PARAMETERS AFFECTING THE BIFURCATION POINT OF UNSYMMETRIC LAMINATES

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ABSTRACT

When an un-symmetric laminate (such as that made of carbon/epoxy with lay-up sequence [0/90]) is cooled from cure temperature to room temperature, it can take up either one of the three shapes: saddle shape, cylindrical shape with a curvature in the x-direction, and cylindrical shape with a curvature in the y-direction. This is due to competition between bending along the x- and y-directions. This competition takes place as the material stiffens during cooling. When both directions have the equal capacity, the saddle shape occurs. At a certain point during the cooling process, one side takes over (this is due to inherent defects or variability within the composite, which makes one side stronger or weaker than the other side). When one side takes over, the bending leans more and more toward that side, and the major mode is cylindrical in that direction. The point of taking over is called the bifurcation point. The determination of the shape of the laminate at room temperature depends on the value of the bifurcation temperature with respect to room temperature. There are many parameters that affect the bifurcation temperature of a laminate. These include the material properties, lay-up sequence, the ratio between the sides a and b (a/b) of the laminate, and the ratio between the thickness and side of the laminate (t/a). This paper examines the effect of t/a over the value of the bifurcation temperature (T_b). This is done by using finite element analysis with special treatment for stability. The results are compared with the extended laminate theory by Hyer.

1 INTRODUCTION

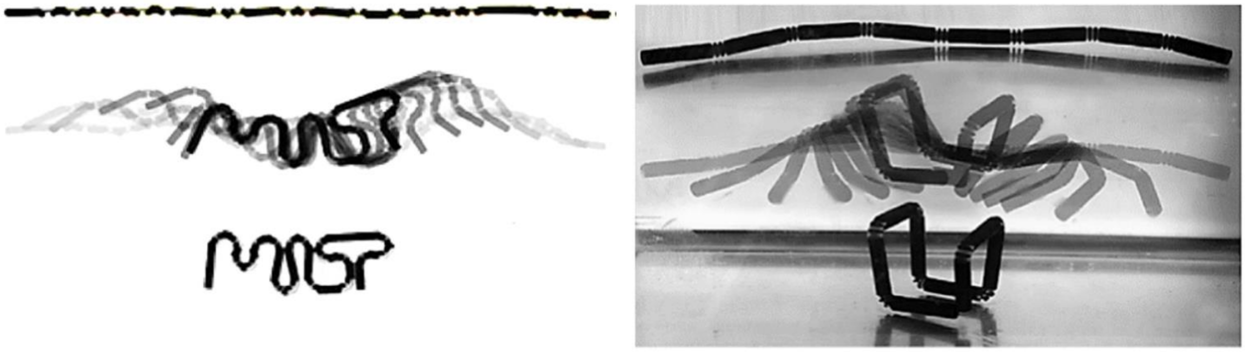
The use of composite structures has seen a significant rise in recent times due to their advantageous properties, such as being lightweight, having high stiffness and strength, good resistance to fatigue and corrosion, etc. Among the various types of composite materials, those made of fibre-reinforced polymers are the most widely used in industrial applications [1]. Composite materials are employed across several industries, serving a variety of purposes. For instance, they are utilized as stiffeners, rudders, and engine intake manifolds in the aerospace industry and as automotive gas and clutch pedals in the automobile industry. Additionally, they are used as elevator cables and for reinforcing beams. The method of production for composite structures varies depending on the application and components; some examples include filament winding for the creation of pressure vessels and cylinders, Hand-Lay-Up, and Autoclave Molding techniques for the production of aircraft structures such as flaps and rudders [2]. When it comes to complex-shaped designs and components, the need for complex-shaped molds or the AFP machine is essential.

