

ADVANCED MULTIFIELD MODELS FOR WAVE PROPAGATION ANALYSIS IN SMART COMPOSITE PANELS

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ABSTRACT

The use of non-destructive techniques (NDTs) is essential in health monitoring applications. This ensures the inspection of the examined structure without inflicting damage. As ultrasonic wave propagation has been widely used in such applications, it is crucial to understand the modelling parameters for such phenomenon. To accomplish this, higher order finite element models based on the Carrera Unified Formulation (CUF) have been used to study the Lamb wave propagation in composite panels, in a coupled electro-mechanical model where the surface mounted actuators and sensors were modelled. Both fundamental symmetric (S₀) and antisymmetric (A₀) Lamb wave propagations were considered. The convergence of the plate model was studied under different modelling parameters, including mesh density, to-the-thickness kinematic model and the number of timesteps. The results show that the use of higher order models is necessary to obtain a systematic low error in comparison with the analytical group velocity obtained from the material dispersion curves.

1 INTRODUCTION

Composite materials are characterized by high specific mechanical properties, which have led to a wide and increasing use of these materials in the aeronautical, aerospace, wind energy, automotive as well as in other sectors worldwide. However, it is also well-known that the failure of the structures made of said materials tend to be abrupt due to the absence of the plastic behaviour region before final failure. Thus, active health monitoring of such structures is essential throughout their life cycle due to the complex damage mechanisms involved compared to metals [1] in the absence of accurate analytical models that permits the prediction of their failure.

A suitable option for the detection of damage can be by capturing and studying the generated lamb waves throughout the structure [2] via the use of piezoelectric sensors. A passive wave-based structural health monitoring (SHM) approach would be where, as a result of the damage phenomena, for example impact and cracks, acoustic waves are generated, that propagate in the structure to be captured by sensors. As an alternative, active SHM is where the wave is generated by the use of an actuator. Two different configurations in active SHM are frequently mentioned and used in literature [3], the Pulse-echo method and the Pitch-catch method. The difference between them is emphasized in (Figure 1).

On the other hand, the use of higher order beam [4] and plate models for wave propagation in transient state [5, 6] have been studied to depict the relation between the accuracy of numerical modelling lamb wave propagation and the use of different model kinematics in isotropic structure.

In this framework, a multifield layer-wise finite element plate model, based on the Carrera Unified Formulation (CUF) was introduced to model the waves propagating through a smart composite carbon fiber slender structure with a hybrid sensor-actuator network. These waves are generated by a couple of surface-fitted piezo-ceramic actuators on one end and received by a piezo-ceramic sensor on the other end, in pitch-catch phenomena. The group velocity of the propagated wave was evaluated for the different layer-wise model kinematics employed. This velocity was compared with the analytical group velocity of the corresponding wave propagating in the layered specimen.



Figure 1: Actuator-sensor in (a) pitch-catch and (b) pulse-echo configurations

In order to further improve the efficiency of the proposed model, a Node Dependent Kinematic (NDK) approach [7, 8], was employed, where the kinematics of the model were reduced in the noncritical areas, to further decrease the computational cost of the model, bringing to light the desirability and convenience of using the proposed numerical tool in modelling wave propagation in bigger and more complex structure.

2 NUMERICAL MODEL

The finite element model used is the one developed by the MUL2 Group, which is based on the Carrera Unified Formulation. Thanks to this model, it is possible to use one and two-dimensional models with three-dimensional accuracy. In the framework of plate 2D models, the electro-mechanical 3D displacement vector of unknowns u (x, y, z, Φ), with Φ as the electric potential, can be split into two terms; the first term, along the x-y plane, corresponds to the unknown FEM 2D mesh, and a second term, along the thickness z deals with the order of kinematics used in the model.

$$u = u_{\tau}(x, y).F_{\tau}(z) \tag{1}$$

The usual convergence study is done by the refinement of the first term which can be carried out using any commercial FEM tool by refining the mesh of the 2D model, either by increasing the number of elements or by changing the type of element used. On the other hand, the second term deals with the kinematics of the model, which allows to use refined models to describe the stresses to-the- thickness instead of adopting the classical laminate theory (CLT) or the first order shear deformation theory (FSDT). This option is not available in commercial FEM software, which means that they are limited to the kinematics used. i.e. no matter how refined the 2D mesh, it will still be bounded by the used kinematics. Using a higher order expansion to-the-thickness allows for the modeling of 3D problems using plate models. This expansion can be in two forms, either in the form of a Layer-wise expansion (LW) referred to as LE, where Lagrange Polynomials (linear B2, quadratic B3, or cubic B4) are used in each layer successively, maintaining a continuous profile of the strains to-the-thickness, or in the form of an equivalent single layer (ESL) expansion referred to as TE, where the whole plate thickness is considered as one layer and Taylor Expansion polynomials are used to the chosen order;

