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STRUCTURAL-DYNAMIC AND ACOUSTIC DESIGN OF ANISOTROPIC MULTILAYERED COMPOSITE STRUCTURES

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SUMMARY: Lightweight structures made of composite materials are increasingly used in many industrial fields for high-technology applications due to their versatile property profile. Especially for dynamically loaded structures, a high material damping combined with low constructive weight and adequate stiffness is required. But in today's sophisticated applications, lightweight structures will also have to meet very high acoustic (low noise) standards. Therefore, an acoustic analysis has to be included in the design process.

The structural-dynamic and acoustic behaviour of anisotropic multilayered composite structures can not be described by classical models. Here, advanced methods have been developed at the ILK, which take into account the special mechanical properties of the fibre-matrix compound. In advanced structural-dynamic and acoustic experimental investigations using laser scanning vibration interferometer technology and sound power measurements within a specially constructed reverberation chamber, the structural-dynamic and acoustic property profile of different lightweight materials has been determined. For the principal analysis of the acoustic radiation of these materials, additional numerical simulations using the Finite Element Method (FEM) and the Boundary Element Method (BEM) have been performed. On the basis of these extensive investigations, a vibro-acoustic model was developed, which takes into account the characteristic structural-dynamic properties such as eigenforms and eigenfrequencies as well as the important structure-fluid coupling.

KEYWORDS: Structural-dynamics, anisotropic material behaviour, sound radiation, lightweight structures

INTRODUCTION

Monolithic materials like magnesium, aluminium or titanium, which are mainly used in today's lightweight applications, often reach their limits with respect to the high demands concerning their structural-dynamic and especially acoustic behaviour. They offer high specific stiffness and strength, but a relatively low damping, which may lead to intense acoustic radiation. Here, composites or compound materials open the possibility to synergistically fulfill the requirements concerning stiffness and strength as well as damping and acoustic quality [1].

The structural-dynamic characteristics of especially anisotropic composite materials such as fibre-reinforced polymers can show a complicated coupling of bending and torsion dependent on the fibre orientation, the matrix composition and the lay-up. Thus, classical models can not describe the vibrations and the acoustic radiation of these structures. Here, advanced methods have been developed, which on the one hand offer the calculation of eigenfrequencies and eigenforms as well as material damping of modern lightweight structures [2]. On the other hand, the modelling of the acoustic radiation has been included. This vibro-acoustic design concept provides the possibility to calculate the eigenforms and the radiated sound power of plates, which are excited by an acoustic sound source inside a reverberant chamber, dependent on the special material properties.

STRUCTURAL-DYNAMIC ANALYSIS OF ANISOTROPIC COMPOSITE PLATES

In general plates are plane load bearing structures, which have a thickness h significantly smaller than the length a and the width b (Fig. 1).

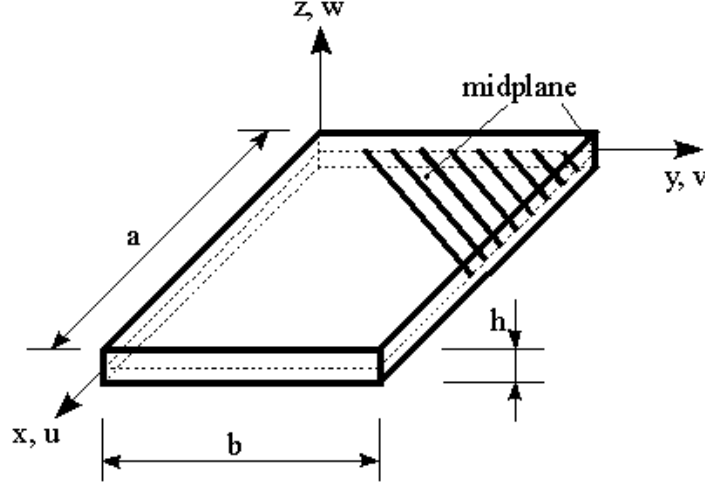


Fig. 1: Plate geometry

The plate of thickness h is here cut into halves by the midplane. As displacements in x -, y - and z -direction, the quantities u , v and w are used, with w being small in comparison to the thickness of the plate. Therefore, the structural-mechanic analysis can be done using a linear theory.