3D printing has garnered significant attention in the manufacturing industry as it enables the production of complex geometric structures that were previously unachievable; this process is also referred as additive manufacturing. The introduction of 4D printing, a concept developed by Skylar Tibbits and his colleagues at MIT in 2014 [3], has taken additive manufacturing to an advanced level. 4D printing is a novel aspect of the conventional 3D printing technology, invented in the early 1980s and this technique allows materials to change shape or properties over time. This advancement has the potential to change the way in which objects are designed, produced and utilized. [4]. Regular 4D printing is defined as a process in when a 3D-printed object is subjected to a stimulus and changes its shape. Considering a three-dimensional box flattens itself when it is impacted by some stimuli or a flattened paper folds itself to create a box, these are some examples of the regular 4D printing process, when the shape and property of a component change as a function of time[5][6]. The main drawback of traditional 4D printing is that the materials typically used, such as polymer or plastic, have low stiffness and strength, making them unsuitable for carrying loads or engineering applications. However, 4D Printing of Composites (4DPC) addresses this limitation by utilizing the same principle as 4D printing but with materials made of long continuous fiber composites, which have been proven to have high strength and stiffness and have been used in the construction of critical engineering structures such as aircrafts, automobiles, and wind turbine blades. [7].

Tibbits studied the self-folding process, which occurs when a structure is built using two different materials. One is rigid, while the other is active, like hydrogel, which can absorb water and change its shape, while the rigid component maintains its shape. An example of this can be seen in

Figure 1-a, where a piece made of active and rigid materials is immersed in water, and the final shape of "MIT" appears. Additionally, self-folding can also take the form of 1D-to-3D, where a single line can change its shape to a cube, as depicted in

Figure 1-b [5].



(a) (b)

Figure 1: Self-folding process: a: 1D-to-2D and b: 1D-to-3D [5]

In 2017, Hoa [7] presented a new method, known as 4D Printing of Composites (4DPC), using unsymmetric lay-up for a laminate constructed from carbon-reinforced composites. The shape of the flat piece changes as it cools from the curing temperature to room temperature; this is due to the difference in mechanical properties of unidirectional prepreg in different directions. By applying Classical Laminate Theory (CLT) and accounting for the effect of shrinkage in the matrix and thermal coefficient in different directions, Hoa was able to calculate the final curvature of the unsymmetric laminate after curing. Hoa [8] applied this method to produce 4D-printed leaf springs; he created two springs with a width of 3 inches and length of 12 inches and 24 inches, respectively. Both samples were made of carbon/epoxy, CYTEC 977-2, and the lay-up sequence used was $[0_{16}/90_{24}]$.

Figure 2 shows the 24-inch spring and two strain gages to measure deflection during the three-point bending test, in which the 0° layers are on the convex side, and the 90° layers are on the concave side. He calculated the spring constant and compared it to that of conventional springs and theoretical predictions. He also performed a fatigue test on the 24-inch sample, and the results showed that the spring constant remained unchanged after 175000 cycles.

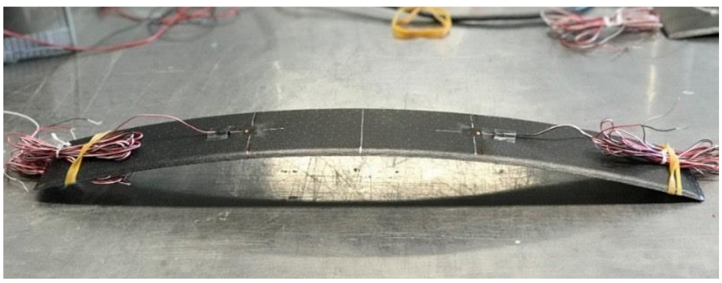


Figure 2: 24-inch long composite leaf spring [8]

Hoa et al. [9] also investigated another application of 4D Printing of Composites; they used the technique to create letters using different lay-up sequences. They formed the word "Concordia" as shown in

Figure 3.