For TE to the nth order denoted by (TE*n*), the second term $F_{\tau}(z)$ is written in the terms of z^n where n is the order of the expansion, reserving n= -1 and n=0 for CLT and FSDT respectively. Taking the fourth order as an example, the general displacement is given in equation (2), where U_i are the 20 unknowns of the problem with U the displacement vector {u, v, w, Φ },

$$U = 1. U_0 + z. U_1 + z^2 U_2 + z^3 U_3 + z^4 U_4$$
(2)

For LE on the other hand, and taking the linear B2 element as an example, the displacement vector U can be written in terms of linear Lagrange polynomials F_1 and F_2 as:

$$U = F_1 U_1 + F_2 U_2$$

Where U_1 and U_2 are the actual displacements at the top and the bottom of the plate or of the layer element, and the polynomials F_1 and F_2 are given by equation (3), where $-1 < \zeta < 1$. On the top, $\zeta = 1$, $F_1 = 1$ and $F_2 = 0$. On the bottom, $\zeta = -1$, $F_1 = 0$ and $F_2 = 1$

$$F_1 = \frac{1+\zeta}{2}$$
 and $F_2 = \frac{1-\zeta}{2}$ (3)

For additional information regarding the used refined plate models, it can be referred to [9, 10] where ESL and LW models were presented in detail.

3 COMPOSITE BENCHMARK PROBLEM

Unlike the previous study in [6], where the parameters of modeling lamb wave propagation were studied for an isotropic aluminum structure, the studied material in this study is a unidirectional carbon fiber/ epoxy (T650-F584), with the material properties shown in Table 1. The configuration of at which the orientation of the layers is $[0,90,90,0]_s$ with respect to the direction of wave propagation, each layer of thickness 0.125mm for laminate of 1mm thickness, was studied.

E11	E_{22}, E_{33}	G ₁₂ , G ₁₃	G ₂₃	V ₁₂ , V ₁₃	V 23	density
153.67 GPa	9.49 GPa	4.26 GPa	3.44 GPa	0.295	0.381	1528 kg/m ³

Table 1: Elastic properties of the unidirectional CF/EP taken from [11]

The dispersion curves of this material were plotted using an available tool (DC2.0) in order to properly choose the excitation frequencies where only the fundamental Lamb modes are excited. As shown in Figure 2 it was chosen that the best frequency of excitation was at f = 600 kHz.



Figure 2: Dispersion curves of the used material showing in (a) phase velocity and in (b)group velocity curves

At this frequency the symmetric and antisymmetric fundamental lamb waves can be excited, without exciting any higher order modes. From these curves, the phase velocity was extracted and the analytical Time of Flight (TOF) of the wave was calculated. On the other hand, the numerical TOF was calculated by plotting the envelope $e_{A,B}(t)$ in equation (4) of the displacements/ electric potential using Hilbert transform $H_{A,B}(u(t))$ at two points and calculating the difference $t_C = t_B - t_A$ of the centroids of the area formed under each of these envelops.

$$e_{A,B} = \sqrt{H_{A,B}(u(t))^{2} + u_{A,B}(t)^{2}} \quad \text{with } H_{A,B}(u(t)) = \frac{1}{\pi} \int_{-\infty}^{\infty} u_{A,B}(\tau) \cdot \frac{1}{t - \tau} d\tau$$
(4)

$$t_{A,B} = \frac{\int_0^{t_{end}} e_{A,B}(t) \cdot t \, dt}{\int_0^{t_{end}} e_{A,B}(t)}$$
(5)

The excitation of the Lamb wave propagated in the structure was achieved by the use of piezoelectric PZT actuators modelled and situated on the surface of the panel. The individual actuation enables the selective generation of symmetric and antisymmetric waves. As the wave is generated, it is possible to either acquire the displacement at any structural node or to acquire the developed voltage at the surface mounted sensor. As the piezo-elements occupy a certain volume, it was chosen to consider to displacement of the nodes at points A and B in the parametric study (Figure 3), when comparing to analytical results. Whereas in the case when using node-dependent-kinematics, the voltage developed at the sensors was compared.



