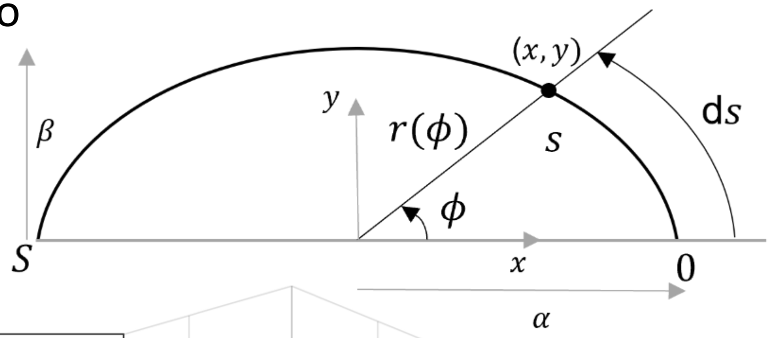


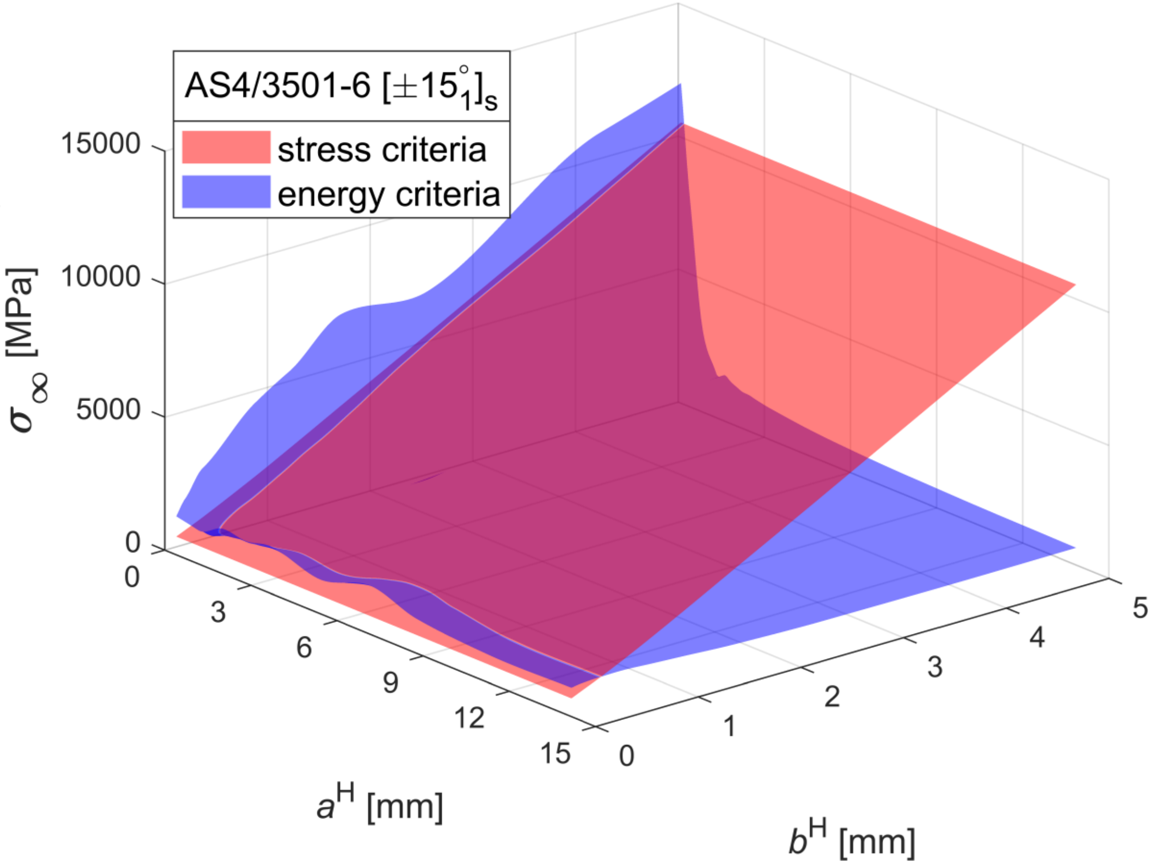



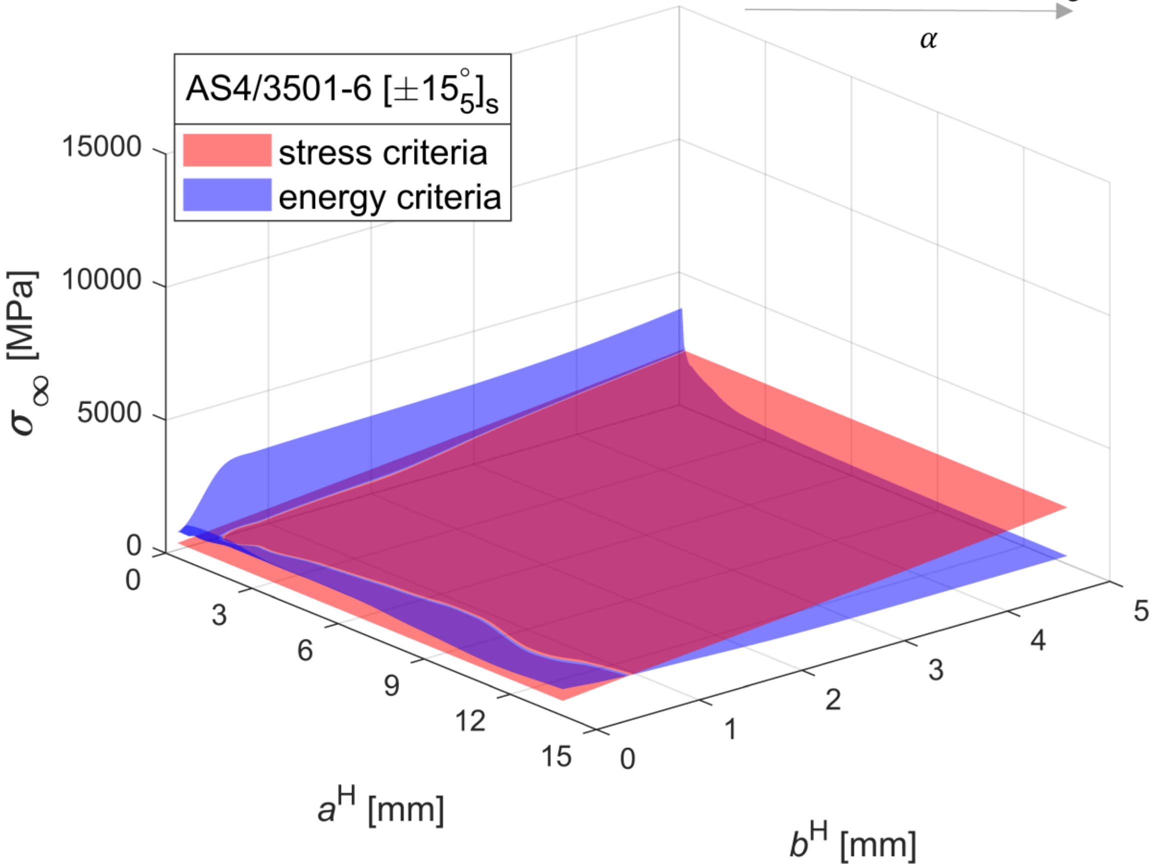

[5] Lagace P, Brewer J, Kassapoglou C. Effect of Thickness on Interlaminar Stresses and Delamination in Straight-Edged Laminates. J Compos Technol Res 1987;9:81–7

# FAILURE PREDICTION of AS1/3501-6

- Failure load is where stress and energy criterion meets which correspond to minimum load.



