SHELL-BASED COMPUTATIONAL HOMOGENISATION WITH IMMERSED METHODS FOR MESO-SCALE MODELLING OF FIBRE REINFORCED COMPOSITES

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Contents

- Immersed methods
  - Application to meso-scale models of fibre composites
- Homogenisation of shells/plates
- Results
- Conclusions

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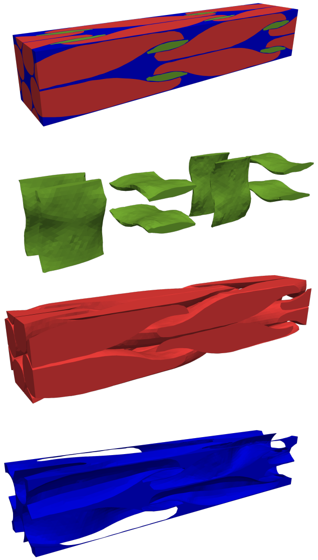
Meso-scale models of advanced fibre composites

Why meso-scale models?

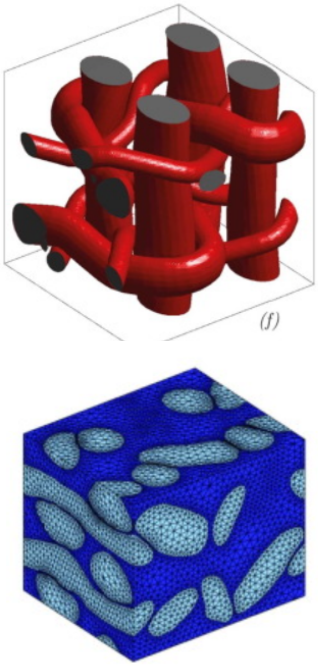
- Estimating stiffness properties
  - Parameter study
- Monitor stresses
- Study damage mechanisms and evolution

Challenges

- Generating bundle geometry
  - Interpenetration
- Discretisation
  - Poor mesh quality/condition number
  - Manual/Semi-automatic meshing
  - Voxel methods



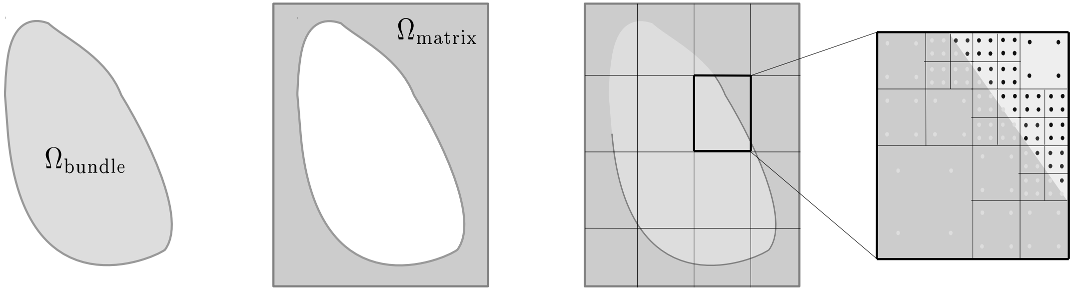
Stig et al. (2012)



Wintiba et al. (2017)

Investigation of Immersed methods!

Immersed methods?



Immersed methods: Finite cell method, Cut-FEM, Immersogeometric, c.f. *Rank et al. (2012)*

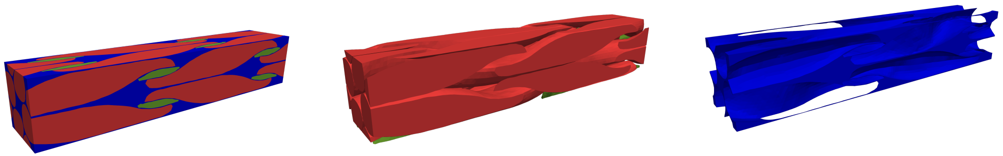
+ Benefits

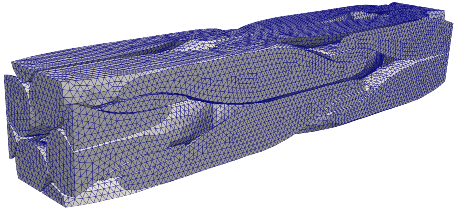
- Fully automatic discretisation of the matrix