Figure 4-a illustrates a schematic of the lay-up sequence of the letter “C”, which includes a symmetric base with lay-up sequence $[0/90]_s$ and a 20-inch long unidirectional prepreg, as well as five sections with 0º and 90º lay-up on top of this unidirectional prepreg. In sections 1, 3 and 5, the lay-up sequence is $[0_4]$, which leads to a straight shape both before and after curing. In contrast, the lay-up sequence of sections 2 and 4 is $[0/90_3]$, which results in a curved shape for these sections.

Figure 4-b shows the final shape of the letter, which is quite similar to the ideal shape, but with some deformation. For instance, section 3 should be straight, but a kind of twist can be observed. This occurs because the laminate is very thin and has to support the upper sections, which impart a bending moment.

Figure 3: Final configuration of the word “Concordia” [9]

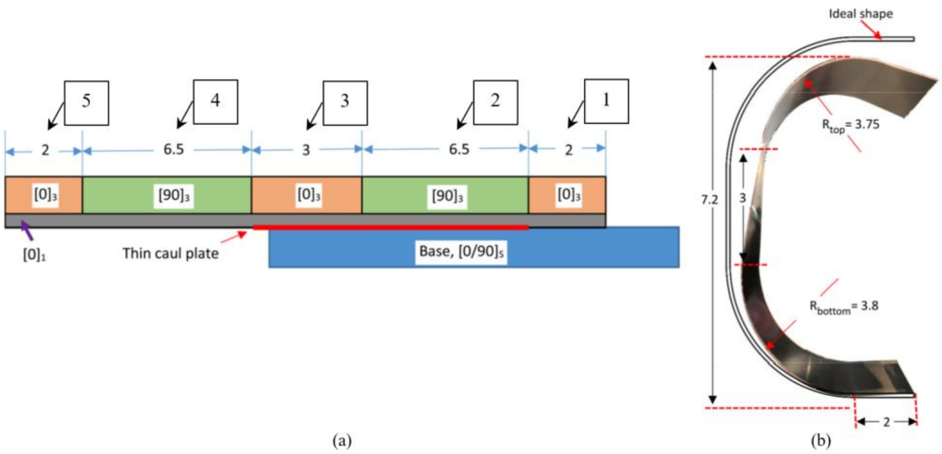
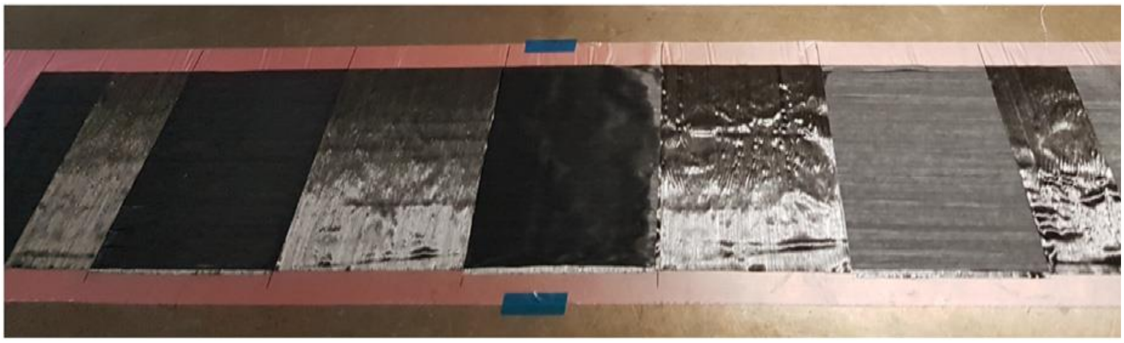


Figure 4: (a) the lay-up sequence of letter “C” and (b) the ideal and produced shape of letter “C” [9]

Filipovic and his colleagues [10] examined the behavior of high-amplitude corrugated thin-walled laminates. They created a sample using a type of prepreg called NTPT thinpregtm 513, and conducted simulations using ANSYS software.

