COUPLED STRESS ENERGY CRITERION FOR COMPOSITE MATERIALS

Rami Hamze[1], Thiago V Birro[2] and Benoit Gely[3]

[1] Technology & Engineering Center, Capgemini Engineering, 4 Av. Didier Daurat, Blagnac, France, rami.hamze@capgemini.com
[2] Technology & Engineering Center, Capgemini Engineering, 4 Av. Didier Daurat, Blagnac, France, Thiago.vasconcellos-birro@capgemini.com
[3] Technology & Engineering Center, Capgemini Engineering, 4 Av. Didier Daurat, Blagnac, France, benoit.gely@capgemini.com

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ABSTRACT

Spacecraft structures are considered to be the ideal applications for the usage of laminated composite materials as they offer high specific stiffness, low coefficient of thermal expansion, and dimensional stability. With all the benefits that composite materials provide, there are always difficulties and hardships in modelling their behaviour and trying to predict their failure. Going into practice, applications in aircraft structures require panels to be riveted together for the purpose of load transfer, structural integrity, and ease in part replacement. These joints often lead to high stress concentrations around the drilled zones, which requires in-depth studies to avoid catastrophic consequences. In fact, along with the brittleness of most composite materials and the possibility of using highly orthotropic laminates which promotes high stress concentrations, the anisotropy in both stiffness and strength properties have to be taken into account. Several methods to predict the failure of notched composites are presented in the literature and commercial software, most importantly the stress criterion, that assumes that the failure of the material occurs when the stress over a certain distance from the discontinuity (the hole) is equal or greater than the strength of the unnotched material. Whereas the energy criterion states that failure occurs when the energy release rate over a certain crack length, has reached the critical fracture toughness of the material. Both these criteria require previous experimental tests to obtain the empirical parameter of length. To resolve this problem, a coupled criterion is applied to eliminate the dependency on such empirical parameters, where both the stress and energy criteria are considered as necessary conditions for the fracture but neither of them is sufficient alone. This means that the coupled criterion aims to introduce a characteristic length, dependent on the composite's properties and geometry, that is used to satisfy both criteria simultaneously. For this purpose, a new tool based on python scripts was created on Abaqus to solve the open and/or filled holed composite plates. This tool applies the coupled criterion for any composite laminate, geometry, loading condition, and crack propagation direction. The results obtained by solving the coupled criterion indicate that the methodology proposed can accurately and efficiently predict ultimate failure loads in composite plates, with an error that does not exceed 5%.

1 INTRODUCTION

The strength of composite laminates has been investigated extensively over the past 30 years because of their huge importance to designers. In open-hole and notched specimens, the complexity of the damage and failure mechanisms present during the loading cycle of a laminate are exaggerated due to the presence of stress concentrations. The sensitivity of a laminate to notches and open holes, in terms of its mechanical performance, is dependent on many factors. The most important factors mentioned in the literature are the laminate size and thickness, notch size and geometry, ply orientation and thickness, machining quality, and material constituents. All these factors affect the mechanical behaviour of the specimen under any kind of loading applied [1].

Spacecraft structures are considered to be the ideal applications for the usage of laminated composite materials. Typical spacecraft structures such as trusses, equipment panels, optical benches, and radiators should have high specific stiffness, low coefficient of thermal expansion and dimensional stability during the operational lifetime. High-performance composites satisfy these requirements and offer the minimum weight material solution for these structures. Going into practice, most of the composite applications in aircraft structures contain notches and joints.

With all the benefits that composite materials provide, there are always difficulties and hardships in modelling their behaviour and trying to predict it. In fact, along with the brittleness of most composite materials and the possibility of using highly orthotropic laminates which promote high-stress concentrations, the anisotropy in both stiffness and strength properties must be taken into account. The stress concentration raisers are often the critical part of a composite structure; therefore, the soundness of the structure design procedure used is reflected in the overall weight and cost of the composite structure [2].

In this context, a new module named NextCompo has been developed by Capgemini Engineering. It aims to predict the failure of open and filled hole composites having different stacking sequences and failure modes. The present study provides the failure prediction of open-hole specimens under pure or mixed mode, which does not possess analytical solutions The main advantage of this tool is that it successfully utilizes the coupled stress-energy criterion [3], that led to avoiding the necessity of experimental tests to determine empirical coefficients, as traditionally done for other failure criteria [4].

