UNIVERSAL FAILURE CRITERIA FOR FIBROUS COMPOSITES BASED ON MULTISCALE APPROACH

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ABSTRACT

This paper presents a reliable, versatile, and efficient modeling technique and associated failure criteria for laminated fibrous composite structures. A multiscale modeling technique was utilized to trace progressive damages in the fiber and matrix materials subjected to either monotonic or cyclic loading. Failure modes are fiber failure, matrix failure, and fiber-matrix interface failure in terms of the perspective of the constituent materials. Thus, there are three sets of failure criteria for the three failure modes. The failure criteria are based on both stress and stress gradient such that they can be applied to composite structures with or without sharp or blunt notches in a uniform manner.

1 INTRODUCTION

Reliable failure predictions are very critical for the safety of load-carrying structures regardless of their applications. As the material behaviors become more complex, failure criteria tend to become also more complex. This is the case for laminated fibrous composite materials whose material properties are neither isotropic nor homogeneous. There are many different failure modes which include fiber fracture, fiber splitting, transverse matrix cracking, interlayer delamination, etc. In addition, there are multiple parameters under control to tailor the material properties depending on the desired designs. In this case, the material properties vary depending on the selected parameters, and those new material properties must be available to apply failure criteria. Furthermore, composite structures usually do not fail at once but have a gradual failure. Thus, it is desirable to have failure criteria that can track progressive damage or failure for various kinds of loadings such as monotonically or cyclically changing loads. Furthermore, the progressive failure criteria should apply to any geometric features like sharp or blunt notches.

To address the problems mentioned above, research has been conducted by the author's research team. As a result, a reliable, versatile, and efficient modeling technique and associated failure criteria have been developed. In the next section, a brief review is presented of the currently available failure criteria. Then, the failure criteria proposed by the author's team are presented along with some validation examples.

2 REVIEW OF FAILURE CRITERIA

2.1 Failure criteria without considering notches

Many different failure criteria have been proposed for predicting the failure of fibrous composite materials. Those failure criteria were for the composite materials without considering any geometric feature in composite structures such as notches. Those failure criteria can be classified into two groups as shown in Fig. 1. The first group is the macroscale failure criteria. Almost all failure criteria belong to this group. The failure criteria of this group require strengths of the lamina of fibrous composite. Because the lamina is not isotropic, multiple failure strengths are required in those failure criteria. The macroscale failure criteria were also divided into different subgroups depending on how the failure criteria were expressed mathematically. Those subgroups were called the limit failure criteria, interactive failure criteria, and discrete mode failure criteria [1-4].

The second group of failure criteria is the microscale failure criteria that use stresses and strains at the fiber and matrix materials [5-8]. Those are called micro-stresses and micro-strains. The failure criteria of this group use the strengths of the fiber and matrix materials. There are three failure criteria from the microscale perspective. They are fiber failure, matrix failure, and fiber-matrix interface failure. In a simplified way, fiber-matrix interface failure may be considered as a part of the matrix failure because both affect the load sharing among fibers in a similar way. However, in most materials, the strength of the interface is quite different from the strength of the matrix material. Therefore, it is more appropriate to consider interface failure as a separate failure criterion.

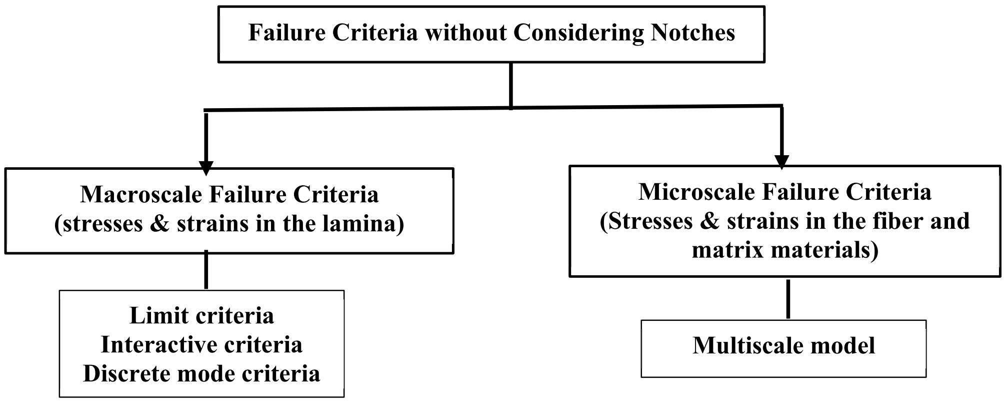

Figure 1: Classification of failure criteria without considering notches.