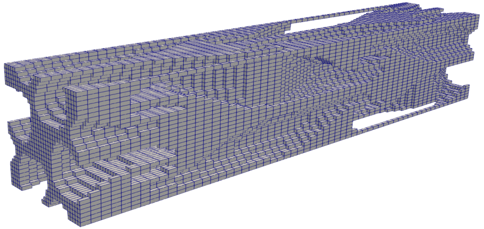
- Drawbacks

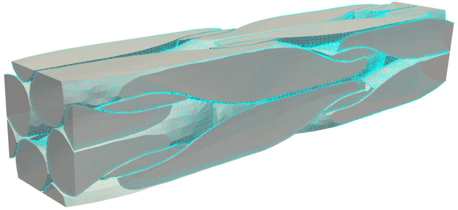
- Many quadrature points (~50-100M quadrature points for large 3d problems)
- Elements with low volume fraction (ill-conditioned matrix system)
  - (solved with stabilization methods)

Prototype - Immersed methods for 3D woven composite









Bundles
Boundary-fitted mesh with e.g FE-elements/IGA.

Matrix
Immersed method

Interface
Couple with e.g. penalty or Nitsche’s method

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Multi-scale modelling with shells/plates

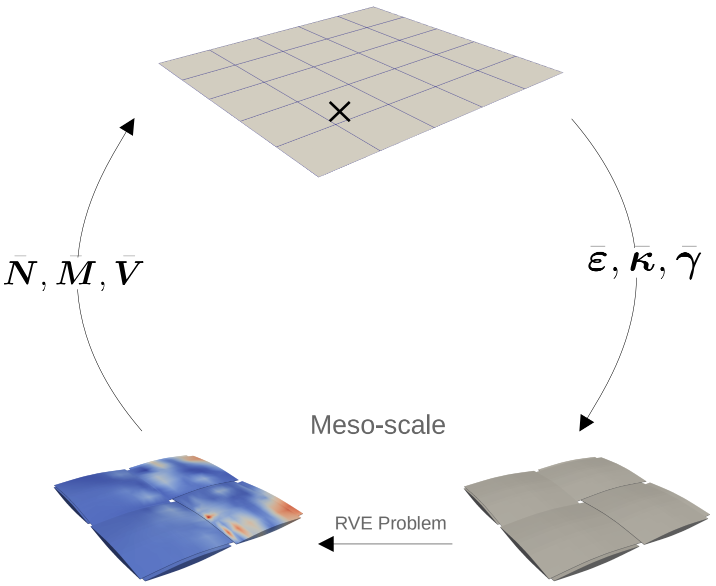
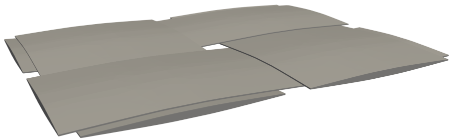
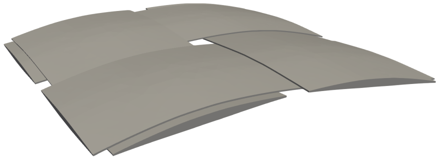


Figure. *Illustration of homogenisation of plates.*

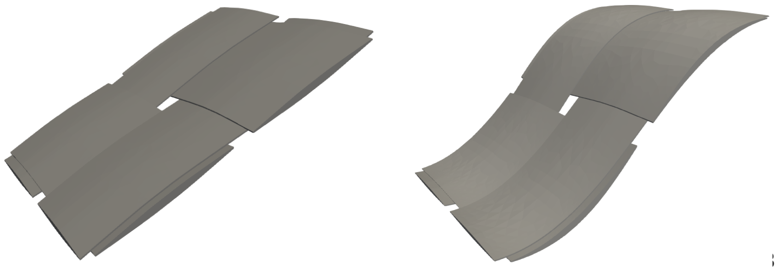
Membrane $\bar{\boldsymbol{\varepsilon}}$



Bending $\bar{\boldsymbol{\kappa}}$



Transverse shear $\bar{\boldsymbol{\gamma}}$



Multi-scale modelling with plates

Variational consistent homogenisation

A framework for deriving the two scale formulation from the “fully resolved form”.