Figure 5-a shows the sample before curing, and

Figure 5-b shows the sample after curing and at service temperature. The study focused on understanding the properties and behavior of these laminates when subjected to different load and temperature conditions. They aimed to investigate the potential of using this type of laminate in various engineering applications, such as aerospace and automotive. The use of simulation software like ANSYS allowed the team to model the performance and predict the behavior of the laminate under different scenarios, providing valuable insights for design and optimization.



(a)

(b)

Figure 5- corrugated laminates (a) before and (b) after cure [10]

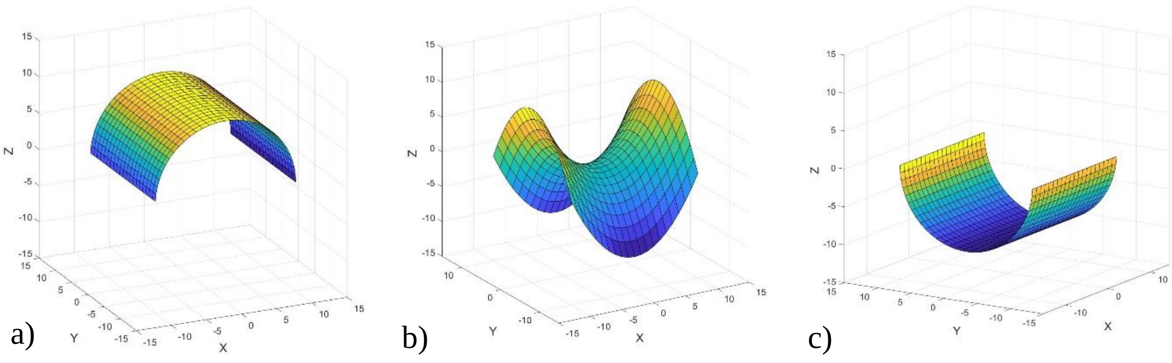


Figure 6- Three possible configurations of an unsymmetric laminate at room temperature based on the extended CLT a) cylindrical with curvature in x-direction b) saddle shape c) cylindrical with curvature in y-direction

Most, if not all, laminates in the above applications are un-symmetric laminates. They usually have the lay-up sequence of $[0_m/90_n]$. It has been observed by many [7–14] that the laminate may take up different shapes depending on the bifurcation point. Based on the conducted studies, Figure 6 illustrates the three fundamental configurations of unsymmetrical laminates at room temperature. It is evident that the final shape can either be cylindrical with curvature in the x- or y-direction or a saddle shape. To develop further applications using the technique of 4D printing of composites (which utilizes mainly unsymmetric laminates), it is necessary to understand the effect of different parameters on the value of the bifurcation temperature. This paper examines the effect of the thickness over length ratio (t/a). The method used is finite element analysis with some special treatment to allow for instability.

2 Extended laminate theory by Hyer et al. [11]

Back in 1981, Hyer [15] observed the existence of the different shapes of the unsymmetric laminates made of lay-up sequence $[0_m/90_n]$. He proposed an extended laminate theory to explain this effect. For the sake of completeness, salient features of these works are repeated here. This approach involves calculating the total potential energy of the laminate under plane stress conditions, which can be represented by the combination of its mechanical and geometrical properties, the change in temperature, and the total strains.

According to Hyer and followers [11–13,15–18] the final shape of a laminate at room temperature can vary depending on its size and material properties.

Figure 7 shows the result for laminate with material properties shown in Table 1 and lay-up sequence $[0_2/90_2]_T$. The laminate was cured at a temperature of 177 °C and the final shape at room temperature is reported. It is possible to determine that if the laminate length is greater than a certain critical point, there are three potential outcomes for its room-temperature shape. For laminates with small lengths, the final shape is a saddle shape where two curvatures exist simultaneously. As the length of the laminate increases, the final shape of the saddle becomes smoother until it reaches a critical length (such as 35mm for $[0_2/90_2]_T$ and 71mm $[0_4/90_4]_T$). Beyond this point (Point B in Figure 7), the solution splits into three possibilities. The first one continues the previous solution and indicates that there are two possible curvatures in the x- and y-direction, resulting in a saddle shape. The second one suggests that there is only one curvature in the x-direction and no curvature in the y-direction, resulting in a cylindrical shape. The last one implies that the curvature in the x-direction is zero and in the y-direction is non-zero, resulting in a cylindrical shape as well.