2 THE STATE OF THE ART

Leguillon (2002) [3] mentioned that both stress and energy criteria are necessary conditions for the fracture but neither of them is sufficient alone. In his paper, he discussed that thanks to the singularity on the tip of the notch, the incremental form of the energy criterion led to a lower bound, and the stress criteria provided the upper bound [3,5].

The stress criterion provided good results for crack-free bodies, whereas the energetic criterion was physically sound for bodies containing a sufficiently large crack [6]. To add more, the stress criterion provided a null failure load in a body containing a crack, due to the singularity of the stresses in front of the crack tip. On the other hand, the energy criterion provided an infinite failure load for a crack-free body having the stress intensity factor as 0 due to the absence of the crack. Thus, these criteria mentioned were effective for the presented extreme cases, but not for the cases of short cracks or sharp notches.

The aim of the Coupled Criterion (CC) is to introduce a material length that allows the stress-based criterion to consider the fracture toughness, and the energy-based criterion to take into account the tensile strength of the material. This means that this criterion takes into account the interaction between the finite crack extension and the geometry of the specimen. Thus, the crack extension becomes a structural variable and not a constant to be calculated or correlated. To physically explain this criterion, it is important to go back to the definition of each of the two criteria used in the coupled criterion. The coupled criterion is based on two conditions: a stress condition that describes the initiation of the micro-cracks and an energy criterion that solves the propagation of these micro-cracks on a finite length to create a macro-crack [7].

Denoting the characteristic length mentioned by Δa, the two different approaches of the coupled criteria are discussed. The coupled criterion is the coupling of stress and energy criteria, and as there are two different stress methods, this generates two coupled criteria approaches. The analysis is conducted with the assistance of Figure 1, which illustrates a plate with dimensions L x W, a hole of radius R, and σ_∞ representing the stress applied to the boundaries. The stress criterion is first computed using a non-cracked plate, along with a fictitious crack Δa (as shown in Figure 1a). Conversely, the energy criterion is computed using a cracked plate (as shown in Figure 1b), with a crack length of Δa. Therefore, the average stress-energy criteria can be described as follows:

$$\frac{1}{\Delta a}\int_{R}^{R+\Delta \mathrm{a}} \sigma_{eq}(x)\, dx \geq \ \sigma_c \tag{1}$$

$$K_{inc} = \frac{1}{\Delta a}\int_{0}^{\Delta a} {K_I}^2 \ d\widetilde{\Delta a}x \geq \ {K_{IC}}^2. \tag{2}$$

where σ_c and K_C are mechanical properties of the laminate and denote the material strength and the fracture toughness, respectively. Besides, σ_{eq} and K_{inc} are the equivalent stress and the incremental stress intensity factor.

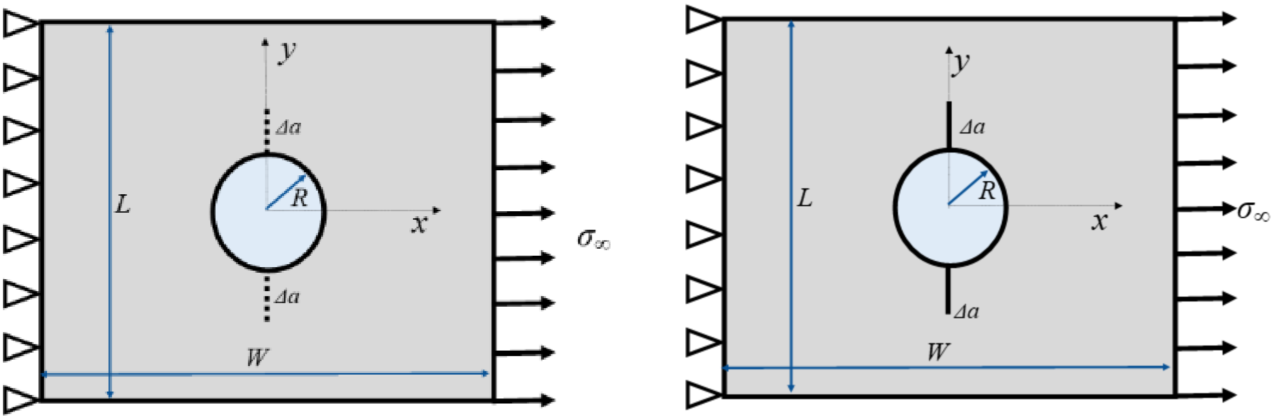

Figure 1: Open-hole plate configurations: (a) computing the equivalent stress (stress criterion) without a crack (b) computing the stress intensity factor (energy criterion) with the crack.

