MICROMECHANICAL JUSTIFICATION OF THE INTERACTIVE TERM IN THE TSAI-WU FAILURE CRITERION

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ABSTRACT

A micromechanical finite element analysis was conducted in this paper to reveal the interaction effect between the direct stresses along and transverse to the fibres in unidirectional (UD) fibre-reinforced composite materials under biaxial loading conditions, which could have a great influence on the failure behaviour of UD composites under in-plane biaxial loading conditions. The analysis was performed on a hexagonal unit cell (UC) and a representative volume element (RVE) with fibre distributed randomly generated in the software UnitCells©, and rational boundary conditions were introduced to the RVEs. The influence of the types of RVEs on the calculated stress fields was analysed by comparing the maximum longitudinal stress and average longitudinal stress in fibres in two RVEs under uniaxial transverse compression loading. Then, uniaxial longitudinal tension loading and in-plane biaxial transverse compression and longitudinal tension loading were imposed on the RVE with fibre distributed randomly. As a characterization of longitudinal stress filed in fibres, the corresponding maximum and average longitudinal stresses in fibres in the RVE under the two loading conditions were extracted and compared, respectively. The comparison results show that the interaction effect could lead to a 31.71% and 12.30% increase for maximum and average longitudinal stress in fibres, respectively, indicating that transverse compression loading could produce significant longitudinal tensile stress in fibres, and hence greatly influences the longitudinal failure of composites. According to the quantitative justification results, the interaction between longitudinal and transverse stresses should be taken into consideration when predicting the failure of composites under multiaxial loading conditions.

1. INTRODUCTION

As the increase use of fibre reinforced polymer (FRP) composites in industries, failure prediction becomes a very important but challenging issue for the application of composite materials due to the complicated mechanical behaviour and various service conditions. Quantities of failure criterions were proposed to predict the failure of UD fibre-reinforced composites under different loading conditions, several popular criterions include Hashin failure criterion, Tsai-Wu failure criterion, Puck's failure criterion and so on. Among these criterions, some physical-based failure criterions assume that the failure is only determined by the stresses which are on the fracture plane and neglect the interactions between the direct stresses along and transverse to the fibres [1] as was included in the Tsai-Wu failure criterion [2]. However, the influence of such interaction effect on the failure of fibre-reinforced UD composites is still unclear, therefore, it is necessary to quantitively justify this interaction effect for the assessment of the assumption made in the failure criterions.

Currently, micromechanical analysis is extensively used for the analysis of mechanical behaviour of composite materials and proved to be an efficient method [3]. Wan et al. [4] reviewed the development of micromechanics-based RVE modelling for the failure analysis of UD fibre-reinforced composites by means of the finite element method (FEM) and discrete element method (DEM). Numerous researchers have dedicated to some important issues in the micromechanical RVE modelling of composite materials for more accurate results of failure prediction, including the generation of random fibre distribution, constitutive modelling of UD composite constituents, definition of periodic boundary conditions as well as failure criterion. The micromechanical analysis is also adopted to assess different failure criterions of UD composites under multiaxial loading conditions since experimental research could be high-cost and difficult to perform [5–7]. Among these currently proposed micromechanical models, the quantitively study on interaction effect between the direct stresses along and transverse to the fibres in UD composites under multiaxial loading conditions has drawn little attention, which could have a great influence on the final failure prediction of UD composite materials under multiaxial loading conditions.

In this paper, micromechanical finite element analysis of unidirectional fibre reinforced composites was carried out in the software UnitCells© [8] to quantitively justify the interaction effect between the direct stresses along and transverse to the fibres. In the analysis, two typical RVEs were adopted, including a hexagonal unit cell and a RVE with fibre distributed randomly over cross-section, and properly prescribed boundary conditions deduced on the basis of symmetry consideration and St Venant's principle were generated in the software so that accurate results could be obtained. In order to investigate the influence of different types of RVE on the obtained stress fields, a uniaxial transverse compression loading was applied on the two RVEs, then maximum longitudinal stress and average longitudinal stress in fibres were calculated in hexagonal unit cell and compared to the ones from the RVE with fibre distributed randomly. Two other loading conditions, including uniaxial longitudinal tension loading and in-plane biaxial transverse compression and longitudinal tension loading were imposed on the RVE with fibre distributed randomly, which is considered to be more practical, and longitudinal stresses in fibres under the two loading conditions were compared to quantitively justify the influence of transverse compression loading on the longitudinal stresses in fibres, as a result, the influence of interaction effect on the failure of unidirectional composites could be quantitively characterized.