Table 1- mechanical properties of the prepreg

Properties	Value
E_1	155 GPa
E_2	12.1 GPa
G_{12}	4.4 GPa
ν_{12}	0.248
α_1	-0.018 e-6 /°C
α_2	24.3 e-6 /°C

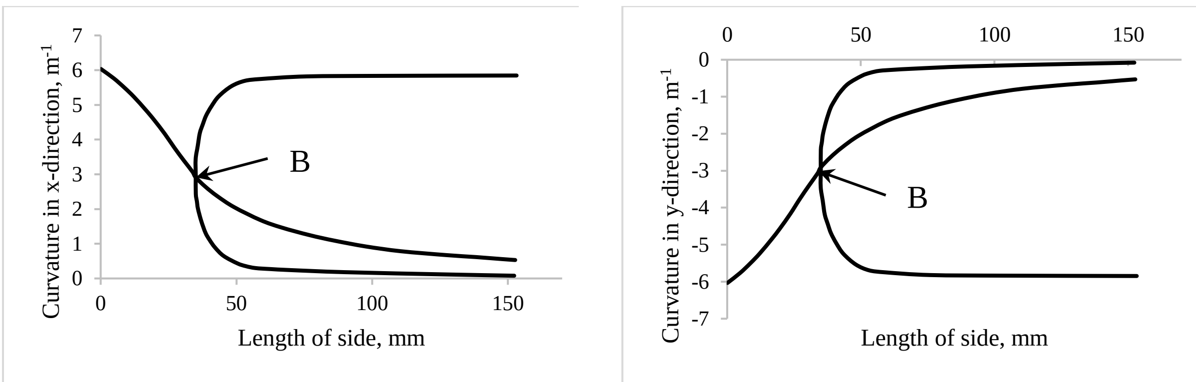


Figure 7- Room-temperature shapes of square $[0_2,90_2]_T$ graphite-epoxy laminates

By taking into consideration the stability theory, minimizing the total potential energy, it was observed that two of the solutions are stable, and one of them is unstable. The stable solutions indicate that the final shapes of cured unsymmetric laminates at room temperature are cylindrical, one solution has a curvature in the x-direction ($R_y = 0$), and the other one has a curvature in the y-direction ($R_x = 0$). The unstable solution is the one that said the laminate could have two possible curvatures at room temperature (saddle shape). By comparing the results obtained from the extended classical laminate theory to experimental results, it can be seen that the theory correctly predicts the final shape of unsymmetric laminates [7].

3 Finite element method:

While the extended laminate theory may be used to determine the value of the bifurcation point, it can only be used for laminates with rectangular shapes. Another technique, such as the finite element method, may be required for laminates with irregular shapes. In order to provide assurance on the accuracy of the finite element method, the procedure for the finite element method is developed using laminates of rectangular shape. The results are compared against those from the extended laminate theory. The following is a detailed description of the Finite Element Method (FEM) procedure used in this study. The analysis was performed using ANSYS software on a 12 mm×12 mm laminate, with a lay-up sequence of $[0_2/90_2]_T$. The four-node element SHELL 181 was utilized for modeling, as it is suitable for linear, large rotation, and/or large strain nonlinear applications. A total of 900 fully integrated shell elements with a mesh size of 4mm were used in the model. The material properties for each layer are listed in Table 1.

Only the central node was fixed to simulate the cooling down process for the laminate, and an external temperature difference between 177°C and 20°C was applied. In real-world scenarios, one curvature overcomes the other side at room temperature due to some unsymmetric conditions in material properties or orientation. However, the finite element software cannot incorporate these unsymmetric conditions. Thus, to create this instability, the fixed point was moved to one adjacent node so that one side curvature could overcome the other.