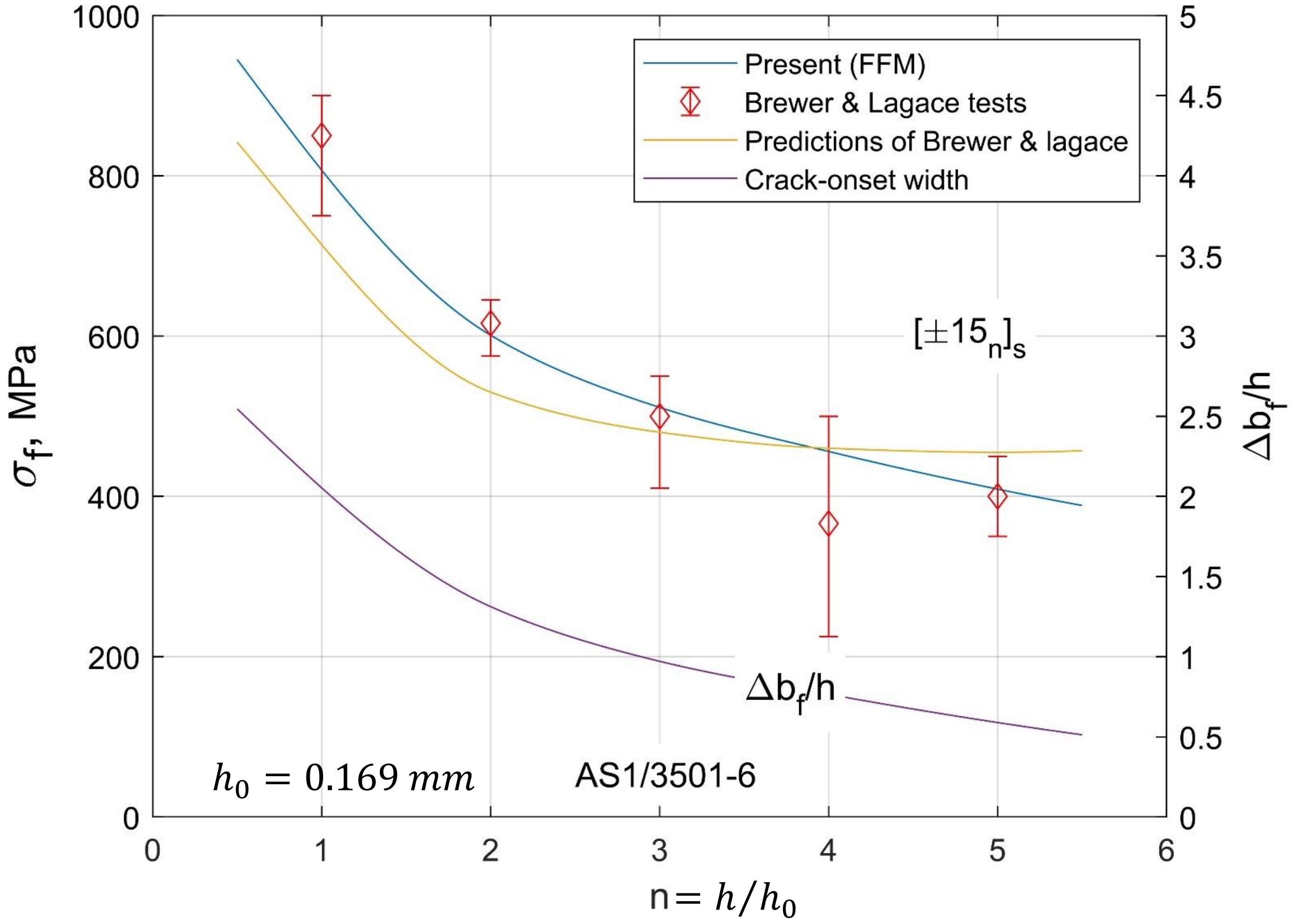



# Validation of FFM criterion of AS1/3501-6

- FFM predicts precisely the delamination onset evolution within the error bars of tests [6] for each ‘n’.

- FFM predicts normalised crack onset width $0.5 \leq \Delta b_f/h \leq 2$ close to 1 which is in accordance to [7].





[6] Brewer JC, Lagace PA. Quadratic stress criterion for initiation of delamination. J Compos Mater 1988;22:1141–55.
[7] Diaz AD, Caron J-F. Prediction of the onset of mode III delamination in carbonepoxy laminates. Compos Struct 2006;72(4):438–45.

# SUMMARY

- Conclusions
  - A method of calculating ERR distribution of semi-elliptical cracks was developed by using 3D-VCCT.
  - Mode-mixity was studied and it was observed that it not only is influenced by polar angle along the crack front but crack profile as well.
  - Similar approach was performed for interlaminar stresses to develop a FFM criterion.
  - Validation of FFM criterion with experimental values indicated very close agreement.

- Future Work
  - Extend current FFM criterion to fatigue scenario (Experimental campaign).

# THANK YOU

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