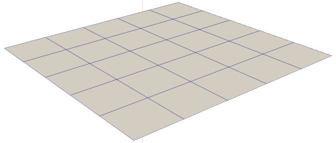
- $\int_\Omega \boldsymbol{\sigma} : \boldsymbol{\varepsilon}(\delta \boldsymbol{u}) \mathrm{d}\Omega = \int_{\Gamma_N} \delta \boldsymbol{u} \cdot \boldsymbol{t}_p \mathrm{d}\Gamma + \int_\gamma \delta \boldsymbol{u} \cdot \boldsymbol{t}_p \mathrm{d}\Gamma$
- $\boldsymbol{u} = \boldsymbol{u}^M + \boldsymbol{u}^s$
- $\boldsymbol{u}^{RM} = \bar{\boldsymbol{u}} + \bar{w}\boldsymbol{e}_3 - z\bar{\boldsymbol{\theta}}$
- First order homogenisation
- ...

Macro-scale formulation

$$\int_A \bar{\boldsymbol{N}} : \delta\bar{\boldsymbol{\varepsilon}} + \bar{\boldsymbol{V}} \cdot (\delta\bar{\boldsymbol{g}} + \delta\bar{\boldsymbol{\theta}}) - \bar{\boldsymbol{M}} : \delta\bar{\boldsymbol{\kappa}} \; \mathrm{d}A = \text{external forces}$$

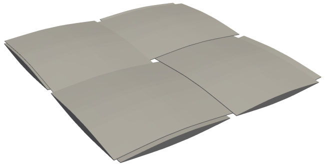
$$\bar{\boldsymbol{N}} = \frac{1}{A_\square}\int_{\Omega_\square} \boldsymbol{\sigma} \; \mathrm{d}\Omega \qquad \bar{\boldsymbol{V}} = \frac{1}{A_\square}\int_{\Omega_\square} \boldsymbol{\sigma} \cdot \boldsymbol{e}_3 \; \mathrm{d}\Omega$$

$$\bar{\boldsymbol{M}} = \frac{1}{A_\square}\int_{\Omega_\square} z\boldsymbol{\sigma} + (\boldsymbol{\sigma} \cdot \boldsymbol{e}_3) \otimes (\boldsymbol{x} - \bar{\boldsymbol{x}}) \; \mathrm{d}\Omega$$



Micro-scale formulation

$$\int_{\Omega_\square} \boldsymbol{\sigma} : \boldsymbol{\varepsilon}(\delta \boldsymbol{u}^s) \; \mathrm{d}\Omega = \int_{\Gamma_\square} \boldsymbol{t}_p \cdot \delta \boldsymbol{u}^s \; \mathrm{d}\Gamma$$



$$\bar{\boldsymbol{u}}_\square(\boldsymbol{u}^s) = 0, \quad \bar{\boldsymbol{w}}_\square(\boldsymbol{u}^s) = 0 \qquad \text{(Rigid body constraints)}$$

$$\bar{\boldsymbol{h}}_\square(\boldsymbol{u}^s) = 0, \quad \bar{\boldsymbol{\kappa}}_\square(\boldsymbol{u}^s) = 0, \quad \bar{\boldsymbol{\gamma}}_\square(\boldsymbol{u}^s) = 0 \qquad \text{(Boundary conditions)}$$

$$\bar{\boldsymbol{\theta}}_\square(\boldsymbol{u}^s) = 0 \qquad \text{(Volume constraint)}$$

Multi-scale modelling with plates

Variational consistent homogenisation

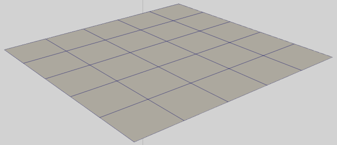
A framework for deriving the two scale formulation from the “fully resolved form”.