Since the model is nonlinear and significant deformation is expected, an incremental finite element analysis was utilized. This process involves dividing the solution process into several increments, with each increment responsible for a small difference in temperature. The output of each increment is used as input for the next increment, resulting in a more accurate representation of the deformation process. This incremental analysis is instrumental when the deformation is large and the structure is nonlinear.

The incremental analysis gradually reduces the temperature of a flat configuration by 1 °C in 157 increments. The first decrement of 1 degree is applied to the flat configuration, and the output, or deformed configuration, is used as the input for the next increment when the second 1 °C is imposed. This process continues until the final increment, where the deformed configuration of the unsymmetric laminate at room temperature is obtained.

Figure 8-a shows the configuration of the flat plate after the first step. It can be observed that there are two modes of deformation competing to be dominant.

Figure 1-b shows the configuration in the second increment when the temperature reduced from 177°C to 175°C. It can be observed that one side's curvature is going to overcome the other sides based on

Figure 8-c and d; there are steeper curvatures in the x-direction rather than z-direction as it can be seen that the area of the deformation is less in the edges facing x-direction. By the 10^{th} increment which is shown in

Figure 8-e the overcome curvature is the one in the x-direction and no saddle shape is observed here. Finally,

Figure 8-f illustrates the final configuration of the laminate after 157 increments, i.e. the shape of the laminate at room temperature.