There are pros and cons in two different groups of failure criteria. The macroscale failure criteria are easy to be applied and do not require extensive computations. The microscale failure criteria require more computations because a multiscale approach should be adopted as sketched in Fig. 2. The multiscale model consists of two processes in opposite directions in terms of the length scale. One is called the upscaling process while the other is called the downscaling process. The upscaling process is the one from microscale to macroscale, and the downscaling process is the opposite. The former process is to determine the effective stiffness of composite materials from the properties of the fiber and matrix materials. Because of that, this is also called the stiffness process. The downscaling process is to determine the micro-stresses and micro-strains from the composite level stresses and strains. That is why this process is also called the strength process.

The upscaling process is important to analyze composite structures subjected to external loading, and the downscaling process is important to apply microscale-based failure criteria to the basic constituent materials. To relieve the computation burden in the multiscale analysis, analytical solutions were developed to be used for the multiscale approach. That is, both upscaling and downscaling processes are conducted analytically without any major computational cost. Therefore, the additional computational cost is very minimal, so the computational time of the multiscale approach is close to that of the standard macroscale analysis. To this end, a unit-cell model was developed for those processes. The three failure criteria for the three different failure modes are discussed in the next section. The mathematical details can be found in Refs. [5,10] so that they are omitted here.

There are many advantages to the microscale failure criteria as listed below:

- The microscale failure criteria can be used to track progressive failure in composite structures because any local damage of any constituent materials can be included in the multiscale analysis model.
- Failure modes are simplified and described intuitively in terms of fiber failure, matrix failure, and fiber-matrix interface failure. For example, interlaminar delamination is an example of matrix failure and/or fiber-matrix interface failure.
- The microscale failure criteria can easily implement residual thermal stresses in predicting failure. For example, thermal residual stresses could be developed in the fiber and matrix materials during the curing process as well as any temperature change because they have different coefficients of thermal expansion. Those microscale stresses can be incorporated into the microscale failure criteria naturally.

- The microscale failure criteria can be applied to composite materials which do not have uniform fiber or matrix volume fractions. For example, if a composite structure has the fiber volume fraction varying through the thickness of a layer to achieve the functionally gradient materials, the microscale failure criteria can be also used.
- The microscale failure criteria do not require new material properties if there is a change in the fiber or matrix volume fraction. On the other hand, the macroscale failure criteria require new strengths because different fiber or matrix volume fractions change the strength properties.

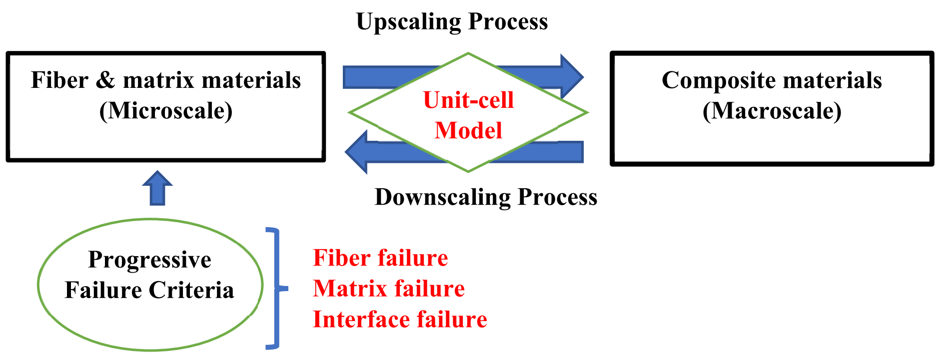

Figure 2: Microscale failure criteria based on multiscale approach.