2. MICROMECHANICAL ANALYSIS MODEL

In order to study the influence of types of RVE on the stress result and quantitively illustrate the interaction effect, two types of analyses have been conducted, one on a hexagonal UC based on an idealised fibre packing and the other on a RVE incorporating the effects of random distribution of fibres over the cross-section of UD (unidirectional) composites, as shown in Fig. 1.

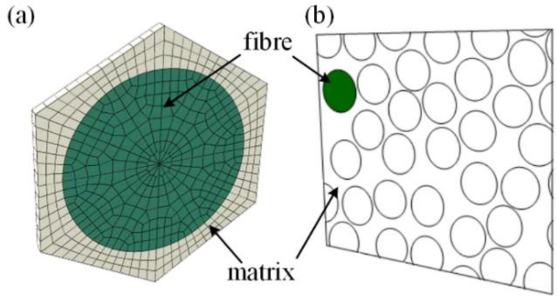


Figure 1: Two typical RVEs used in the analysis. (a) Hexagonal UC (b) Fibres distributed randomly over cross-section

In the analysis, the fibres are considered as transversely isotropic linear elastic material and the matrix is considered as isotropic linear elastic, and their elastic properties are shown in Table 1 and Table 2, respectively [9]. Besides, the volume fraction of fibres is chosen as 60%, and a linear and fully integrated brick element C3D8 is used for both fibre and matrix in the finite element model.

E_1(GPa)	E_2(GPa)	G_{12}(GPa)	v_{12}	v_{23}
214	26	112	0.28	0.445

Table 1: Elastic properties of fibre.

E(GPa)	v
4.1	0.356

Table 2: Elastic properties of matrix.

2.1 Boundary conditions

In order to obtain convincing results in the micromechanical analysis, proper boundary conditions should be defined. For the hexagonal unit cell, relative displacement boundary conditions derived by Li et al. based on rational symmetry consideration were imposed on the free edges of the hexagonal unit cell in the UnitCells©, as introduced in references [8,10,11].

For RVE with fibres distributed randomly over the cross-section, a uniformly displacement boundary conditions were assumed. Although the prescribed boundary condition is inaccurate, the results obtained from an inner sub-domain of the RVE will be sufficiently accurate when the boundary of the sub-domain is sufficiently far from the external boundary according to the St Venant's principle, as revealed in [12].

2.2 Loading conditions

Three different loading conditions were considered in the micromechanical analysis, *viz.*, uniaxial transverse compression, uniaxial longitudinal tension and in-plane biaxial transverse compression and longitudinal tension loading, as illustrated in Fig. 2, and the values of applied stresses were summarized in Table 3. The direction '1' denotes the direction parallel to the fibres, while the direction '2' denotes the direction transverse to the fibres, and the direction '3' indicates the thickness direction.

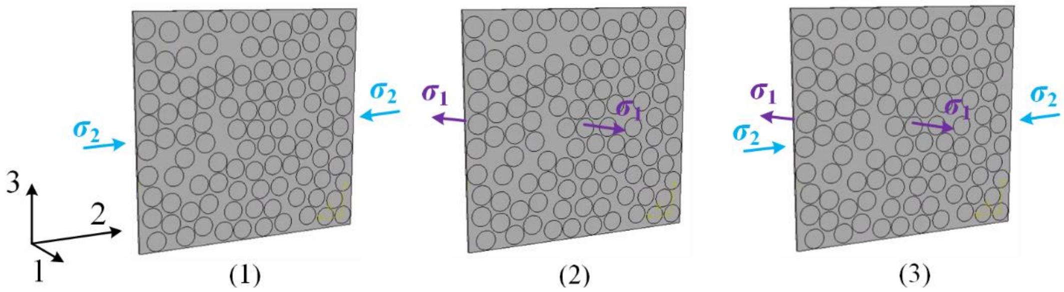


Figure 2: Illustration of three different loading conditions

Loading Cases	1	2	3
Longitudinal stress σ_1 (MPa)	0	100	100
Transverse stress σ_2 (MPa)	-100	0	-100

Table 3: Values of three loading conditions.

