ASSESSMENT OF EFFECTIVE ELASTIC PROPERTIES OF HONEYCOMB CORES BY FINITE ELEMENT ANALYSIS OF SANDWICH PANELS

Altan Kayran
Dept. of Aerospace Eng. Middle East Technical University, Ankara, 06531,Turkey
E-Mail: akayran@metu.edu.tr

and

İlke Aydıncak
Scientific and Technological Research Council of Turkey
Defense Industries Research and Development Institute, 06261, Ankara, Turkey,
E-Mail: iaydın@sage.tubitak.gov.tr

SUMMARY

Continuum models of regular hexagonal aluminum honeycomb cores are evaluated by finite element analysis. The evaluation is based on the comparison of total in-plane and out-of-plane reaction forces determined by the finite element analyses of the sandwich panels with the actual honeycomb core geometry and with the existing equivalent core models.

Keywords: Sandwich structure; Honeycomb core; Equivalent elastic properties; Homogenization; Finite element analysis

INTRODUCTION

Sandwich panels with honeycomb core are widely used in different structural applications such as aircraft floor panels, control surfaces, external radomes, access panels, rocket fins, civil engineering structures and many more. The main problem in analyzing honeycomb core sandwich structures using the finite element method lies in the substantial computational effort that has to be spent in modeling and analyzing a sandwich structure with a multi-cell construction core by maintaining the actual honeycomb core geometry. Therefore, the common practice in the finite element modeling of honeycomb core sandwich structures is to replace the core by an equivalent two or three dimensional orthotropic material. Replacement of the actual honeycomb core by an equivalent continuum model works well especially in problems involving global structural analysis such as deflection, vibration or aero elastic analysis of structures made of sandwich construction. In these problems, the global stiffness match of the sandwich structure with the equivalent continuum model and the sandwich structure with the actual honeycomb core is the main goal. However, by using a finite element model with an equivalent continuum core, it is not possible to determine the local stress distribution in the core and in the face-sheet material interacting with the core since the actual geometry is not preserved in the equivalent model.

In the literature many works have been performed on the homogenization of the elastic properties of honeycomb cores. In the following a few of these works will be referenced. The initial work on the determination of the transverse shear moduli of honeycomb foils was performed by Kelsey et.al. by employing the energy approach [1]. Vinson studied the optimum design of honeycomb sandwich panels [2]. The book by Gibson and Ashby [3] is a systematic study which presents the equivalent in-plane and out-of-plane elastic properties of uniform thickness honeycomb cores by means of the standard beam theory and energy approach, respectively. Grediac [4] determined the equivalent transverse shear moduli of honeycomb cores by the finite element analysis of a representative unit cell. The equivalent transverse shear modulus is described by an alternative relation which is written in terms of the lower and upper bounds of the transverse shear modulus determined by Gibson and Ashby. Masters and Evans obtained refined in-plane elastic constants of the honeycomb core by incorporating the flexure, stretching and hinging effects [5]. Burton and Noor [6] assessed the accuracy of the predictions of an equivalent core model for square cell honeycomb core made of titanium by using the finite element free vibration responses of sandwich panels, with composite face-sheets, as the standard for assessing the accuracy of the predictions. Nast [7] determined all nine orthotropic material constants of the honeycomb core by taking a unit cell which includes the straight wall with double foil thickness and two inclined walls with single foil thickness. The cell walls were modeled as plates and the analytically determined material constants were compared with the experimental results. Schwingshackl et.al [8] reviewed fifteen different approaches of determining the equivalent orthotropic material constants of the honeycomb core.