Figure 3: Benchmark problem with the actuators and sensors positioned, dimensions in mm

4 RESULTS AND DISCUSSION

4.1 Antisymmetric wave propagation/ mesh-size and model kinematics analysis

The propagation of antisymmetric waves was studied in this section. The fixed modelling parameters were the total modelling time $T = 300 \mu s$ and the number of timesteps N = 5000 steps, whereas the studies parameters were the mesh refinement, and the kinematics of the model. The plate element mesh used was the 9 node (Q9) element, whereas the model kinematics were the layer wise linear B2, quadratic B3, or cubic B4 elements in each lamina. Under these modelling parameters, the error of the TOF was plotted as a function of the computational degrees of freedom (DOF). As shown in Fig.Figure 4, one can see the effects of mesh refinement and the model kinematics, as the error is lower the mode refined the model is. It is clear that using a 500-mesh element density is not favourable as the error is extremely high regardless of any model used. However, as the mesh density is increased to 1000 and 1500 elements, it is shown that the results are converging. Although the convergence value is not zero, this is due to the remaining model parameters already fixed, namely the time step as will be shown later.



Figure 4: Antisymmetric wave propagation under refined elements and model kinematics

4.2 Symmetric wave propagation/ time-step and model kinematics analysis

For the symmetric wave propagations, the fixed parameters were the mesh density of 1000 Q9 plate elements, the total modelling time $T = 100\mu s$, whereas the studied parameters were the model kinematics and the number of time steps N. The results of the error of TOF obtained were plotted in Fig.Figure 5, where it is shown the effect of increasing the number of timesteps, i.e. decreasing the time step of the analysis on the error obtained, on the side of also showing the convergence of the results with respect to the model kinematics used. It is obvious that using B3 and B4 to-the-thickness kinematics is more desirable, as the model converges. Moreover, it can be seen that the higher the number of timesteps N the closed the model converges to reach a value of a null error. It is also important to mention that the effect of a less refine timestep is a negative error which causes the whole curve to shift down, thus converging to a negative error.



Figure 5: Symmetric wave propagation showing the number of timestep and model kinematics effect on the obtained error

5 NODE DEPENDENT KINEMATICS

In order to decrease the computational cost of the dynamic analysis, a Node Dependent Kinematics (NDK) approach was adopted. Using this approach, it is possible to locally refine the problem in certain critical areas. The areas of refinement chosen are the areas where the actuator and the sensor are placed. The kinematics of the rest of the model was changed, adopting an ESL kinematics to-the-thickness, as seen in Figure 6.



Figure 6: NDK benchmark showing the different models used, dimensions in mm

The error of the A0 Lamb wave propagating was calculated as the difference between the TOF in a

refined LE model and compared to the corresponding model to the order of the TE*n* used. The used model in the LE was cubic B4 elements in each layer to the thickness, of 1000 elements with 459,573 DOFs. The TOF was calculated between the centroids of the envelops of both the actuation voltage and the voltage produced at the sensors. The graph in Figure 7 shows the results of the error as well as the reduction in the DOF for each case.



Figure 7: NDK results for a propagating A0 Lamb wave

From these results, it is possible to see that using the classical theories produce a high error, with around 70% error for CLT. Using FSDT, TE1 and TE2 produce almost the same results, with an error of around 3%, but with a reduction of 83% in the computational cost. In order to acquire satisfactory results, of an error less than 1%, TE5 and higher must be used. The reduction in DOF for using TE5 is up to 60%, which is quite interesting, and can be exploited to model accurately with lower computational cost.

9 CONCLUSIONS

The lamb wave propagation in composite slender plate was studied by the use of a refined plate models based on the CUF unified formulation. Modeling parameters were assessed for the dynamic modelling of fundamental lamb wave propagation in a coupled electro-mechanical case. The studied parameters were focused on the element mesh size, the time step and most importantly on the kinematics of the plate model to-the-thickness. It was found that in order to get satisfactory results, higher order model kinematics need to be used. Complimentary to that, the time step refinement must be performed, as it is apparent that the already recommended values are those calibrated when using low order models. Using low order models, the obtained error is a positive value, while using a non-refined timestep gives a negative value error. Even though sometimes, the resulting error is close to zero, it still is unsystematic and problem dependent.

Moreover, the node dependent kinematics approach was implemented to show the effect of local refinement of the model. It was shown, in this case, that the use of equivalent single layer model kinematics in non-critical areas reduces the computational cost by more than 60%, while only contributing with an error of less than 0.15% compared to the fully refined LW model.

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