THREE-DIMENSIONAL ANALYTICAL MODEL FOR PREDICTING EFFECTIVE STIFFNESS OF COMPOSITE LAMINATES WITH PLY CRACKING

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ABSTRACT

In this study, a model for the three-dimensional effective compliance of composite laminates with transverse cracks is developed based on continuum damage mechanics. Three-dimensional laminate theory is used to reproduce all the thermoelastic properties of the damaged laminate. The damage variable, which describes the degree of stiffness reduction caused by transverse cracking, is formulated based on a three-dimensional micromechanical model, with a loose boundary condition and assuming parabolic crack opening. These assumptions contribute to the analytical accuracy of the stiffness reduction model, while simplifying the damage variable expression. The effective thermomechanical properties of cross-ply, angle-ply, and quasi-isotropic composite laminates are predicted using the proposed model and compared with finite element analysis (FEA) and experimental results. We found that the proposed model with derived damage variable successfully reproduce the FEA and experimental results of stiffness degradation of damaged composite laminates.

1 INTRODUCTION

Polymer matrix composite laminates, which consists of unidirectional plies with various fibre angle, are widely used in the aerospace and aircraft industries. They have high specific rigidity and strength, which enables weight reduction of the structure. However, composite laminates exhibit nonlinear behaviour due to ply cracking, delamination, and fibre breakage. Ply cracking (or transverse cracking), which run parallel to the fibres in a ply, is typically the first damage mode. The ply cracks cause the reduction of the laminate stiffness as well as subsequent damage phenomena like delamination and fibre breakage. Therefore, it is essential to investigate the mechanical behaviour of composite laminates with ply cracking. In addition, the in-plane and out-of-plane stiffness properties of the damaged laminate are required to evaluate stress distributions in the composite structures under triaxial loading.

In this study, we analytically formulated a three-dimensional effective compliance of composite laminates with ply cracks based on continuum damage mechanics (CDM). The effective compliance proposed by Lopes et al. [1] was used. Damage variables, which describe the degree of the stiffness reduction caused by ply cracking, were derived as a function of the ply crack density with a loose boundary condition and assuming parabolic crack opening. Three-dimensional laminate theory [2] was used to predict the in-plane and out-of-plane effective elastic modulus.

2 CDM MODEL

2.1 Effective compliance of a ply with ply cracks

The three-dimensional effective compliance [1] was used to describe the ply with transverse cracks, as shown in Figure 1 (a). The material coordinates are defined as (1,2,3), where "1" is the fibre direction, "2" is the transverse direction, and "3" is the through-thickness direction. The origin of the material coordinate system is placed at the centre of the ply. In the material coordinate system, the effective compliance for orthotropic damaged ply of the Lopes model is described as follows:

$$\boldsymbol{S}_{\mathrm{mat}} = \begin{bmatrix} \frac{1}{(1-d_1)E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{12}}{E_1} & 0 & 0 & 0 \\ & \frac{1}{(1-d_2)E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ & & \frac{1}{(1-d_3)E_2} & 0 & 0 & 0 \\ & & & \frac{1}{(1-d_4)G_{23}} & 0 & 0 \\ & & & & \frac{1}{(1-d_5)G_{12}} & 0 \\ & \text{sym.} & & & & \frac{1}{(1-d_6)G_{12}} \end{bmatrix} \tag{1}$$

where E is Young's modulus, ν is Poisson's ratio, G is shear modulus, and d_i is the damage variable. The thermal expansion coefficient of the ply in the material coordinate system is expressed as

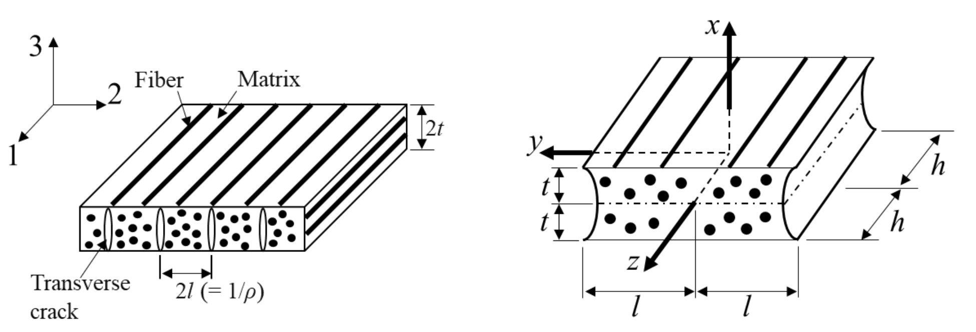
$$\boldsymbol{\alpha}_{\mathrm{mat}} = \{\alpha_1 \quad \alpha_2 \quad \alpha_2 \quad 0 \quad 0 \quad 0\} \tag{2}$$