In most of the previous studies the main emphasis was the determination of the effective elastic constants of the honeycomb core. In some of these works finite element analysis of only the unit cell was performed in order to verify the effective honeycomb-core elastic constants which are mostly determined by different analytical approaches. In practical applications involving the design and analysis of sandwich structures with complex geometries and loading conditions, the use of the finite element method is inevitable. Therefore, it is assessed that more work on the evaluation of the effective elastic constants of the honeycomb cores based on the finite element analysis of the sandwich structure with the face-sheets and the honeycomb core is necessary. Thus, the effect of face-sheet-honeycomb core interface can be taken into consideration properly. Finite element analyses of the honeycomb core only would not include the skin effect. For this purpose, in the present study different finite element model alternatives are developed to come up with the most reliable and feasible finite element model of the sandwich panel with the actual honeycomb core geometry. Finite element models of the sandwich panel with effective elastic constants of the honeycomb core are also generated based on the existing equivalent continuum models of the honeycomb core. In order to assess the effective elastic constants of honeycomb cores, finite element analyses are performed with both models for all possible combinations of boundary conditions and in-plane and out-of-plane input displacements applied to the different faces of the sandwich panel. The assessment is based on the comparison of the total reaction forces, calculated by both finite element models, on the supported faces of sandwich panels. The results show that the reliability of the individual in-plane and out-of-plane effective elastic constants of the existing continuum models of the honeycomb cores can be successfully evaluated based on the comparative study.

FINITE ELEMENT MODELS OF SANDWICH PANELS WITH ACTUAL CORE GEOMETRY AND EQUIVALENT CORE

To assess the effective elastic constants of honeycomb cores an accurate finite element model of the sandwich panel with the actual honeycomb geometry and the face-sheets is required. In this study accurate finite element model of the sandwich panel is named as the reference model. Figure 1 depicts the typical reference and equivalent core finite element models of sandwich panels which are used in the assessment study.

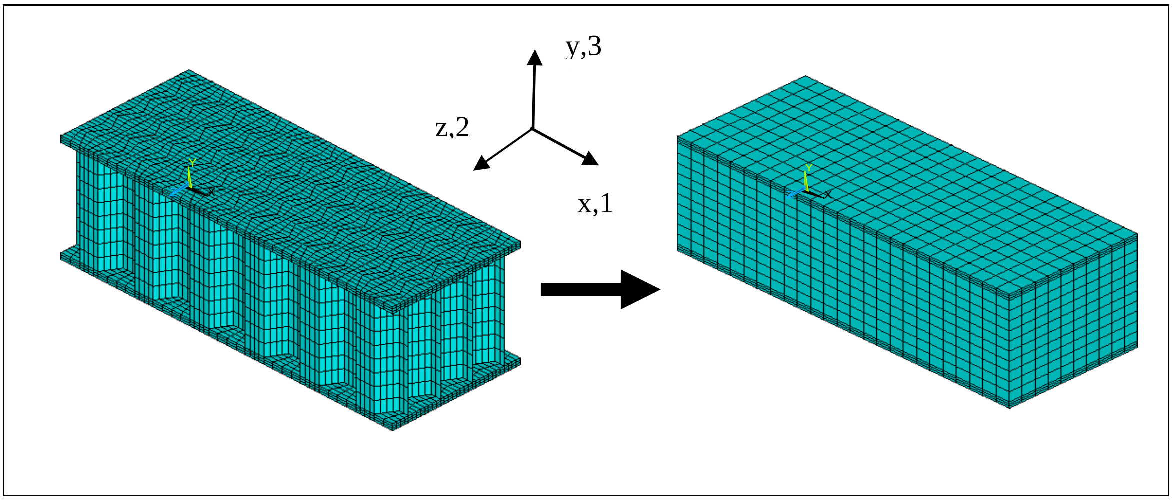

Figure 1 Finite element models of the honeycomb and equivalent core sandwich panels

The equivalent honeycomb core models are evaluated based on finite element analyses of the sandwich panels. Finite element analyses are conducted by imposing input displacements to different faces of the sandwich panel in different directions. Figure 2 shows the faces of the sandwich panel with letters assigned to each of the six faces. Table 2 gives the nine different load cases for which the finite element analyses are performed. For each fixed face case, three different uniform displacements are applied in the in-plane and out-of-plane directions on faces B, D and F and the total reaction forces on the opposite fixed faces A, C and E are calculated. In Table 2 the last column gives the total reaction force calculated on the opposite face of the input displacement face. The nomenclature for the total reaction force is FIJ, where I denotes the direction of the normal of the face which is fixed, and J denotes the direction of the input displacement on the opposite face of the fixed face. In the finite element model of the sandwich panel with the actual honeycomb core geometry it is assumed that the nodes on the edges and side walls of the honeycomb foil which intersect with the faces A, C and E are fixed. On the other hand, in the finite element model of the sandwich panel with the equivalent honeycomb core all the nodes on the faces A, C and E are fixed.