These relationships are alone distinct. This means that the fulfilment of one is not usually the fulfilment of the other. So, applying the stress criterion and reaching the critical strength of the material at a specific characteristic distance Δa from the edge of the hole, does not mean that the energy released in the crack extension Δa yields the fracture toughness of the specimen and vice versa.
Using the relations shown before, and considering the average point stress–energy criteria for simplicity, the final form of the relation that consists of squaring the stress equation yields the following:

$$\frac{\Delta a \int_0^{\Delta a} \hat{K}^2 \, d\tilde{\Delta a}}{\left(\int_R^{R+\Delta a} \widehat{\sigma_{eq}} \; dx\right)^2} = \frac{{K_C}^2}{{\sigma_c}^2} \tag{3}$$

with:

$$\widehat{\sigma_{eq}} = \frac{\sigma_{eq}(x)}{\sigma_\infty} \tag{4}$$

$$\hat{K} = \frac{K(x)}{\sigma_\infty} \tag{5}$$

Since the applied force does not depend on any variable, it was cancelled when dividing the two criteria. Note that this equation cannot be solved algebraically, particularly because the nominator must be integrated numerically. This equation can easily be solved with standard methods for root search to calculate the characteristic length that causes the stress and energy criterion to be fulfilled. Finally, the corresponding material strength can then be calculated by using the obtained finite crack size in one of the two equalities.
In a summary, each equation of the coupled criterion whether it is stress or energy are necessary conditions for the failure of the specimen. The fulfilment of both of them is necessary and sufficient condition for failure initiation. Physically, the coupled criterion states that the fracture energy is driven but a sufficiently high-stress field must act to trigger crack propagation. And thus, refer to Figure 2 which shows the upper and the lower bounds, Δa_{min} and Δa_{max} respectively. In such a case, the lower bound is given by the fulfilment of energy criterion, while the upper bound is given by the fulfilment of the stress criterion. Therefore, referring to Figure 2 which shows the upper and the lower bounds imposed by both criteria on the crack length, the application, bounds, and role of the coupled criterion are clear.

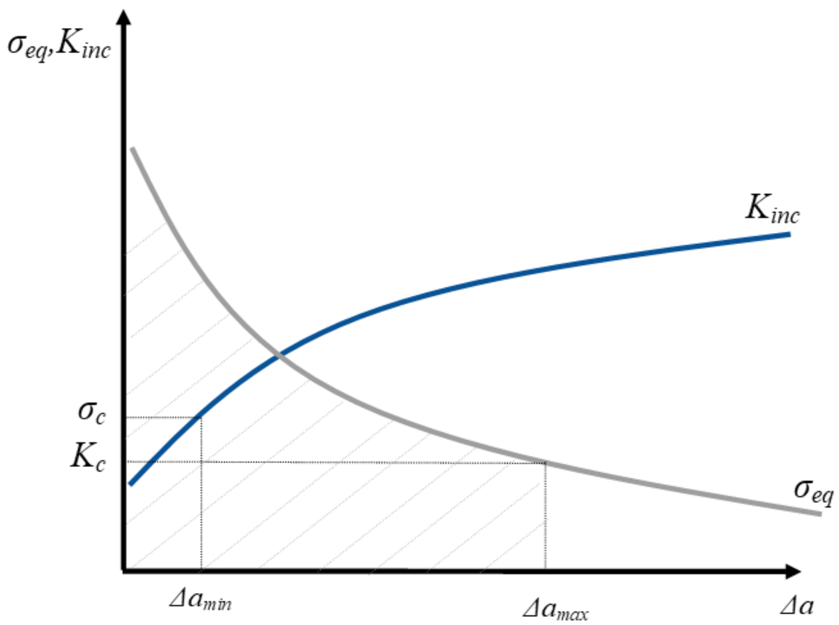

Figure 2: The application of the coupled criterion is terms of its consisting criteria.