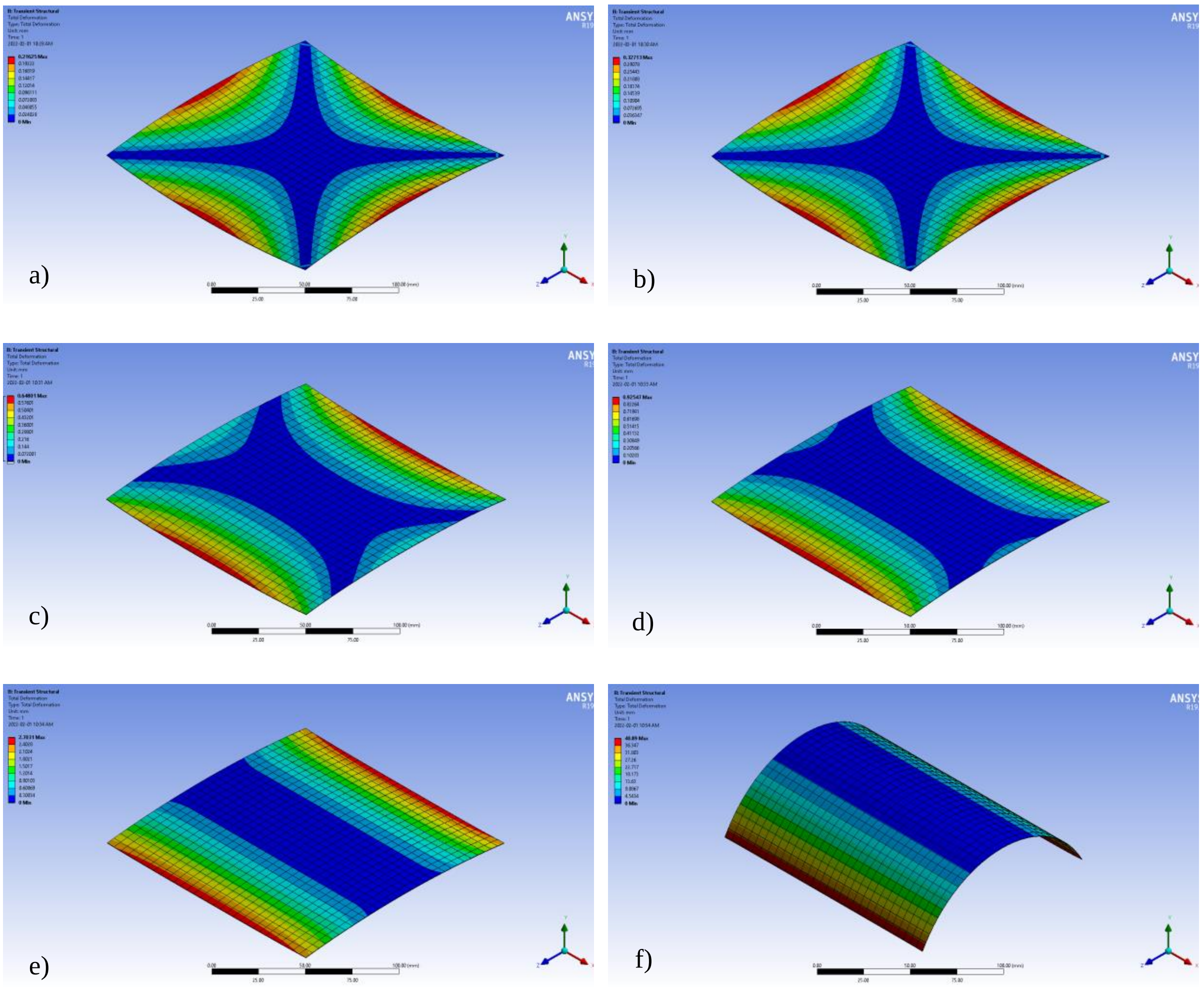


Figure 8- The configuration of 12 mm×12 mm and [0/90] laminate at different temperature increments during curing – a) after the 1st increment, b) after the 2nd increment, c) after the 3rd increment, d) after the 4th increment, e) after the 10th increment, f) at 20 °C.

The bifurcation point for different laminates can be found using the FEM analysis and the result from the extended laminate theory.

Figure 9 presents the data on the temperature at which bifurcation begins in a square laminate. The figure is constructed using a unitless term "$\frac{t}{a}$", where t represents the thickness of the laminate and a represents the size of the square. It can be inferred that the line $T = 177 - 1.23 * 10^6 \left(\frac{t}{a}\right)^2$ provides a good fit for the data. This implies that instead of using more complex methods such as Finite Element Method (FEM) or extended laminate theory, one can simply substitute the value of "$\frac{t}{a}$" into the equation $T = 177 - 1.23 * 10^6 \left(\frac{t}{a}\right)^2$ to determine the bifurcation temperature for the square laminate with the properties from Table 1.

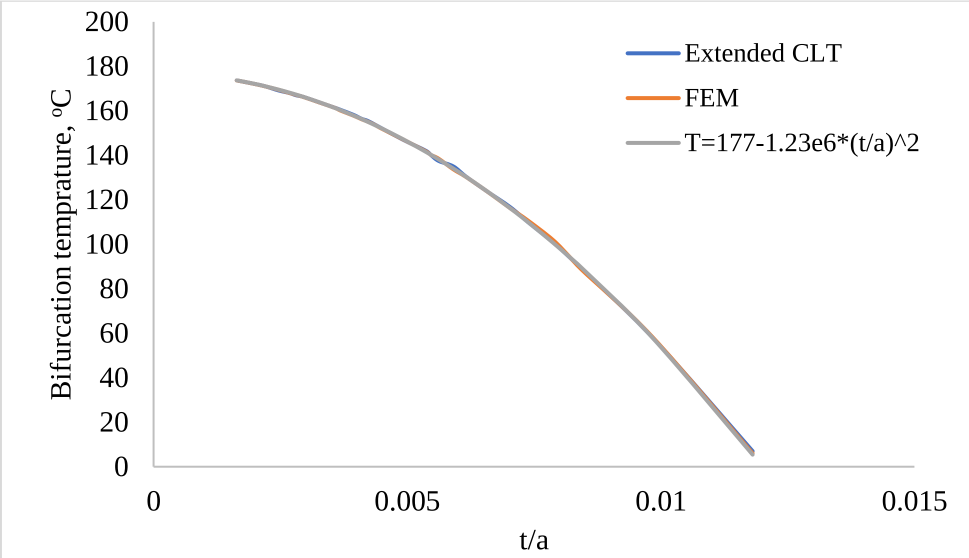


Figure 9- Bifurcation temperature for a square laminate with side “a” and thickness “t”, with properties shown in Table 1.