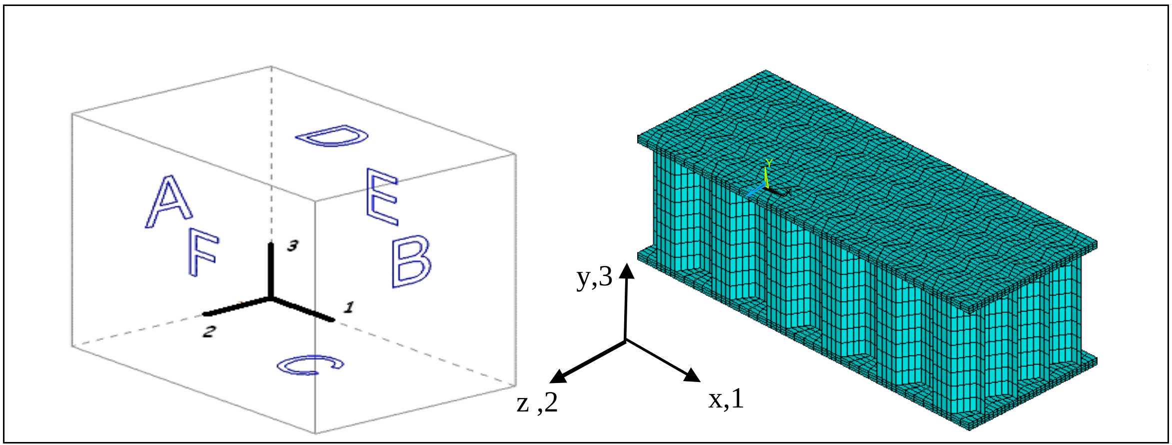

Figure 2 Faces of the sandwich panel with letters assigned to each face

Table 1 Load cases and boundary conditions used in finite element analyses

Load Case	Fixed Face	Input Displacement Face	Direction of the Input Displacement	Total Reaction Force
1	A	B	1	F11
2	C	D	3	F33
3	E	F	2	F22
4	A	B	3	F13
5	A	B	2	F12
6	C	D	1	F31
7	C	D	2	F32
8	E	F	1	F21
9	E	F	3	F23

Finite Element Model Alternatives of the Reference Model

Before the assessment study of equivalent honeycomb core models, different finite element model alternatives are tried to come up with a reliable reference finite element model to be used in the calculation of total reaction forces. Two alternative finite element models of the sandwich panel are compared to decide on the reference model. In both models, cell walls of the honeycomb core are modeled by shell elements. The first alternative utilized shell elements in the face-sheets and in the second alternative solid elements are used in the face-sheets. Finite element analyses have been performed by using Shell 93 and Solid 186 elements of ANSYS, for shell and solid modeling, respectively [9]. In this study aluminum sandwich panels having a face-sheet thickness of 1 mm, with six cells in the ribbon direction and four cells in the transverse direction to the ribbon direction, are used in the calculations. The details of honeycomb geometry and material property used are given in Ref. 10. The first comparison of the full shell and mixed shell/solid finite element models of the sandwich panels is made by determining the total reaction force F33 by the two models. Table 2 shows the comparison of the total reaction forces calculated by two different finite element models and the mechanics of material approach. Results are obtained for a sandwich panel with a face-sheet thickness of 1 mm, a core height of 15.875 mm and a cell size of 4.76 mm [10].

Table 2 Total reaction force calculated by three different approaches

Approach	Shell FEM	Mixed FEM	Mechanics of Material
F33 (N)	22,04	18,21	18.15

As one would expect the finite element model with the face-sheets meshed with shell elements overestimates the reaction force. Overestimation of the reaction force is due to the rigidity of the shell elements in the thickness direction. Since the face-sheets are in series connection with the honeycomb core, the equivalent stiffness of the sandwich panel with the face-sheets meshed with shell elements becomes higher than the true stiffness in the thickness direction, resulting in a higher total reaction force. On the other hand, finite element model with the face-sheets meshed with solid elements estimates the thickness direction stiffness of the sandwich panel very accurately, and this is reflected in the total reaction force calculated at the supported face. For the same sandwich panel configuration total reaction forces determined by the full shell and mixed shell/solid finite element models are compared in Table 3 for all load cases given in Table 1.