3 METHODOLOGY

In the present work, a methodology inspired by the analytical approach developed by Martin et al. (2012) [5] is undertaken. First, the geometry of the plate is created, and the mechanical properties and the stacking sequence of the plate are defined. Subsequently, two different branches are created separately. In the first branch, different crack lengths are inserted along a given direction to evaluate the energy release rate via the J-integral approach. According to Aboudi et al. (2021) [8], the values of J-integral are then transformed into stress intensity factor (SIF), using Equation 6.

$$J = \frac{K^2}{\sqrt{2E_x E_y}} \sqrt{\sqrt{\frac{E_x}{E_y}} + \frac{E_x}{2G_{xy}} - \upsilon_{xy}} \tag{6}$$

The elastic coefficients of the plate in x and y axes are given as follows:

$$E_x = \frac{A_{11} + A_{12}\left(\frac{A_{26}A_{16} - A_{12}A_{66}}{A_{22}A_{66} - A_{26}^2}\right) + A_{16}\left(\frac{-A_{16}}{A_{66}} + \frac{A_{26}A_{12}A_{66} - A_{26}A_{16}^2}{A_{22}A_{66}^2 - A_{26}^2 A_{66}}\right)}{h} \tag{7}$$

$$E_y = \frac{A_{22} + A_{12}\left(\frac{A_{16}A_{26} - A_{12}A_{66}}{A_{11}A_{66} - A_{16}^2}\right) + A_{26}\left(\frac{-A_{26}}{A_{66}} + \frac{A_{16}A_{12}A_{66} - A_{16}A_{16}^2}{A_{11}A_{66}^2 - A_{16}^2 A_{66}}\right)}{h} \tag{8}$$

$$G_{xy} = \frac{A_{66} + \frac{A_{26}^2}{A_{22}} + \frac{2A_{12}A_{16}A_{26}A_{22} - A_{12}^2A_{26}^2 - A_{16}^2A_{22}^2}{A_{11}A_{22}^2 - A_{22}A_{12}^2}}{h} \tag{9}$$

$$v_{xy} = \frac{A_{12} - \frac{A_{16}A_{26}}{A_{66}}}{A_{22} - \frac{A_{26}^2}{A_{66}}} \tag{10}$$

where A_{ij} denotes the extensional stiffness matrix [9] and h is the thickness of the plate. Finally, the SIF is evaluated for different crack lengths and a fifth-order polynomial fit is proposed to represent the SIF in terms of Δa.

A second branch is then created, using a non-cracked plate (see Figure 3). A path is created along a virtual crack direction to determine the stress tensor along the path. Finally, the mixed mode stress along the virtual crack path is determined using Equation 10 [10]. Thus, the transformation is done in the script proposed once the direction of the crack is specified, as follows:

$$\sigma_{eq} = \sqrt{{\sigma_\theta}^2 + {\sigma_{r\theta}}^2} \tag{11}$$

with:

$$\sigma_r = \sigma_x \cos\theta^2 + \sigma_y \sin\theta^2 + 2\,\sigma_{xy} \cos\theta \sin\theta \tag{12}$$

$$\sigma_\theta = \sigma_x \sin\theta^2 + \sigma_y \cos\theta^2 - 2\,\sigma_{xy} \cos\theta \sin\theta \tag{13}$$

$$\sigma_{r\theta} = \left(\sigma_y - \sigma_x\right) \cos\theta \sin\theta + \sigma_{xy} \cos\theta^2 - \sigma_{xy} \sin\theta^2 \tag{14}$$

where the subscripts x and y denote the global stress in the global coordinate system and the subscripts r and θ denote the stress in the polar coordinate system, aligned with the crack direction.

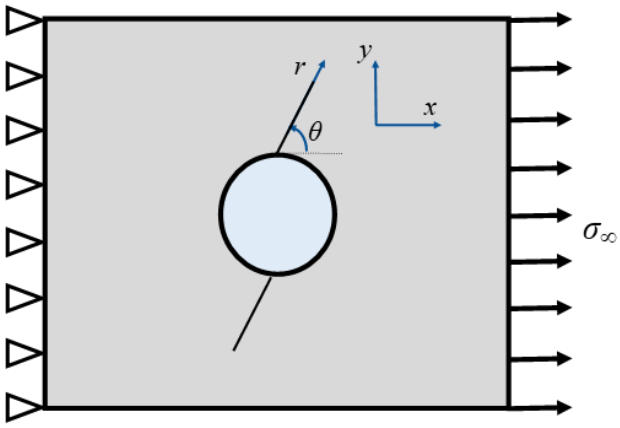

Figure 3: The principal and local coordinate systems in the plate.