3. RESULTS AND DISCUSSIONS

Considering the longitudinal failure of UD composites is ultimately related to the breakage of fibres, which is greatly influenced by local stress concentration and local strength distribution in fibres [13], the longitudinal stress distribution and maximum longitudinal stress in fibres were extracted from the two RVEs under different loading conditions for the quantitative justification of the influence of transverse loading on the fibre breakage failure, and an average longitudinal stress in fibres was also calculated according to the following equation,

$$\bar{\sigma}_f^{11} = \frac{\sum_{i=1}^{n} \sigma_i^{11} S_i}{\sum_{i=1}^{n} S_i} \tag{1}$$

where n is the total number of fibre elements, σ_i^{11} is the longitudinal stress of i-th fibre element and S_i denotes the area of front face of i-th fibre element, which could be calculated from the coordinates of its four nodes.

3.1 Results of two RVEs under uniaxial transverse compression loading

Longitudinal stress fields in fibres in the two RVEs under uniaxial transverse compression loading (loading case 1) were shown in Fig. 3. Also, the maximum longitudinal stress and average longitudinal stress in the fibres were extracted and calculated as listed in Table 4.

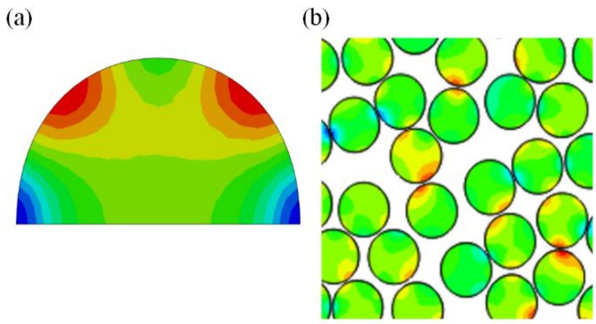


Figure 3: Longitudinal stress field in fibres. (a) Hexagonal UC, (b) RVE with fibres distributed randomly

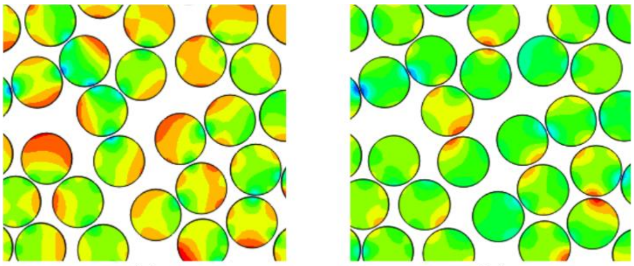
	Hex	Random
Maximum longitudinal stress (MPa)	16.81	53.33
Average longitudinal stress (MPa)	13.62	20.91

Table 4:. Comparison of longitudinal stresses in fibres in two RVEs under transverse compression loading

According to results of hexagonal UC, 100 MPa transverse compression would result in a tension stress in fibres whose maximum and average value are 16.81 MPa and 13.62 MPa, respectively. For RVE with randomly distributed fibres, the maximum and average value of induced longitudinal stress in fibres are 53.33 MPa and 20.91 MPa, respectively. As an idealized packing, hexagonal UC assumes that fibres are all uniformly distributed over the transverse cross-section, however, fibres are often randomly distributed in the practical UD fibre-reinforced composites, and the distances between two adjacent fibres would vary randomly. As a result, stress concentration in fibres would be more serious while two adjacent fibres are getting close, leading to a larger local longitudinal stress in fibres, as shown in Figure 3(b). Hence, the maximum and average longitudinal stress in RVE with fibres distributed randomly are much larger than the ones in hexagonal UC when suffering the same transverse loading condition. The results obtained from RVE with fibres distributed randomly are considered to be more convincing as well, since the fibre distribution is more in accordance with practical situation. Taking this into consideration, the RVE with fibres distributed randomly was chosen to perform the analysis with uniaxial longitudinal tension loading and in-plane biaxial loading.

3.2 Results of RVE with fibres distributed randomly under different load conditions

Longitudinal stress fields in fibres in RVE with fibres distributed randomly under uniaxial longitudinal tension loading and im-plane biaxial loading condition were shown in Fig. 4, and the magnitude of maximum longitudinal stress and average longitudinal in fibres under the two loading conditions were summarized in Table 5.



(a) (b)

Figure 4: Longitudinal stress fields in fibres under two loading conditions.(a) Loading Case 2; (b) Loading Case 3

	Case-2	Case-3	Increasing percentage
Maximum longitudinal stress(MPa)	170.08	224.01	31.71%
Average longitudinal stress (MPa)	170.03	190.94	12.30%

Table 5: Comparison of longitudinal stresses in fibres under two loading conditions.