Table 3 Total reaction forces calculated by the two different finite element models

Total Reaction force	F11 (N)	F12 (N)	F13 (N)	F22 (N)	F21 (N)	F23 (N)	F33 (N)	F31 (N)	F32 (N)
Shell FEM	5.72	0.19	0.25	40.48	10.71	1.9	22.04	3.61	1.72
Mixed FEM	5.69	0.19	0.24	40.34	10.71	2.0	18.21	3.19	1.63

Table 3 shows that the largest difference between the total reaction forces determined by the full shell finite element model and mixed shell-solid finite element model occurs in F33, which is the total reaction force in the thickness direction. Besides F33, the largest percent differences among the total reaction forces occur in F13, F23, F31 and F32, which are the total reaction forces in the out-of-plane direction. It should be noted that these reaction forces are very much dependent on the out-of-plane shear moduli of the sandwich panel. Based on the initial comparison studies it can be concluded that the use of the full shell model may not be an adequate reference finite element model to assess the effective elastic constants of the honeycomb core. The discrepancy in the thickness direction total reaction force F33 calculated by the full shell finite element model and the mixed finite element model is mainly attributed to the rigidity of the shell elements in the thickness direction. Also, in sandwich panels face-sheets are usually much thicker than the cell walls of the honeycomb core and this fact further justifies the use of solid elements in the face-sheets.

Finite Element Model of Sandwich Panel with Equivalent Core

As shown in Fig. 1, finite element model of the sandwich panel with the equivalent core is generated by meshing the core with Solid 186 elements and assigning three dimensional orthotropic material properties to the solid elements. Effective elastic constants of the honeycomb core are based on the existing continuum models of the honeycomb core reported in the literature. The face-sheets of the equivalent models are also meshed with Solid 186 element just like in the mixed shell-solid model of the sandwich panels with the actual core geometry. Since the face-sheets of the mixed element reference model and the equivalent model use the same solid element topology, the differences between the total reaction forces determined by the mixed element model and the equivalent model may be primarily attributed to the differences in the in-plane and out-of-plane stiffnesses of the honeycomb core. Seven different equivalent core models are selected to demonstrate the assessment of the equivalent elastic constants of honeycomb cores. The equivalent models that are used in the current study are listed below together with explanations of the orthotropic elastic constants used. Care is taken to select models which complement each other. Models which only give in-plane elastic constants and models which only give out-of-plane elastic constants are combined to demonstrate the effect of elastic constants of the honeycomb core on the total reaction force which is used as the main parameter to assess the continuum models of the honeycomb core. It should be noted that the models used in the present study are not claimed to be the best performing models. They are selected simply to demonstrate the assessment of the effective elastic constants of honeycomb cores based on the comparisons of total reaction forces determined by the finite element analyses of sandwich panels by the reference and equivalent core finite element models.

<u>Model #1 (M1):</u> This equivalent model uses the in-plane elastic constants (E_1; ribbon direction, E_2; transverse direction, G_{12}, ν_{12}) of Masters and Evans [5]. For normal loading in the thickness direction E_3 simply reflects the solid modulus of the honeycomb core, E_{hc}, scaled by the area of the load-bearing section, and for the double foil thickness in the ribbon direction it is given by Eq.(1) [3]. In all the models the out-of-plane Young's modulus E_3 is based on Eq. 1 which is a universally agreed relation. In this model the two Poisson's ratios ν_{31} and ν_{32} are taken as the Poisson's ratio of the core material itself [3].

$$E_3 = \frac{2E_{hc}}{\cos\varphi.(1+\sin\varphi)}\left(\frac{t}{a}\right) \qquad \nu_{31} = \nu_{32} = \nu_s \tag{1}$$

The Poisson's ratios ν_{13} and ν_{23} are then found from the reciprocal relations

$$\nu_{13} = \frac{E_1}{E_3}\nu_{31} \approx 0 \qquad \nu_{23} = \frac{E_2}{E_3}\nu_{32} \approx 0 \tag{2}$$

To see the effect of the out-of-plane shear moduli on the total reaction forces, the out-of-plane shear moduli G_{13} and G_{23} are neglected, and very small values are assigned to the out-of-plane shear moduli in the material definition of the equivalent core in the finite element model.