The effective compliance $\boldsymbol{S}^i$ and thermal expansion coefficient $\boldsymbol{\alpha}^i$ of the *i*-th ply in the laminate coordinate system were obtained by rotating the fiber orientation.

$$\boldsymbol{S}^i = \boldsymbol{R}(\theta^i)\boldsymbol{S}_{\mathrm{mat}}\boldsymbol{T}(-\theta^i), \tag{3}$$

$$\boldsymbol{\alpha}^i = \boldsymbol{R}(\theta^i)\boldsymbol{\alpha}_{\mathrm{mat}}, \tag{4}$$

θ^i is fiber angle of the *i*-th ply, and $\boldsymbol{T}(\theta^i)$ and $\boldsymbol{R}(\theta^i)$ are the coordinate conversion matrices of the stress and strain, respectively. Once the damage variables are formulated, the effective compliance and thermal expansion coefficient can be obtained using Eqs. (3) and (4), respectively.



(a) Cracked ply (b) Representative volume element

Figure 1: (a) A ply with transverse cracks and (b) RVE of the damaged ply; (1, 2, 3) is the material coordinate system, while (x, y, z) is the RVE coordinate system.

2.2 Damage variables

The damage variables d_2 and d_6 associated with transverse cracks are derived from a three-dimensional micromechanics model by assuming a parabolic crack opening of transverse cracks [3]. According to [3], the damage variables are formulated as follows.

$$d_2 = \frac{2\lambda_1 t\rho}{\sqrt{3}} \tanh \frac{\sqrt{3}}{2\lambda_1 t\rho} \text{ (Parabolic Solution)} \tag{5}$$

$$d_6 = \frac{2t\rho}{\sqrt{3}} \tanh \frac{\sqrt{3}}{2t\rho} \text{ (Parabolic solution)} \tag{6}$$

$$\lambda_1 = \sqrt{\frac{E_2}{G_{23}} \frac{1-\nu_{12}\nu_{21}+(\nu_{12}\nu_{21}+\nu_{23})a+\nu_{12}(1+\nu_{23})b}{(1+\nu_{23})(1-\nu_{23}-2\nu_{12}\nu_{21})}}, \tag{7}$$

$$a = -\left(\nu_{23} + \frac{G_{23}}{E_2}(1-\nu_{23}^2) - \frac{G_{12}}{E_1}\nu_{12}(1+\nu_{23})\right), \tag{8}$$

$$b = -\left(\nu_{21} - \frac{G_{23}}{E_2}\nu_{21}(1+\nu_{23}) + \frac{G_{12}}{E_1}(1-\nu_{12}\nu_{21})\right), \tag{9}$$

Here, $2t$ is the ply thickness, ρ is the ply crack density, E is Young's modulus, ν is Poisson's ratio, and G is shear modulus. The remaining damage variables are d_1, d_3, d_4, and d_5. The ply has transverse cracks only; therefore, d_1 and d_3 are postulated as follows:

$$d_1 = 0, d_3 = 0. \tag{10}$$

The shear damage variables d_4 and d_5 are calculated using the following approximate expressions [4]:

$$\frac{1}{1-d_4} = \frac{1}{2}\left(\frac{1}{1-d_2} + \frac{1}{1-d_3}\right),\ \frac{1}{1-d_5} = \frac{1}{2}\left(\frac{1}{1-d_1} + \frac{1}{1-d_3}\right). \tag{11}$$

Substituting Eqs. (5)-(11) into Eq. (3), the effective compliance $\boldsymbol{S}^i$ of the ply is calculated as a function of transverse crack density ρ.