The eigenfrequencies of multilayered anisotropic plate structures are calculated on the basis of the *HAMILTONIAN* principal as an extremum principle in elasto-dynamics. Assuming a harmonic vibration, the kinetic energy is then determined by

$$T = \frac{1}{2} \int_V \mathbf{r} \left(\frac{\partial \underline{u}_f}{\partial t} \right)^2 dV \quad (1)$$

with the first-order time-derivative of the displacement vector \underline{u}_f which describes a vibration consistent with the boundary conditions. The potential energy of the multilayered composite can be derived from the deformation energy functional

$$U = \frac{1}{2} \int_{-h/2}^{+h/2} \int_0^a \int_0^b \left[\tilde{Q}_{11}^{(k)} \mathbf{e}_x^2 + 2\tilde{Q}_{12}^{(k)} \mathbf{e}_x \mathbf{e}_y + 2\tilde{Q}_{16}^{(k)} \mathbf{e}_x \mathbf{g}_{xy} + \tilde{Q}_{22}^{(k)} \mathbf{e}_y^2 + 2\tilde{Q}_{26}^{(k)} \mathbf{e}_y \mathbf{g}_{xy} + \tilde{Q}_{66}^{(k)} \mathbf{g}_{xy}^2 + \tilde{Q}_{44}^{(k)} \mathbf{g}_{yz}^2 + \tilde{Q}_{55}^{(k)} \mathbf{g}_{xz}^2 + 2\tilde{Q}_{45}^{(k)} \mathbf{g}_{yz} \mathbf{g}_{xz} \right] dy dx dz, \quad (2)$$

which offers the possibility to include a shear-elastic displacement approach using shear deformation theory as well as the classical *KIRCHHOFF* plate theory [3].

For the case of steady-state conditions, the components of the displacement vector can be described by harmonic functions. Here, the eigenforms are approximated by using the corresponding solutions of the beam problem, which fulfill the boundary conditions [4]. Minimising the energy functional leads to a system of homogeneous linear equations for the

determination of the coefficients of the *RITZ* approach, which can be written as a characteristic equation of an eigenvalue problem

$$\left(\underline{\underline{K}} - \underline{\underline{I}}_f \underline{\underline{M}}\right) \underline{a}_f = 0. \quad (3)$$

The calculation of the eigenvalues $\underline{\underline{I}}_f$, the eigenfrequencies f_f and the eigenvectors \underline{a}_f is numerically done by using standard algorithms.

For anisotropic multilayered composites, the analytical determination of the loss factor d_f is based on the concept of complex moduli, which results in complex parameters in the material laws. According to the basic assumption that the complex moduli are transformed like their elastic correspondancies, a separation of the reduced complex stiffnesses is possible

$$\tilde{Q}_{ij}^{*(k)} = \tilde{Q}_{ij}^{(k)} + i\tilde{Q}_{ij}''^{(k)}. \quad (4)$$

Due to the small values of material damping of fibre-reinforced composites, the resulting complex characteristic equation of the eigenvalue problem can approximately be separated in its real and imaginary part using the *RAYLEIGH* quotient, which leads - after elimination of $\underline{\underline{I}}_f$ - to the modal loss factor

$$d_f = \frac{\underline{a}_f^T \underline{\underline{K}}'' \underline{a}_f}{\underline{a}_f^T \underline{\underline{K}} \underline{a}_f}, \quad (5)$$

where \underline{a}_f denotes the real part of the complex eigenvector.

EXPERIMENTAL INVESTIGATIONS

The determination of the vibro-acoustic property profile of anisotropic composite structures involves experimental investigations, which have to include structural-dynamic and acoustic measurements.

The structural-dynamic measurements were carried out using the Laser Scanning Vibration Interferometer (LASVI) technology, which is an advanced measuring system for the analysis of vibrating structures. Compared with conventional techniques using electromechanical sensors, the mass and eigenfrequencies of which have negative influence on the accuracy of the measurements, the LASVI method allows a contactless and highly exact determination of vibrational quantities and conditions.

In order to determine the eigenmodes of the investigated plate structures, a scanning system was used, which allows measuring the velocity of different surface positions (net points) without mechanically moving the optical measuring head. Depending on the chosen resolution, the number of net points is variable. For the performed investigations, a resolution of 16×38 with 608 measuring points was used according to the numerical calculations.

The FFT analysis of the measured surface velocity is performed for every net point. The summation of the frequency-dependent data of each point is used to build the resonance diagram of the vibrating plate, which is the basis for the identification of the eigenfrequencies. In Fig. 2a, the resonance spectrum of the surface velocity of a CFRP plate with a thickness of 2.0 mm in the frequency interval from 150 Hz to 250 Hz is shown.