Model #2 (M2): Equivalent model M2 uses the effective elastic constants given by Nast [7]. The model given by Nast provides all nine components of the orthotropic properties of the equivalent core. The Poisson's ratios ν_{13} and ν_{23} of the equivalent core model of Nast are also very small and can be neglected.

Model #3 (M3): Model 3 uses the out-of-plane elastic constants of Grediac [4] and the in-plane constants (E_1, E_2, ν_{12}) are taken from model 1. To investigate the effect of the in-plane shear modulus, G_{12} is neglected and a very small value is assigned to the in-plane shear modulus in the material definition of the equivalent core in the finite element model. The out-of-plane Poisson's ratios are taken as described in model 1. In this model the lower limit of the out-of-plane shear modulus G_{13} of Grediac [4] is used.

Model #4 (M4): Model 4 neglects the in-plane elastic constants ($E_1, E_2, G_{12}, \nu_{12}$) and uses the same out-of-plane shear moduli from Grediac [4]. In this model as opposed to model M3, the upper limit of the out-of-plane shear modulus G_{13} of Grediac is used.

Model #5 (M5): Model 5 is the same as model 3 with the inclusion of the in-plane shear modulus G_{12} from Masters and Evans [5], and the out-of-plane elastic constants are taken from Grediac [4]. In this model the lower limit of the out-of-plane shear modulus G_{13} is used as in model 3.

Model #6 (M6): Model 6 uses the same elastic constants of model 5 but, unlike model 5, it uses the upper limit of the out-of-plane shear modulus G_{13} of Grediac.

Model #7 (M7): Model 7 is based on the effective elastic constants of Nast but to see the effect of the in-plane Poisson's ratio ν_{12} on the results, it is neglected.

ASSESSMENT OF EFFECTIVE ELASTIC PROPERTIES

The assessment of the effective elastic constants of honeycomb cores is demonstrated by comparing the total reaction forces which are determined by finite element analyses of sandwich panels with the reference finite element model of the sandwich panel with the actual honeycomb geometry, and finite element models of the sandwich panels with equivalent cores M1-M7. Four different honeycomb cell sizes and four different cell heights are selected, and finite element models are generated based on sixteen different honeycomb configurations whose details are given in Ref. 10. Because of the vast amount of data, only a portion of the results pertaining to a few equivalent models is presented in this article. The comparisons are performed based on the percent differences of total reaction forces calculated by the reference and the equivalent core finite element models. The percent difference in the total reaction force is calculated by

$$\text{Percent difference in total reaction force} = \frac{\left|\Delta FIJ_{reference-equivalent\ \mathrm{mod}el}\right|}{(FIJ)_{reference\ \mathrm{mod}el}} \tag{3}$$

Table 4 presents the percent differences of total reaction forces for single cell height and core geometry. The percent differences for the other cell heights and core geometries show similar behavior and they are tabulated in detail by Aydıncak [11].

Table 4 Percent differences between reference and equivalent core FE models[1]

Total Reaction force	F11	F12	F13	F22	F21	F23	F33	F31	F32
M1	1.74	0.47	99.37	3.43	0.37	96.8	0.29	99.98	99.96
M2	0.95	0.05	29.35	2.7	0.27	84.6	0.27	64.67	51.52
M3	0.84	0.08	0.84	1.48	0.19	4.5	0.29	0.96	0.36
M4	1.93	0.65	5.86	3.7	0.44	4.45	0.29	10.42	0.09
M5	0.61	0.02	0.93	0.08	0.07	4.53	0.29	0.98	0.44
M6	0.6	0.02	6.23	0.11	0.05	4.53	0.29	10.58	0.44
M7	1.88	0.6	29.07	3.66	0.41	84.51	0.29	64.5	51.28