2.3 Effective compliance of damaged laminates

The effective compliance $\boldsymbol{S}^{\mathrm{L}} = \{S_{ij}^{\mathrm{L}}\}$ and thermal expansion coefficient $\boldsymbol{\alpha}^{\mathrm{L}} = \{\alpha_i^{\mathrm{L}}\}$ of damaged composite laminate with arbitrary lay-ups are calculated using the effective compliance $\boldsymbol{S}^i$ of the ply and three-dimensional laminate theory [2]. The three-dimensional laminate theory assumes that the in-plane strains and out-of-plane stresses of the laminate are identical to that of each ply, and in-plane and out-of-plane elastic moduli can be calculated. The thermomechanical properties of the laminate can be obtained from $\boldsymbol{S}^{\mathrm{L}}$ and $\boldsymbol{\alpha}^{\mathrm{L}}$:

$$E_X^{\mathrm{L}} = \frac{1}{S_{11}^{\mathrm{L}}}, E_Y^{\mathrm{L}} = \frac{1}{S_{22}^{\mathrm{L}}}, E_Z^{\mathrm{L}} = \frac{1}{S_{33}^{\mathrm{L}}}, \tag{12}$$

$$G_{YZ}^{\mathrm{L}} = \frac{1}{S_{44}^{\mathrm{L}}}, G_{XZ}^{\mathrm{L}} = \frac{1}{S_{55}^{\mathrm{L}}}, G_{XY}^{\mathrm{L}} = \frac{1}{S_{66}^{\mathrm{L}}}, \tag{13}$$

$$\nu_{YZ}^{\mathrm{L}} = -\frac{S_{23}^{\mathrm{L}}}{S_{22}^{\mathrm{L}}}, \nu_{XZ}^{\mathrm{L}} = -\frac{S_{13}^{\mathrm{L}}}{S_{11}^{L}}, \nu_{XY}^{\mathrm{L}} = -\frac{S_{12}^{\mathrm{L}}}{S_{11}^{\mathrm{L}}}, \tag{14}$$

$$\alpha_X^{\mathrm{L}} = \alpha_1^{\mathrm{L}}, \alpha_Y^{\mathrm{L}} = \alpha_2^{\mathrm{L}}, \alpha_Z^{\mathrm{L}} = \alpha_3^{\mathrm{L}}, \alpha_{YZ}^{\mathrm{L}} = \alpha_4^{\mathrm{L}}, \alpha_{XZ}^{\mathrm{L}} = \alpha_5^{\mathrm{L}}, \alpha_{XY}^{\mathrm{L}} = \alpha_6^{\mathrm{L}}, \tag{15}$$

where E is Young's modulus, G is the shear modulus, ν is Poisson's ratio, and α is the thermal expansion coefficient. The subscripts X, Y, and Z indicate the laminate coordinate-directions.

3 RESULTS AND DISCUSSION

The effective thermoelastic properties of CFRP and GFRP composite laminates with various layups were predicted by the three-dimensional stiffness reduction model presented in the previous section. The predicted results were compared with the FEA and experimental results from previous studies to validate the proposed model.

Unlike conventional shear-lag analyses, the present model uses the three-dimensional laminate theory; therefore, all effective thermoelastic properties of composite laminates with various layups can be determined. Figure 2 shows the effective Young's moduli, shear moduli, Poisson's ratios, and thermal expansion coefficients of the angle-ply $[55/\text{-}55]_s$ GFRP laminate. The FEA results [2] are plotted for comparison. The present model with a parabolic solution (Eqs. (5) and (6)) can reproduce the change in all effective thermoelastic properties due to transverse cracking. The effective laminate Young's modulus E_Z^{L} in the thickness direction is slightly reduced due to Poisson's effect.

Figure 3 depicts the normalized effective axial Young's modulus of the $[0/\theta_8/0]$ (θ = 90°, 60°, 45°) CFRP laminate based on the present and experimental results [5] to verify the present model for cracking in off-axis plies. The laminate layups were $[0/90_8/0]$, $[0/60_8/0]$, and $[0/45_8/0]$. The present model quantitatively agrees with the experimental results regardless of the fiber angle in the middle plies. Therefore, the proposed model can be applied to express the effective mechanical properties of cracked composite laminates in structural components subjected to multiaxial loading.