Comparing the longitudinal stress fields in fibres from Case-3 with those from Case-2, it could be noted that the transverse compression leads to a 31.71% and 12.30% increase of maximum and average longitudinal stress in fibres, respectively, indicating that transverse compression loading would produce significant longitudinal tensile stress in fibres, especially the maximum stress has a greater increase due to the interaction effect. As a result, when under im-plane biaxial tension-compression loading, the transverse compression-induced tension stress will be placed on top of the applied longitudinal tension loading, resulting in prediction of reduced strength comparing with the prediction without such interaction. Alternatively, if the composite is loaded in compression in the fibre direction, i.e., when suffering in-plane biaxial compression-compression loading, a part of the transverse compression-induced longitudinal tension stress will be consumed in cancelling the tension in fibres, resulting in prediction of increased strength comparing with the prediction without such interaction.

Since the transverse tensile strength of UD composites is usually much smaller than the transverse compression strength, the magnitude of transverse tensile-induced stress would be much smaller than the one of transverse compression-induced stress at the failure point of UD composites under biaxial loading conditions, so the transverse tension loading would make little difference on the longitudinal stress in fibres compared to the transverse compression loading, and hence little influence on the longitudinal failure of UD composites. It should be noted that although this conclusion is drawn based on elastic analysis of UD composites, it would be still valid for the final longitudinal fibre failure prediction under the assumption of linear elastic behaviour of fibres.

As shown in Fig. 5, failure envelops of typical unidirectional carbon fibre reinforced composite and unidirectional glass fibre reinforced composite [14] under in-plane direct stress state with and without the interaction term F_{12} according to rationalised Tsai-Wu failure criterion [2] were compared, and the comparison results clearly show that the transverse compression loading could lead to pronounced increase and decrease of longitudinal compression failure stress and tension failure stress, respectively, while the transverse tension loading has little influence, which is in accordance with the model results and discussions presented above.

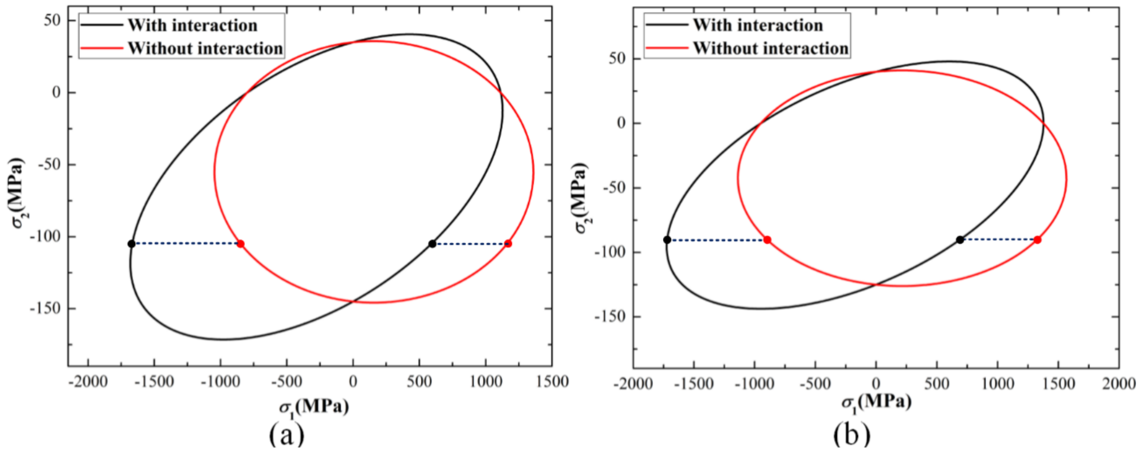


Figure 5: Comparison of failure envelops with and without interaction term in Tsai-Wu failure criterion. (a) carbon fibre reinforced composite; (b) glass fibre reinforced composite

4. CONCLUSION

A micromechanical analysis was performed to quantitively justify the effect of interaction between the transverse and longitudinal stress in UD composites under multiaxial loading conditions in this paper. The analysis was conducted on two typical RVEs in the software UnitCells©, including a hexagonal unit cell and a RVE with fibre distributed randomly over the cross-section. Proper boundary conditions with rational considerations were imposed on the two RVEs in the micromechanical analysis to ensure the accuracy of analysis results.