[1] Cell size d=4.76 mm ; Cell height: 15.875 mm

A number of conclusions can be inferred from the results presented in Table 4. Firstly, it should be noted that all the models have almost the same percent differences in the total reaction force F33. For the particular cell size and height, it was shown in Table 2 that the total thickness direction reaction force was calculated as 18.21 N by the mixed finite element model of the sandwich panel. On the other hand, all the equivalent models gave a total thickness direction reaction force of about 18.16 N and this result is almost the same as the reaction force determined by the mixed finite element model and the reaction force determined by the mechanics of material approach. Therefore, it can be concluded that Equation (1) estimates the equivalent modulus of the honeycomb core in the thickness direction very accurately.

Comparison of the results of models M1, M3 and M4 reveals that by incorporating the out-of-plane elastic constants in the equivalent model, the percent differences in the out-of-plane total reaction forces between the mixed finite element and equivalent models are significantly reduced, as expected. A further remark can be made with regard to the use of the lower and upper limit of the out-of-plane shear modulus G_{13} of the Grediac's model [4] for the regular hexagonal honeycomb core. The percent differences in the out-of-plane total reaction forces F13 and F31 of model 3, which uses the lower limit of G_{13} proposed by Grediac is smaller than the differences determined by model 4, which uses the upper limit of G_{13}. Model 3 gives percent difference within 1%, whereas the percent difference in F31 of model 4 is about 10 %. Thus, based on the comparison of the differences in the total reaction forces, it can be concluded that the lower limit of the out-of-plane shear modulus G_{13} proposed by Grediac is more reliable. The effect of using the lower and upper limit of the out-of-plane shear modulus G_{13} of Grediac is also clearly seen in the results of models M5 and M6. Model M5 uses the lower limit and model M6 uses the upper limit of the out-of-plane shear modulus G_{13}. The percent differences in the total shear forces F13 and F31 are much lower in model M5 compared to model M6, and this result again supports the reliability of the lower limit of the out-of-plane shear modulus G_{13} of Grediac.

Comparison of the percent differences in the total in-plane reaction forces (F11, F12, F21, F22) of models M3 and M4 reveals that the differences are lower in model M3, which does not neglect the in-plane elastic constants of the equivalent honeycomb core. It is noticed that inclusion of the in-plane elastic constants in model M3 results in a definite improvement in the percent differences in the total in-plane reaction forces. However, because the in-plane elastic moduli (E_1, E_2, G_{12}) of the honeycomb core is very small compared to the out-of-plane elastic moduli (E_3, G_{13}, G_{23}) the effect of neglecting the in-plane elastic constants of the honeycomb core on the percent differences in the total in-plane reaction forces is also small unlike the situation in the out-of-plane reaction forces. The in-plane stiffness of the face-sheet is much higher than the in-plane stiffness of the honeycomb core resulting in much higher share of the total in-plane reaction force by the face-sheets. Therefore, the in-plane elastic constants of the honeycomb core do not significantly affect the overall in-plane stiffness of the sandwich panel.

Model 2 proposed by Nast [7] gives very low differences for the in-plane reaction forces. However, the out-of-plane shear reaction forces of Model 2 are very different from the corresponding values determined by the mixed finite element model and this indicates the poor prediction of the out-of-plane elastic constants of the honeycomb core by the model proposed by Nast.

The main difference between the models M3 and M5 is the existence of the in-plane shear modulus G_{12} of Masters and Evans [5] in model M5. However, the percent differences in the total in-plane shear forces F12 and F21 are very small with slightly better prediction by the model M5. This result again shows that the in-plane shear modulus of the honeycomb core is not as effective as the out-of-plane shear moduli on the total reaction force. This conclusion is justified because based on the existing studies on the continuum models of honeycomb cores in the literature, the out-of-plane shear moduli are proportional to the ratio of the foil thickness of the core to the length of the cell wall, whereas the in-of-plane shear modulus is proportional to the cube of the same ratio [3,4,5]. Therefore, in-plane shear modulus of honeycomb core is very small compared to the out-of-plane shear moduli and neglecting the in-plane shear modulus in the equivalent model does not cause significant differences in the total in-plane shear reaction forces.