2.2 Failure criteria considering notches

When there is a notch in a structural part, a significant stress gradient develops just in front of the notch tip. If the notch tip has a zero radius, it is called a crack. Then, there is stress singularity at the crack tip, where the stress goes to infinity for linear elastic materials. Then, all the failure criteria mentioned in the last section are not applicable anymore because all of them will state failure even if a very tiny load is applied to a structural part with a crack. This is not the case. Therefore, other failure theories and criteria have been proposed for failure at the notches.

To deal with a crack with stress singularity, fracture mechanics was developed, and the concept of stress intensity factor was introduced [11]. Then, a new material property called fracture toughness was introduced to predict failure initiating from a crack tip. A notch other than a crack does not have stress singularity. Instead, there is stress concentration at the notch tip. In this case, fracture mechanics is not applicable directly. As a result, other alternative ways were proposed to predict failure at the notch tip [12-16].

The failure criteria without considering notches are based on the stresses and strains at critical points, and they do not consider stresses around the points. On the other hand, the failure criteria for notches considered stresses around the critical points. Therefore, instead of evaluating stresses at the critical points, they examined the stresses at some neighbor locations other than the critical failure points. The distance from the notch tip to the neighbor location is called the critical distance. That is, the failure criteria for notches use the stress at or along the critical distance. The critical distance started from the fracture mechanics. The stress at the crack tip σ is expressed for the first mode as below:

$$\sigma = \frac{K_I}{\sqrt{\pi x}} \tag{1}$$

where K_I is the stress intensity factor and x is the distance from the crack tip. The critical distance d_c is determined as K_I is equal to the fracture toughness K_{IC}, and the stress is equal to the failure stress σ_f of the material. That is, the critical distance is

$$d_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_f}\right)^2 \tag{2}$$

To improve the accuracy of the failure prediction, the critical distance was modified. The critical distance is considered independent of the notch size and shape.

3 UNIFIED FAILURE CRITERIA

Recently, the research team of the authors proposed a unified failure criterion that can be applied to both notched and unnotched structural parts [17-21]. This criterion does not require any critical distance to predict failure. Instead, the new criterion examines both stresses and stress gradients at critical locations. The stress gradients are the information on the stresses at the critical and their neighboring points. Thus, the critical distance is not used in this theory.

The unified failure criterion has two parts. Both parts of the failure criteria must be satisfied simultaneously for failure to occur. The two conditions are expressed below:

$$\sigma_{eff} \geq \sigma_f \tag{3}$$

$$\sigma_{eff} \geq \left(2E\kappa_f \left|\frac{d\sigma_{eff}}{ds}\right|^{-1}\right)^{\frac{1}{3}} \tag{4}$$

where σ_{eff} is the effective stress, E is the elastic modulus, and κ_f is called the critical surface energy. In addition, s is the failure path. The effective stress depends on the material characteristics. For a brittle isotropic material, the maximum or minimum normal stress is used for tensile or compressive failure, respectively.

Equation (3) is equivalent to the failure criteria without considering notch effects while Eq. (4) is equivalent to the failure criteria for notches. In other words, if there is no notch or minor stress concentration in a structural part, Eq. (4) is negligible compared to Eq. (3). This means the failure stress from Eq. (3) is greater than that from Eq. (4). Similarly, for a structural part with a notch, the failure stress from Eq. (4) is generally greater than that from Eq. (3).

For a crack with the stress distribution shown in Eq. (1), applying Eq. (4) yields

$$\kappa_f = \frac{K_{Ic}^2}{\pi E} = \frac{G_{IC}}{\pi} \tag{5}$$

Therefore, the critical surface energy is directly related to fracture mechanics' critical energy release rate.