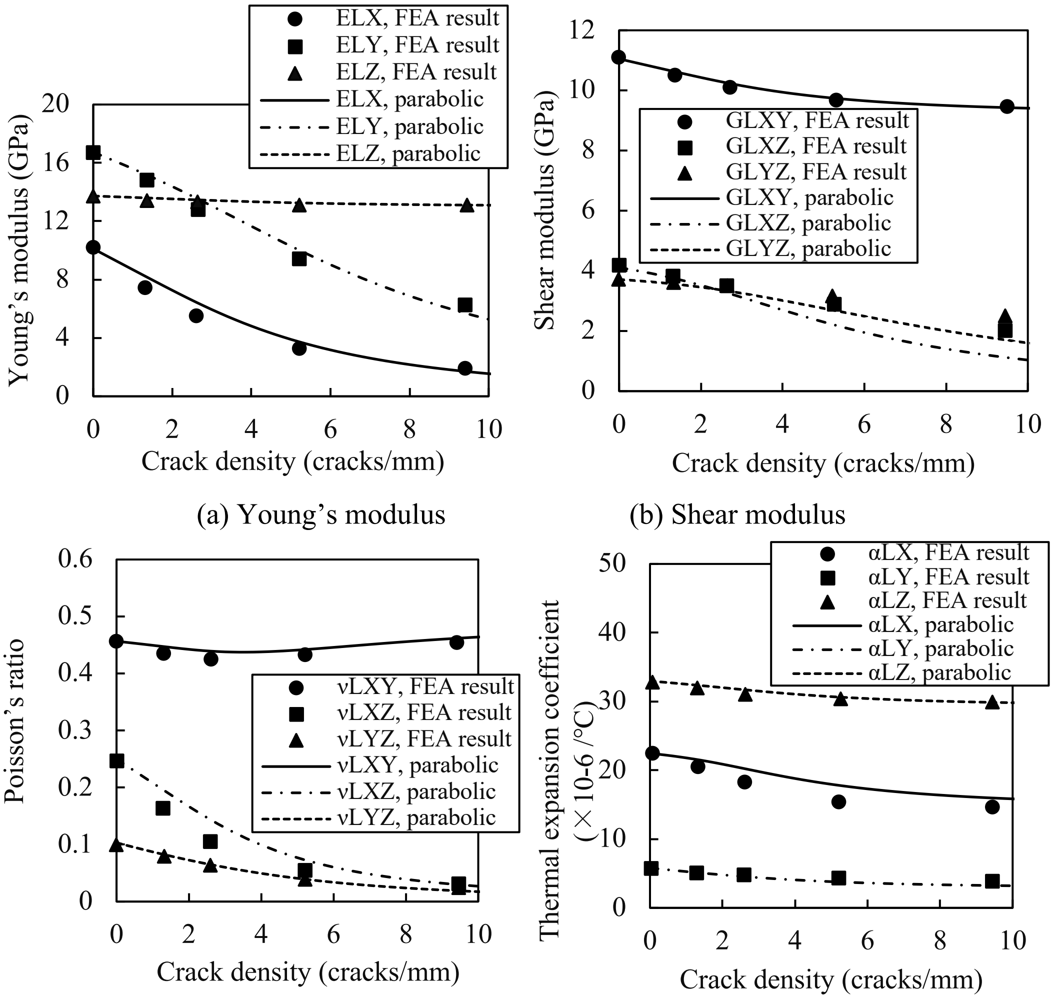
The effective thermal expansion coefficient of the $[0_2/90_2]_s$ CFRP laminate with a ply thickness of 0.125 mm was calculated and compared with experimental results [6]. Figure 4 shows the predicted effective thermal expansion coefficients. The thermal stress of a ply in composite laminate is induced as temperature changes due to mismatch in the thermal expansion coefficients between plies with different orientations [6], [7]. The large crack opening of the damaged 90° ply results in low effective ply stiffness. Therefore, the load bearing capacity of the 90° ply is reduced, and the thermal stress in the 0° layers becomes large. As a result, the effective laminate thermal expansion coefficient approaches the thermal expansion coefficient of 0° plies that have low thermal expansion coefficient. The parabolic solution results are slightly higher than the experimental results at high crack density. However, the present model successfully reproduced the experimental results.

Quasi-isotropic laminates develop transverse cracks in multiple ply orientations. Tong et al. [8] examined the cracks in the $[0/90/\text{-}45/45]_s$ GFRP laminate with a 0.5 mm ply thickness. In Tong's experiments, the cracks in the 90° plies traversed the width and thickness of the 90° plies, while the cracks did not fully propagate in the 45° and -45° plies. Singh and Talreja [9] defined the relative density factor ρ_r as the ratio of the actual surface area for partial cracks to the surface area for full cracks is defined as follows:

$$\rho_r = \frac{\text{Actual surface area for partial cracks}}{\text{Surface area for full cracks}}. \tag{16}$$