Model M7 uses the same elastic constants as M2 except for the in-plane Poisson's ratio ν_{12}, which is set to zero in model M7. The in-plane Poisson's ratio of the regular hexagonal honeycomb core is predicted to be about 1 in most of the studies in the literature [3,5,7]. By neglecting the in-plane Poisson's ratio in model M7, it is seen that the percent differences in the in-plane reaction forces slightly increase, which indicates the use of an improper in-plane Poisson's ratio instead of the commonly used value, which is approximately 1.

Based on the comparison of the percent differences in the total reaction forces given in Table 5, M5 seems to be the best performing model among M1-M7.

CONCLUSION

Continuum models of regular hexagonal aluminum honeycomb cores used in sandwich structures are assessed via finite element analyses of sandwich panels by employing finite element models with the actual honeycomb core geometry, and with the existing equivalent core models. The assessment of the effective elastic constants of honeycomb cores is based on the comparison of the total reaction forces, calculated by both finite element models, on the supported faces of sandwich panels due to different in-plane and out-of-plane uniform input displacements applied to the faces of the panels. It is inferred that to perform a reliable assessment the face-sheets of the reference model, which has the actual core geometry, and equivalent core finite element models should be meshed with solid elements in order to represent the out-of-plane stiffness of the panel most accurately. Based on the comparison of the differences in the total reaction forces calculated by the reference and equivalent core finite element models, it is concluded that the reliability of the individual in-plane and out-of-plane effective elastic constants of the existing continuum models of the honeycomb cores can be successfully evaluated. It is also observed that the sensitivity of a total reaction force to variations in an elastic constant of the equivalent model is sufficient to make a judgment on the reliability of the particular effective elastic constant. It is also concluded that among the existing equivalent core models that have been investigated, model M5 gives the best overall result. For future study, it is recommended that the present study be extended, and that other equivalent continuum core models that have not been investigated be included in a comparative study to come up with better equivalent models.

References

1. Kelsey, S., Gellatly, R.A., Clark, B.W., (1958). The Shear Modulus of Foil Honeycomb Cores, *Aircraft Engineering*, 30: 294-302.
2. Vinson, J.R., (1986). Optimum Design of Honeycomb Sandwich Panels Subjected to Uniaxial Compression, *AIAA Journal*, 24(10): 1690-1696.
3. Gibson L.J., and Ashby M.F., (1988). The Mechanics of Honeycombs, *Cellular Solids: Structure and Properties*, 2nd Edition, 93-173, Pergamon, Oxford.
4. Grediac, M., (1993). A Finite Element Study of the Transverse Shear in Honeycomb Cores, *International Journal of Solids and Structures*, 30(13):1777-1788.
5. Masters, I.G., and Evans, K.E., (1996). Models for the Elastic Deformation of Honeycombs, *Composite Structures*, 35(4): 403-422.
6. Burton, W.S. and Noor, A.K., (1997). Assessment of Continuum Models for Sandwich Panel Honeycomb Cores, *Computer Methods in Applied Mechanics and Engineering*, 145(3): 341-360.
7. Nast, E. (1997). On Honeycomb Core Moduli, 28th AIAA ASME ASCE AHS ASC Structures, Structural Dynamics and Materials Conference, AIAA-97-1178, 2:1035-1044.
8. Schwingshackl, C.W., Aglietti, G.S., and Cunningham, P.R. (2006). Determination of Honeycomb Material Properties: Existing Theories and an Alternative Dynamic Approach, *Journal of Aerospace Engineering*, 19(3): 177-183.
9. ANSYS, Version 11, ANSYS, Inc. Southpointe, PA, USA.
10. Aydıncak, İ., and Kayran, A,. An Approach for the Evaluation of Effective Elastic Properties of Honeycomb Cores by Finite Element Analysis of Sandwich Panels, *Journal of Sandwich Structures and Materials*, to appear.
11. İ.Aydıncak. Investigation of design and analysis principles of honeycomb structures. MSc Thesis, Middle East Technical University, Ankara, Turkey (2007).