Finally, the whole methodology presented in the present section can be summarized in Figure 4, where both branches are presented.

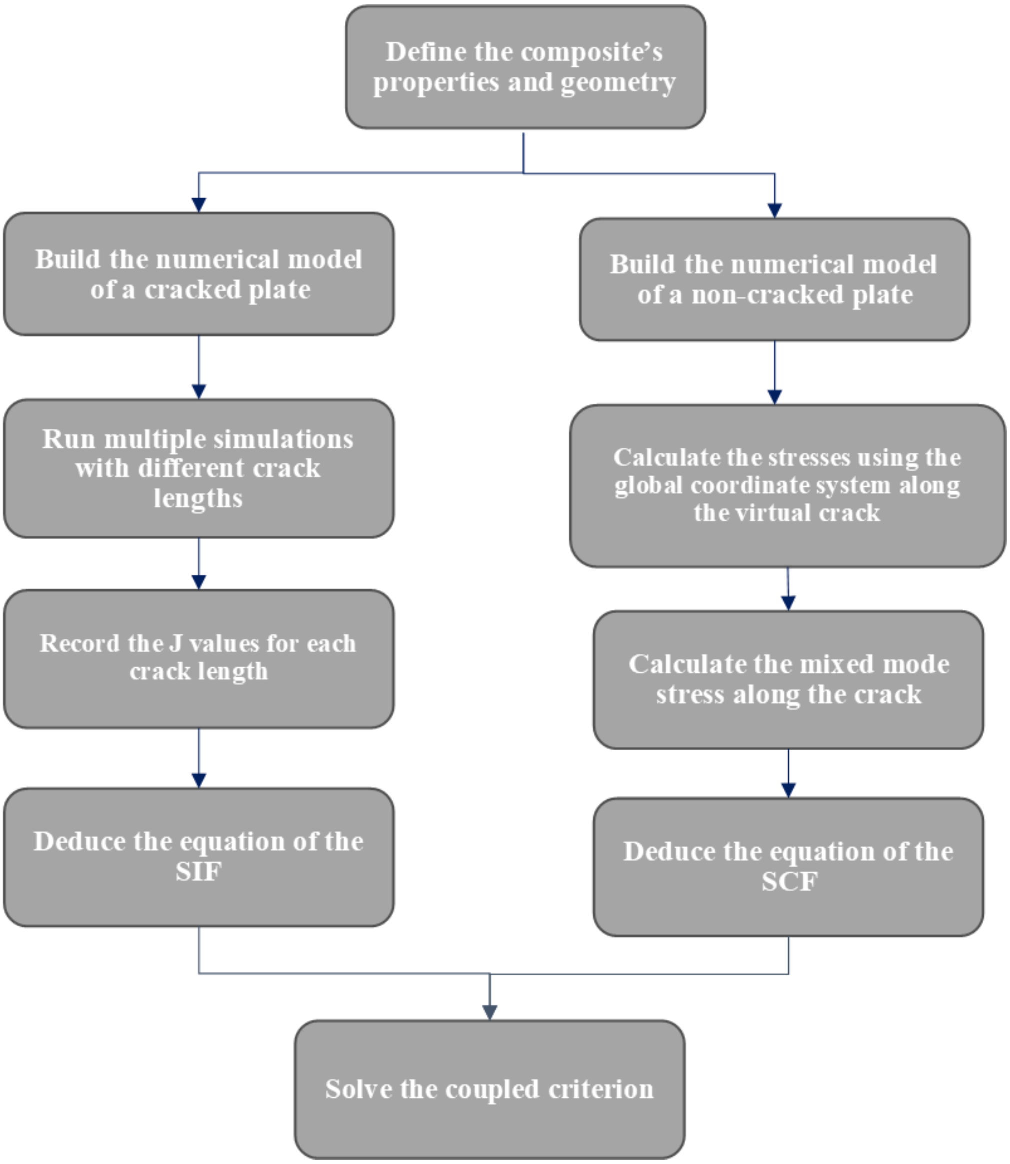

Figure 4: Flowchart summarizing the steps taken to solve the coupled criterion.