By introducing ρ_r, the crack density is defined as the sum of crack area contained per volume of the damaged ply. It means that a one large crack is equivalent to two small cracks with same crack surface area of the one large crack. The relative density factor could not be calculated using the data from Tong's experiments. Therefore, the transverse crack density in -45° and 45° plies was assumed to be a 90° transverse crack density multiplied a relative density factor ρ_r. Three values of ρ_r were considered: 0.25, 0.5, and 1. Figure 5 shows the normalized axial Young's modulus and in-plane Poisson's ratio of the quasi-isotropic GFRP laminate as functions of 90° transverse crack density, as predicted by the proposed model, as well as the experiments of Tong et al. [8]. The present model with $\rho_r = 0.5$ is in the best agreement with the experimental results, which is consistent with the study of Singh and Talreja [9]. The results indicate that the present model can quantitatively predict the stiffness reduction of cracked composite laminate with arbitrary layups.

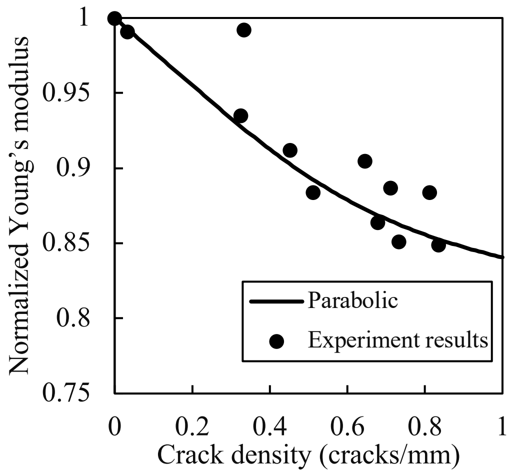
The three-dimensional stiffness reduction model for composite laminates containing transverse cracks is formulated, and thermomechanical properties of damaged composite laminate is predicted as a function of transverse crack density. To discuss crack growth in each layer of laminates with arbitrary layups, formulation of a crack growth analysis method using stress and energy criteria is a future task.



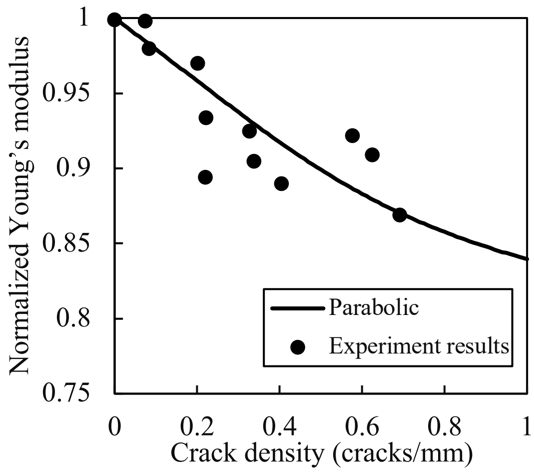
(a) Young’s modulus (b) Shear modulus

(c) Poisson’s ratio (d) Thermal expansion coefficient

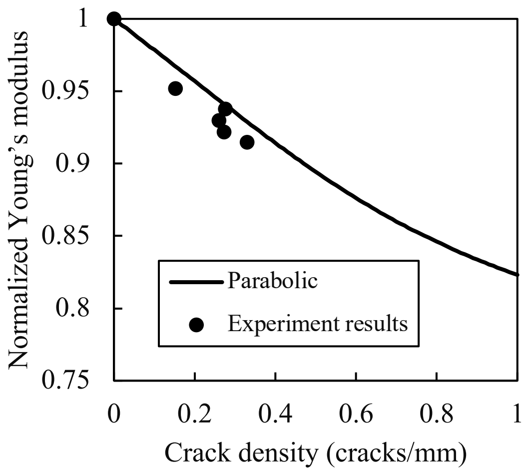
Figure 2: (a) Young’s moduli, (b) shear moduli, (c) Poisson’s ratios, and (d) thermal expansion coefficients as a function of transverse crack density of $[55/-55]_s$ angle-ply GFRP laminate. FEA results [2] are plotted for comparison.



(a) [0/90$_8$/0]



(b) [0/60$_8$/0]



(c) [0/45$_8$/0]

Figure 3: Normalized axial Young's modulus of (a) [0/90$_8$/0], (b) [0/60$_8$/0], and (c) [0/45$_8$/0] CFRP laminates as a function of transverse crack density. Experiment results [5] are plotted for comparison.