The linear elastic, isotropic infinite plate with a circular hole of the radius R and subjected to a remote unit tensile load has the following stress in the loading direction along the section of the minimum cross-section x.

$$\sigma = \frac{1}{2}\left[\left(1+\frac{R^2}{x^2}\right)+\left(1+\frac{3R^4}{x^4}\right)\right] \tag{6}$$

Taking the derivative of Eq. (6) with respect to x and substituting it into Eq. (4) states that the failure stress at the edge of the hole is inversely proportional to the one-third power of the hole radius like

$$\sigma_{eff} \propto R^{-\frac{1}{3}} \tag{7}$$

4 UNIFIED FAILURE CRITERIA FOR MULTISCALE MODEL

The unified failure criterion presented in the last section is applied to the microscale failure model of fibrous composite materials. As stated previously, there are three failure modes for fibrous composites: fiber failure, matrix failure, and fiber-matrix interface failure. Thus, the unified failure criterion is applied to the three failure modes, respectively. That is, the effective micro-stresses are used for Eqs. (3) and (4).

The effective stress for fiber failure is expressed as

$$\sigma^f_{eff} = \sqrt{\left(\sigma^f_x\right)^2 + \left(\frac{E^f_x}{G^f_{xy}}\right)\left[\left(\sigma^f_{xy}\right)^2 + \left(\sigma^f_{xz}\right)^2\right]} \tag{8}$$

where superscript *f* denotes the fiber, *x* is the axis along the fiber direction, *E* is the elastic modulus, and *G* is the shear modulus. This effective stress is applied to the unified failure criterion, Eqs. (3) and (4) simultaneously. Fibers are considered to have different strengths in tension and compression. Thus, different failure strengths are used for the failure criterion.

Some polymer matrix materials are brittle and isotropic. In that case, the maximum or minimum normal stress is used for the effective stress as below:

$$\sigma^m_{eff} = \begin{cases} \sigma^m_{\max} & \text{if tension} \\ \sigma^m_{\min} & \text{if compression} \end{cases} \tag{9}$$

where superscript *m* indicates the matrix stresses.

The effective stress for the fiber-matrix interface failure is expressed as

$$\left(\frac{\sigma^m_{xy} + \sqrt{\nu^f}\left(\sigma^m_y - \sigma^m_x\right)}{\tau^{\text{int}}_{\text{fail}}}\right)^2 + \langle\frac{\sigma^m_y}{\sigma^{\text{int}}_{\text{fail}}}\rangle^2 \geq 1 \tag{10}$$

in which superscript *int* denotes the interface, and ν^f is the fiber volume fraction. Furthermore, the second term $\langle\sigma\rangle = \frac{\sigma + |\sigma|}{2}$ is the Macaulay operator because the interface normal stress is assumed to contribute to failure if it is tensile.

5 FATIGUE FAILURE MODEL

All the failure criteria discussed above are for monotonic loading. However, fatigue failure results from repeated cyclic loading, and this requires tracking progressive damage accumulation in the material. In this case, the previous failure criteria are not applicable as expected. However, the previous multiscale model can be still used for predicting failure under cyclic loading. That is, the microscale progressive damage is used to predict the fatigue failure subjected to cyclic loading. Because fibers are the major load-carrying element, progressive damage is modeled for the fibers as they go through cyclic loading.

Progressive damage in the fibers is modeled such that a fraction of fibers continues to fail along with cyclic loading until all the fibers fail. When all the fibers fail, that determines the number of cycles for the failure of the composites. The fraction of failed fibers after *n* cycles at the applied stress level σ is determined from the following expression.

$$\varphi(\sigma) = \begin{cases} \dfrac{\sigma}{\sigma_n} e^{-a\left(\frac{\sigma}{\sigma_n}\right)^b\left(\frac{\sigma_n}{\sigma}-1\right)} & 0 \le \sigma \le \sigma_{fail} \\ 1 & \sigma > \sigma_{fail} \end{cases} \tag{11}$$

where σ_{fail} is the failure stress, and σ_n is the stress carried by the intact fibers after n cycles. They are related to each other as follows.

$$\sigma_n = r_n \sigma_{fail} \tag{12}$$

Here, r_n is the fraction of intact fibers after n cycles. Thus, r_{n+1} is determined as follows.

$$r_{n+1} = (1 - \varphi(\sigma))\, r_n \tag{13}$$

Furthermore, a and b in Eq. (11) are the material constants controlling the fraction of fibers that failed after each cyclic loading.