3 NUMERICAL ANALYSES OF OPEN HOLE COMPOSITE PLATE

For the present paper, the analysis and the results obtained by Falcó et al. (2018) [11] are considered hereafter for the open-hole composite plate, since it offers a lot of different experimental tests and simulations done on different composites, with different failure models. The elastic material properties of the laminate are given in Table 1:

Ply Elastic Properties	Mean Value
E_{1t} (GPa)	137.1
E_{1c} (GPa)	114.4
E_{2t} (GPa)	8.8
E_{2c} (GPa)	10.1
$G_{12} = G_{13}$ (GPa)	4.9
$\nu_{12} = \nu_{13}$	0.314
ν_{23}	0.487

Table 1: The ply elastic properties.

where E, G and v denote the Young's modulus, the shear modulus and Poisson's ratio, respectively. The subscripts "t" and "c" represent the properties obtained under tension and compression tests, while the subscripts "1", "2" and "3" represent the directions parallel, perpendicular, and transverse to the fibres. Besides, the ply strength properties are listed in Table 2, as follows:

Ply Strength Properties	Mean Value
X^T (MPa)	2106.4
X^C (MPa)	1675.9
Y^T (MPa)	74.2
Y^C (MPa)	322.0
S^L (MPa)	110.4

Table 2: The ply strength properties.

Where X^T,X^C,Y^T,Y^C and S^L correspond to the longitudinal tensile/compressive, transverse tensile/compressive and in-plane shear strengths.

Table 3 also shows the three different composites used in the analysis; the naming of these composites goes back to the number of 0-degree plies they contain. For example, the third presented configuration is named a soft composite, which is due to the presence of only 10% of 0-degree plies in the laminate.

Ply Configuration	Stacking Sequence
Hard Composite	$[0,45,0,90,0,-45,0,45,0,-45]_S$
Quasi-Isotropic Composite (QI)	$[-45,0,45,90]_{2S}$
Soft Composite	$[45,-45,0,45,-45,90,45,-45,45,-45]_S$

Table 3: The ply strength properties.

Then, Table 4 shows the failure of each of the composites in compression and tension when the laminate has the following geometry (see Figure 1): $L = 100$ mm, $W = 38.1$ mm, $R = 3.175$ mm, and the thickness of each ply was given as 0.184 mm. Using the quasi-isotropic configuration, the K_c for tension (48 MPa.m$^{0.5}$) and compression (38 MPa.m$^{0.5}$) is computed using the analytical solution, as previously explained by Camanho et al. (2012) [12].

Solicitation mode	Property	Unnotched Failure Stress (MPa)	Notched Failure Stress (MPa)	The direction of Crack Orientation (Degrees)
Tension	Hard Composite	1105.5	526.7	90
	Quasi Isotropic Composite	651.1	370.9	Not Visible
	Soft Composite	421.9	289.3	135
Compression	Hard Composite	787.2	425.7	Not Visible
	Quasi Isotropic Composite (QI)	554.5	301.8	90
	Soft Composite	414.1	269.8	Not Visible

Table 4: Different failure stresses and crack directions of the presented composite plate.

Proceeding with that, the numerical simulations were run to calculate the predicted failure load. In order to check the validity of the results, the obtained ones were compared to the experimental ones proposed by the author, and the results are shown in Figure 5.