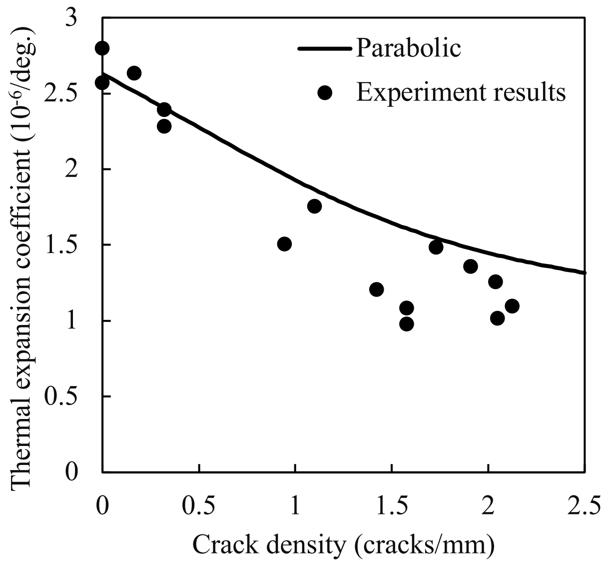
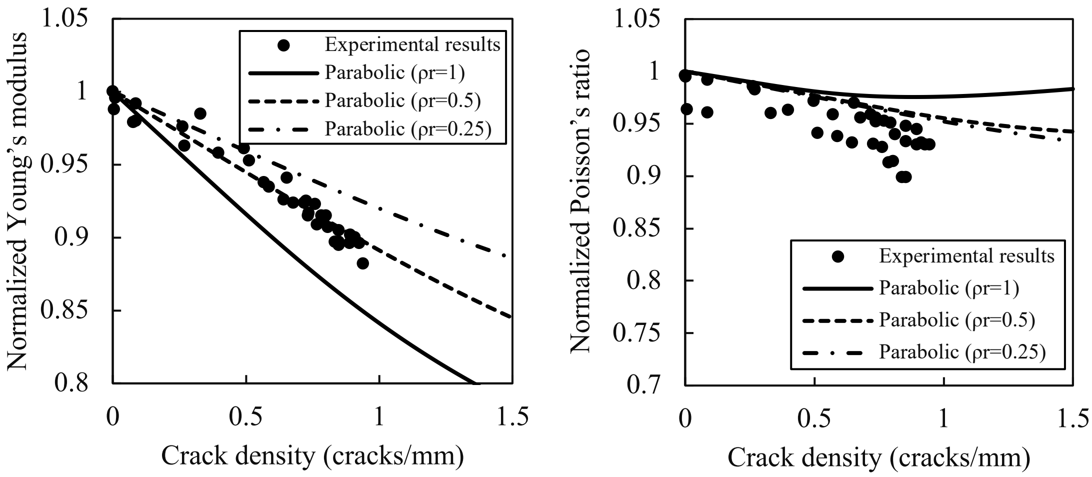


Figure 4: Thermal expansion coefficient as a function of transverse crack density of $[0_2/90_2]_s$ CFRP laminate with surface cracking. Results shown are from the present stiffness reduction model with parabolic and infinite series solutions and the experimental results of Tong et al. [6].



(a) Young's modulus (b) Poisson's ratio

Figure 5: (a) Normalized axial Young's modulus and (b) normalized in-plane Poisson ratio of [0/90/-45/45]$_s$ quasi-isotropic GFRP laminate as a function of 90° transverse crack density. Experimental results [8] are plotted for comparison.

4 CONCLUSIONS

In this study, a three-dimensional effective compliance model is developed for composite laminates with transverse cracking. Three-dimensional laminate theory is used to reproduce all the thermoelastic properties of damaged laminates with various layups. Lopes' effective compliance [1] of the damaged ply is used to represent the stiffness degradation due to transverse cracking based on CDM. The damage variable d_2 and d_6 are formulated based on a three-dimensional micromechanical model, where the boundary conditions are loosened and parabolic crack opening is assumed.

The effective thermomechanical properties of various composite laminates are predicted using the proposed model with simple parabolic solutions (Eqs. (5) and (6)) and compared with FEA and experimental results. The parabolic solution results successfully agreeed with the FEA and experimental results, regardless of the laminate layup configuration. Therefore, this assumption contributes to the analytical accuracy of the stiffness reduction prediction while simplifying calculation of the damage variable.

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