6 RESULTS AND DISCUSSION

Equation (7) was validated against experimental results in Ref. [16] which tested PMMA and GPPS specimens with circular holes. Both materials are considered isotropic and linearly elastic. The test specimens were close to infinite plates with holes if the hole size was much smaller than the size of the specimen. Figure 3 shows the stress concentration factors for four different sizes of holes. The normalized hole size (i.e., hole diameter divided by the specimen width) varied from 0.01 to 0.1. The corresponding stress concentration factors varied from 2.96 to 2.72. That is, the smallest hole with a ratio of 0.01 has a stress concentration factor close to 3.0 which is the value for the infinite plate. Thus, the specimen may be considered as an infinite plate in a close approximation. However, as the hole size increased, the specimen deviated more from the assumption of an infinite plate.

Figure 3 shows the test results against the theoretical curve in Eq. (7). Even though PMMA and GPPS have different failure strengths, their behaviors were almost identical and agreed with the theoretical curve, especially for the smaller holes. However, as the hole size became larger, there was a deviation between the theory of the infinite plate and the experimental results, as expected.

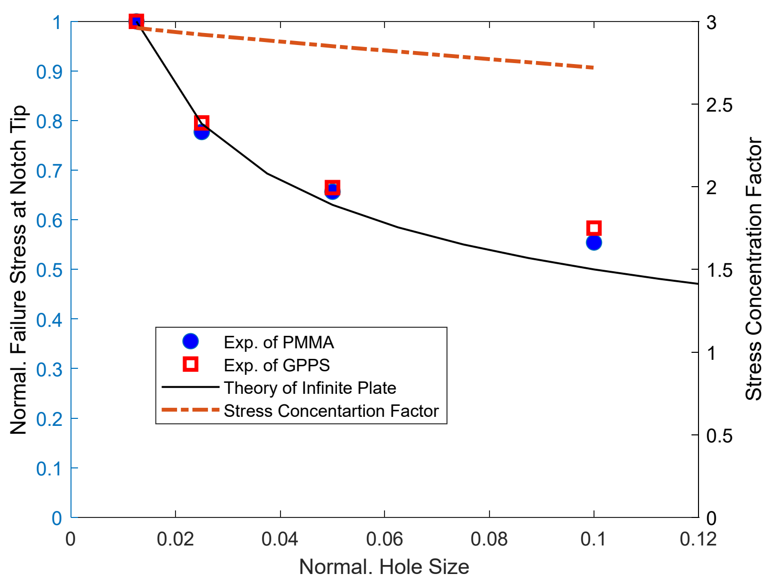

Figure 3: Plot of normalized failure stresses at the edge of the holes.

The next example is the polylactic acid (PLA) specimens made by 3D printing. The printing setting was selected such that the PLA specimens had orthotropic material properties. That is, the strength and stiffness were higher in the printing orientation than those in its orthogonal direction. The specimens had dimensions 140 mm long, 24 mm wide, and 2 mm thick. Specimens were fabricated in the printing angles $\pm\theta^o$, and the angle was varied from 0 to 90 incrementally. All the specimens had center holes of 6 mm diameters. Figure 4 compares the experimental failure stresses to the theoretical predictions. The comparison is very good.

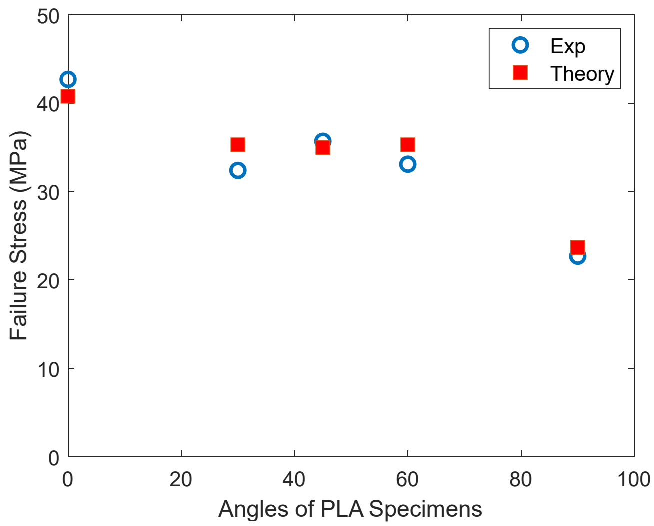

Figure 4: Comparison of failure stresses of PLA specimens with 6 mm diameter holes with different printing angles.