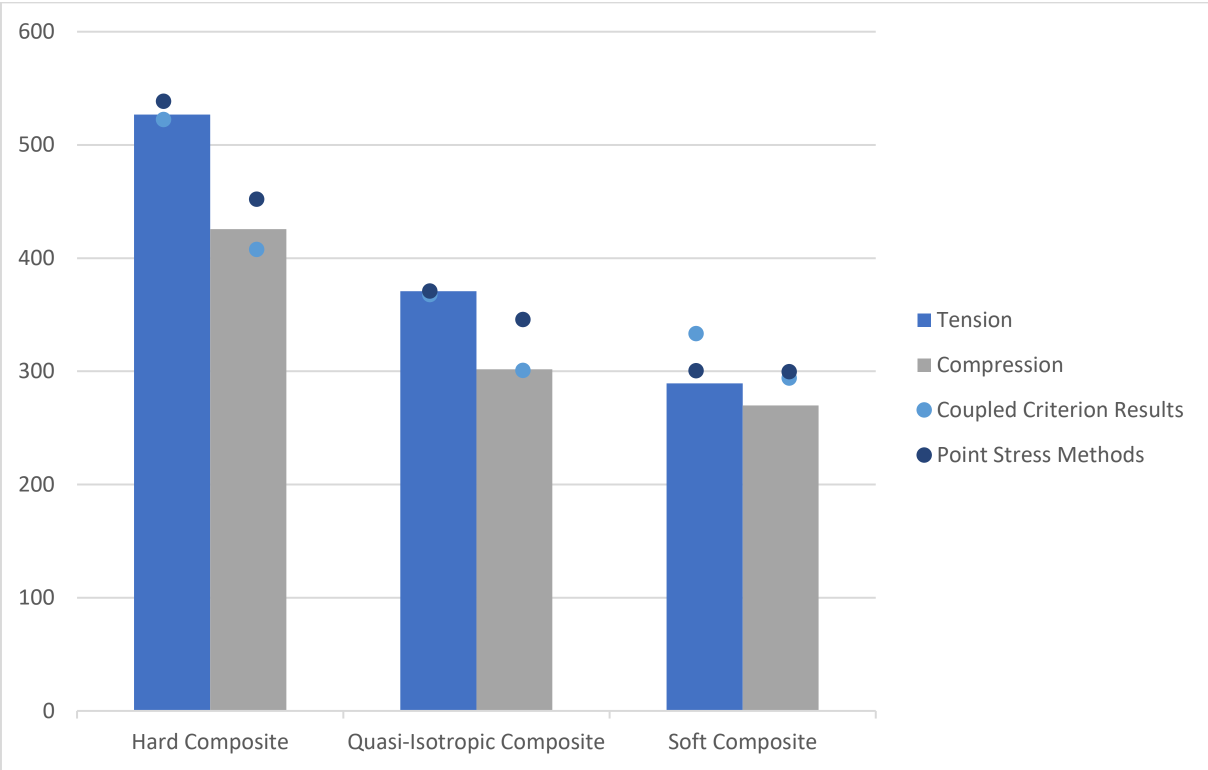

Figure 5: Comparison of the numerical and experimental results for the open hole with Falcó et al. (2018) [11].

To examine the outcomes, it can be insightful to discuss each type of composite individually. Beginning with the Quasi Isotropic composite, it is evident that the results are highly precise, owing to the fact that this composite behaves as an isotropic material with existing stress concentration factor (SCF) and SIF analytical equations, thereby minimizing the potential errors that could arise from numerical simulations. Moving on to the Hard composite, it can also be observed that the outcomes are remarkably accurate, particularly when compared to the author's point stress method, with a maximum error rate of 4.2%. On the other hand, the Soft composite yielded increased errors of 14% in tension and 8% in compression. It is worth noting that the Soft composite only includes two 0-degree plies, accounting for 10% of the entire laminate, which renders the material ductile, as evidenced by the recorded failure crack length of approximately 9 mm. Consequently, based on Camanho and Catalanotti (2011) [12], the coupled criterion, which is founded on Linear-Elastic Fracture Mechanics, should not be employed to analyse materials that may undergo severe plastic deformation in the vicinity of the crack. In other words, the plastic deformation that a ductile laminate undergoes plays a crucial role, and cannot be assessed by the coupled criterion, which may have led to the elevated error rates regarding the failure stress at failure.

4 CONCLUSIONS

Based on the findings presented in this study and the literature review conducted on the coupled criterion, it can be concluded that this criterion is a promising methodology for predicting the failure stress of a composite laminate in both open and filled hole configurations. In contrast to other failure criteria discussed in the literature, the coupled criterion does not rely on empirical parameters to predict composite failure. Moreover, the coupled criterion demonstrates high accuracy when compared to both experimental and numerical results from the literature.

Specifically, the coupled criterion is capable of predicting open hole composite failure for hard and quasi-isotropic composite stacking sequences. Preliminary analyses have also been conducted using 3D shell elements for the filled hole configuration, and the results have been promising using the same methodology.

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