Figure 9 shows that the results from the extended laminate theory and finite element analysis agree well. For very thin laminates (small t/a), the bifurcation is close to the curing temperature of 177 °C. This means that the laminate will take a cylindrical shape at room temperature. For thicker laminate (such as that with t/a = 0.012), the bifurcation temperature is close to 20 °C, indicating that saddle shape will appear at room temperature.

Another study was conducted to examine the bifurcation point of a rectangular laminate with an aspect ratio of a/b = 0.75 and with a lay-up sequence of $[0_n,90_n]$. (n=1 to 7). The result is shown in figure 10. It can be seen that even for thicker laminates (t/a = 0.011), the bifurcation temperature is higher than 25 °C, indicating that the laminate will take a cylindrical shape at room temperature.

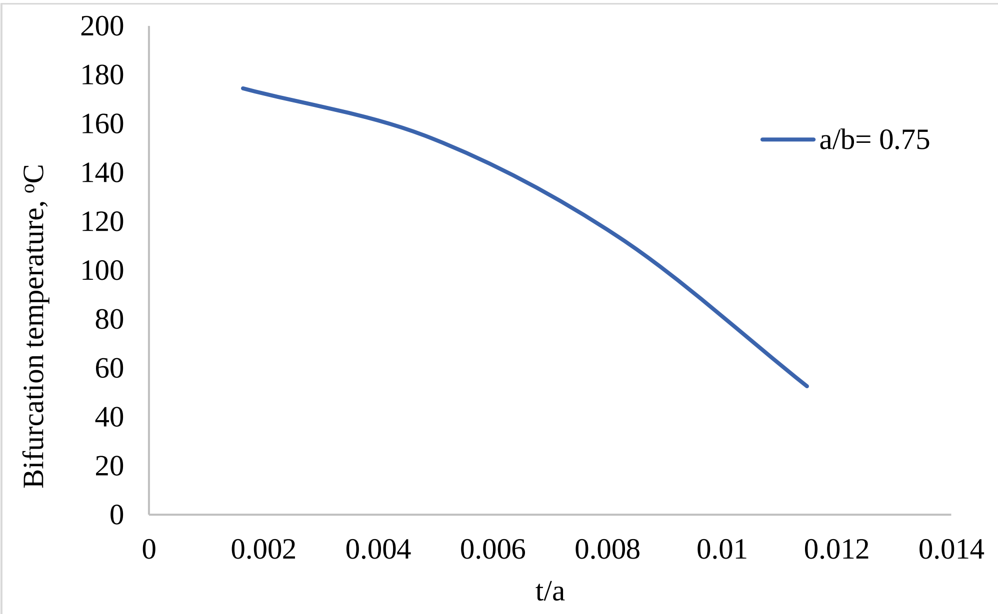


Figure 10- Bifurcation temperature for rectangular laminate based on aspect ratio a/b = 0.75 and properties from Table 1.

4 CONCLUSIONS

In conclusion, the final shape of a laminate at room temperature depends on material properties, lay-up sequence, thickness, and aspect ratio of the sides. This paper examines the effect of the thickness over length ratio and the effect of aspect ratio for laminates with lay-up sequence [0/90]. Both extended laminate theory and finite element have been used to determine the bifurcation temperature of rectangular laminates. For square laminates, thin laminates appear in cylindrical shape while thick laminates appear as saddle shape at room temperature. For laminates with an aspect ratio a/b = 0.75, all laminates with reasonable thickness appear as cylindrical shapes at room temperature.

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