- $\int_\Omega \boldsymbol{\sigma} : \boldsymbol{\varepsilon}(\delta \boldsymbol{u}) \mathrm{d}\Omega = \int_{\Gamma_N} \delta \boldsymbol{u} \cdot \boldsymbol{t}_p$
- $\boldsymbol{u} = \boldsymbol{u}^M + \boldsymbol{u}^s$
- $\boldsymbol{u}^{RM} = \bar{\boldsymbol{u}} + \bar{w}\boldsymbol{e}_3 - z\bar{\boldsymbol{\theta}}$
- First order homogenisation
- $\dots$

Macro-scale formulation

$$\int_A \bar{\boldsymbol{N}} : \delta\bar{\boldsymbol{\varepsilon}} + \bar{\boldsymbol{V}} \cdot (\delta\bar{\boldsymbol{g}} + \delta\bar{\boldsymbol{\theta}}) - \bar{\boldsymbol{M}} : \delta\bar{\boldsymbol{\kappa}} \ \mathrm{d}A = \text{external forces}$$

$$\bar{\boldsymbol{N}} = \frac{1}{A_\square} \int_{\Omega_\square} \boldsymbol{\sigma} \ \mathrm{d}\Omega \qquad \bar{\boldsymbol{V}} = \frac{1}{A_\square} \int_{\Omega_\square} \boldsymbol{\sigma} \cdot \boldsymbol{e}_3 \ \mathrm{d}\Omega$$

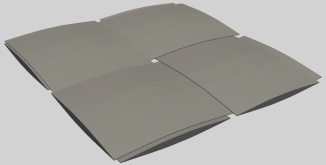


Read more in:

[1] Börjesson, E., Larsson, F., Runesson, K., Remmers, J. J. C., Fagerström, M. Variationally consistent homogenisation of plates, **Computer Methods in Applied Mechanics and Engineering** (2023)

Meso-scale formulation

$$\int_{\Omega_\square} \boldsymbol{\sigma} : \boldsymbol{\varepsilon}(\delta \boldsymbol{u}^s) \ \mathrm{d}\Omega = \int_{\Gamma_\square} \boldsymbol{t}_p \cdot \delta \boldsymbol{u}^s \ \mathrm{d}\Gamma$$



$$\bar{\boldsymbol{u}}_\square(\boldsymbol{u}^s) = 0, \quad \bar{\boldsymbol{w}}_\square(\boldsymbol{u}^s) = 0 \qquad \text{(Rigid body constraints)}$$

$$\bar{\boldsymbol{h}}_\square(\boldsymbol{u}^s) = 0, \quad \bar{\boldsymbol{\kappa}}_\square(\boldsymbol{u}^s) = 0, \quad \bar{\boldsymbol{\gamma}}_\square(\boldsymbol{u}^s) = 0 \qquad \text{(Boundary conditions)}$$

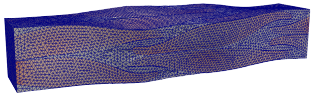
$$\bar{\boldsymbol{\theta}}_\square(\boldsymbol{u}^s) = 0 \qquad \text{(Volume constraint)}$$

Results – Plate stiffness properties

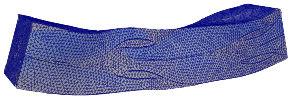
Bundle mesh:
- Linear Tetrahedral ~
- Transversely isotropic material:
  - E_1 = ~115 GPa
  - E_2 = ~8 GPa
  - ν_{12} = ~ 0.29
  - ν_{23} = ~ 0.46
  - G_{12} = ~ 2 GPa

Immersed matrix mesh:
- 30x50x30 elements
- 2^{nd} order IGA-Elements
- Linear elastic material:
  - E = 3100 MPa
  - N = 0.35
- ~30M Quadrature points

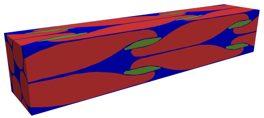
Membrane $\bar{\varepsilon}$



Bending $\boldsymbol{\kappa}$



Shear γ



Effective beam properties in longitudinal direction

	Membrane stiffness EA [N]	Bending stiffness EI [Nmm^2]	Transverse shear KGA [N]
Immersed mesh	127793	434138	
Pure tetrahedral matrix mesh	128382	412725	
Difference [%]	< 1 %	~ 5.1 %	

Conclusion and outlook

Multi-scale homogenisation for plate-elements using VCH.
Automatic discretisation scheme for matrix material for meso-scale models.

- Simple and flexible way of obtaining discretisations for meso-scale modells of advanced fibre compostis.
- Show good agreement with standard FE-models

- Apply the methodology to other complex meso-scale structures.
- Extend the method to non-linear simulation
  - Interface separation
  - Damage in bundles

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