In the analysis, two parameters, *viz.*, maximum longitudinal stress and average longitudinal stress in fibres were calculated to characterize the longitudinal stress filed in fibres under different loading condition. By comparing the two stress parameters calculated from the two RVEs under uniaxial transverse tensile loading, it could be concluded that the RVE with fibre distributed randomly would result in more practical and convincing longitudinal stress fields in fibres. Further, the comparison of the two stress parameters in the RVE with fibres distributed randomly under uniaxial longitudinal tension loading and in-plane biaxial transverse compression and longitudinal tension loading were presented, and the comparison results show that when in-plane biaxial loading is imposed on the fibre-reinforced composite materials, transverse load will result in significant longitudinal stress in fibres, which could contribute to the failure in the longitudinal direction, especially for transverse compression loading. As a result, the interaction effect between longitudinal and transverse stresses should be taken into consideration when predicting the failure of fibre-reinforced composites under multiaxial loadings.

REFERENCE

[1] Z. Hashin, Failure criteria for unidirectional fibre composites, *ASME Journal of Applied Mechanics*, 1980, **47**, pp.329–334 .

[2] S. Li, M. Xu, Sitnikova, Elena, Fully rationalised Tsai-Wu failure criterion for transversely isotropic materials. In *Double-Double—A New Perspective in Manufacture and Design of Composites*; Tsai, S., Ed.; JEC & Stanford, 2022.

[3] S. Li, General Unit cells for micromechanical analyses of unidirectional composites, *Composites: Part A*, 2000, **32**, pp.815–826 .

[4] L. Wan, Y. Ismail, Y. Sheng, J. Ye, D. Yang, A review on micromechanical modelling of progressive failure in unidirectional fibre-reinforced composites, *Composites Part C: Open Access*, 2023, **10** (doi:10.1016/j.jcomc.2023.100348).

[5] R. Gutkin, S.T. Pinho, P. Robinson, P.T. Curtis, Micro-Mechanical Modelling of Shear-Driven Fibre Compressive Failure and of Fibre Kinking for Failure Envelope Generation in CFRP Laminates, *Composites Science and Technology*, 2010, **70**, pp.1214–1222 (doi:10.1016/j.compscitech.2010.03.009).

[6] Q. Sun, G. Zhou, Z. Meng, H. Guo, Z. Chen, H. Liu, H. Kang, S. Keten, X. Su, Failure Criteria of Unidirectional Carbon Fiber Reinforced Polymer Composites Informed by a Computational Micromechanics Model, *Composites Science and Technology*, 2019, **172**, pp.81–95 (doi:10.1016/j.compscitech.2019.01.012).

[7] F. Naya, C. González, C.S. Lopes, S. Van der Veen, F. Pons, Computational Micromechanics of the Transverse and Shear Behavior of Unidirectional Fiber Reinforced Polymers Including Environmental Effects, *Composites Part A: Applied Science and Manufacturing*, 2017, **92**, pp.146–157 (doi:10.1016/j.compositesa.2016.06.018).

[8] S. Li, E. Sitnikova, *Representative volume elements and unit cells: concepts, theory, applications and implementation*; Woodhead Publishing, 2019;

[9] E. Totry, C. González, J. LLorca, Prediction of the failure locus of C/PEEK composites under transverse compression and longitudinal shear through computational micromechanics, *Composites Science and Technology*, 2008, **68**, pp.3128–3136 (doi:10.1016/j.compscitech.2008.07.011).

[10] S. Li, Boundary conditions for unit cells from periodic microstructures and their implications, *Composites Science and Technology*, 2008, **68**, pp.1962–1974 (doi:10.1016/j.compscitech.2007.03.035).

[11] S. Li, General unit cells for micromechanical analyses of unidirectional composites, *Composites Part A:* Applied *Science and Manufacturing*, 2000, **32**, pp.815–826 .

[12] A. Wongsto, S. Li, Micromechanical FE analysis of UD fibre-reinforced composites with fibres distributed at random over the transverse cross-section, *Composites Part A: Applied Science and Manufacturing*, 2005, **36**, pp.1246–1266 (doi:10.1016/j.compositesa.2005.01.010).

[13] R. Talreja, J. Varna, *Modeling damage, fatigue and failure of composite materials*; Elsevier, 2015;

[14] A.S. Kaddour, M.J. Hinton, P.A. Smith, S. Li, The background to the third world-wide failure exercise, *Journal of Composite Materials*, 2013, **47**, pp.2417–2426 (doi:10.1177/0021998313499475).