The next example is laminated carbon fiber composite specimens. The experimental failure stresses were compared to the theoretical failure stresses using the failure criteria discussed in the previous section. Figure 5 shows a very good comparison between the two results.

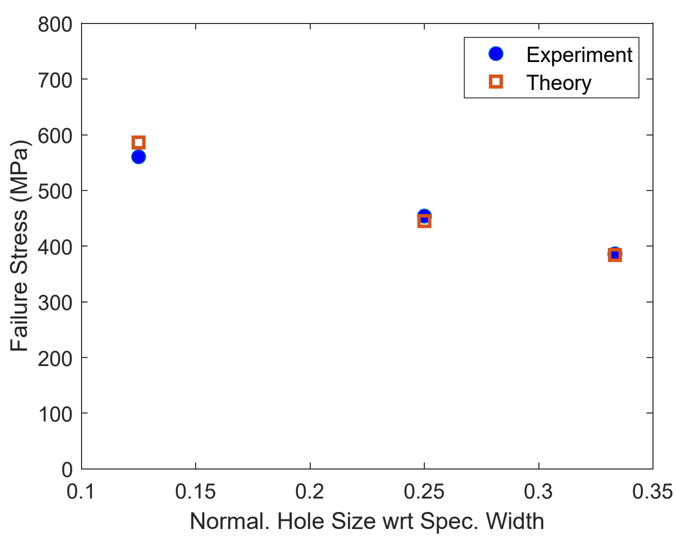

Figure 5: Comparison of failure stresses of quasi-isotopically laminated specimens.

The last example is unidirectional fibrous composite specimens subjected to cyclic loading. Progressive fiber failure was predicted using the failure model discussed in the last section. The residual strength of the unidirectional composite specimens was measured as a function of the number of applied cyclic loading. Then, the same strength was also predicted using the proposed model. Figure 6 shows the comparison. Both results showed that the residual strength had a very minor change until the specimens failed under cyclic loading. Thus, it is not possible to predict the expended life cycles using the residual strength.

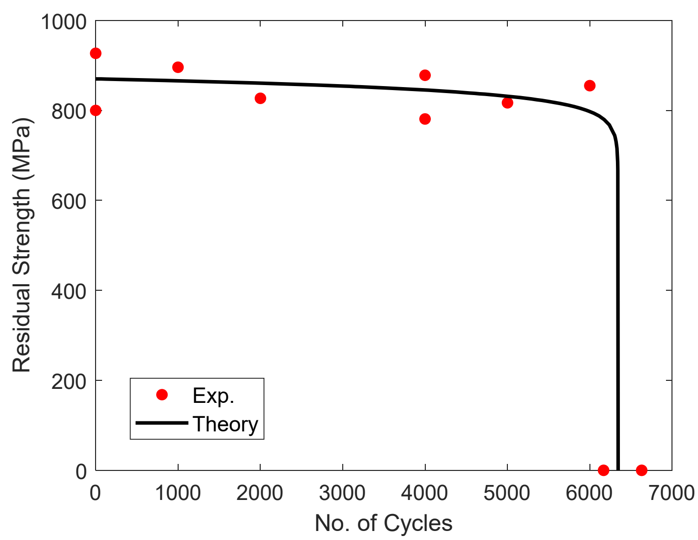

Figure 6: Comparison of residual strength of unidirectional composite subjected to cyclic loading.

7 CONCLUSIONS

Previous failure criteria were briefly but critically reviewed. Then, the unified failure criteria were discussed along with a multiscale model which uses micro-stresses and micro-strains to predict failures. The failure criteria are universal so that they can be applied to structural members with or without sharp or blunt notches. The proposed failure model and criteria would also minimize or eliminate experimental tests which are normally required as the design parameters of laminated composites are varied because their material properties change with those parameters.

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