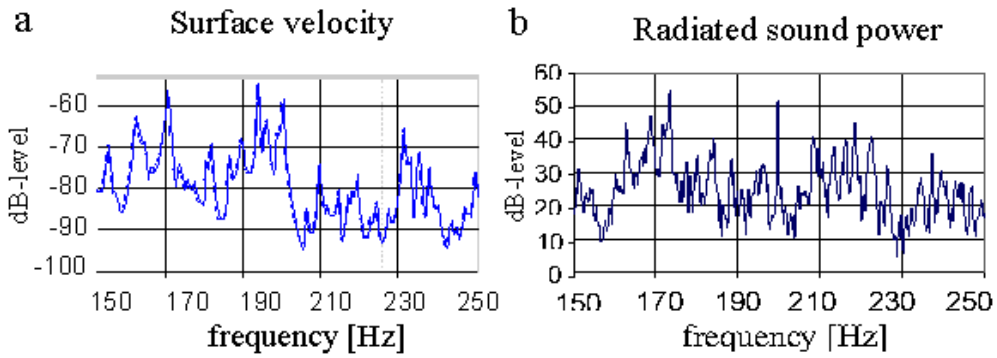


Fig. 2: Measured resonance diagrams

The acoustic investigations were carried out within a reverberant chamber, which is a closed room built by acoustically rigid walls. The sound waves inside the reverberant chamber are reflected on all of the rigid walls, so that this multiple reflection and scattering leads to an ideal mixture of waves of different frequencies. The standing waves are superimposed and the locally varying maxima and minima of the sound pressure are approximately compensated. This creates a sound field, which is characterised by an equal sound intensity at every position inside the reverberant chamber, a so called diffuse sound field.

The diffuse sound field is then taken as the acoustic excitation of a test plate, which is clamped inside a rigid wall. This wall separates the reverberant chamber into two rooms, the so called transmitter and the receiver hall. Inside the transmitter hall, an acoustic sound source creates the diffuse sound field, whereas inside the receiver hall the radiation of the vibrating plate is the sound source (Fig. 3a).

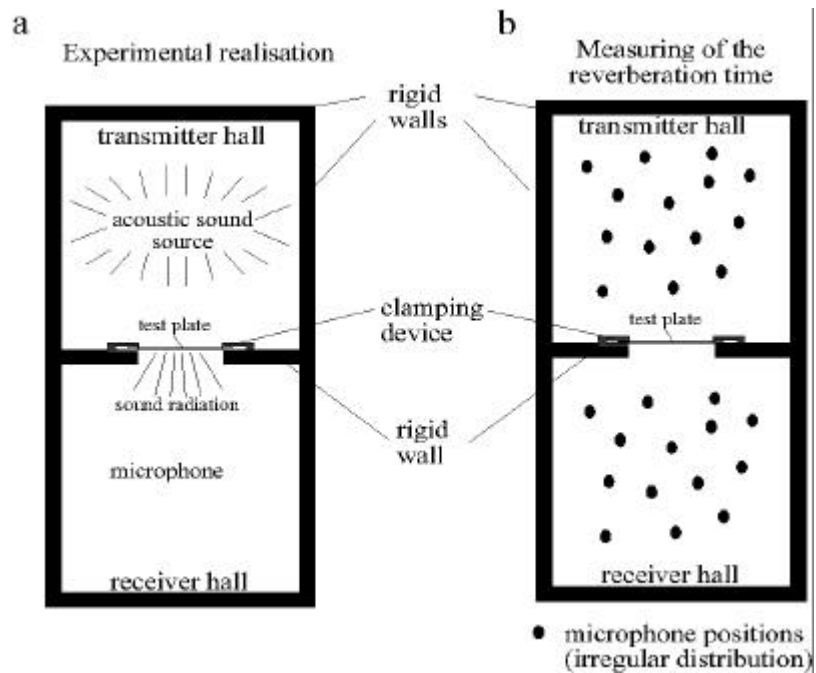


Fig. 3: Principal drawing of the reverberant chamber

Due to the diffuse sound field in both of the halls, the energy of the sound waves is approximately preserved. Thus, a reverberant chamber can be used to determine the radiated sound power of acoustically excited structures inside the receiver hall. This sound power is an integral and spatially constant physical parameter and therefore a suitable acoustic design variable.

The acoustic sound source is a group of speakers, which are positioned in the transmitter hall. The test plate is fixed in a specially constructed clamping device, which guarantees reproducible boundary conditions. In the receiver hall, the radiated sound pressure is measured by a microphone, which rotates on a pivot mounting in a plane with an inclination of 45° . On the basis of the measured sound pressure values, the radiated sound power level is then calculated according to DIN EN 23741 [5].

In order to analyse the quality of the diffuse sound field in the transmitter and the receiver hall, the reverberation time has to be measured. Within this reverberation time, the sound pressure level is reduced by 60 dB after the acoustic sound source is abruptly cut off. Due to the diffuse sound field, this frequency-dependent time period has to show approximately the same behaviour at any position inside the reverberant chamber (see Fig. 3b).

The frequency analysis of the measured sound power is then done by FFT. For the CFRP plate, Fig. 2b presents the measured radiated sound power. A comparison of the frequency-dependent behaviour clearly reveals the mechanism of acoustic radiation, which can be seen by the coupling of the frequency distribution of surface velocity and radiated sound power. The main focus of the performed numerical simulations is on this important coupling between the structural-dynamic and acoustic parameters surface velocity and radiated sound power.

NUMERICAL CALCULATIONS

For a more detailed analysis of the vibro-acoustic coupling between the physical parameters surface velocity and radiated sound power, numerical calculations on the basis of the FEM and the BEM have been performed. These calculations were carried out in the workstation pool of the ILK. For the structural-dynamic analysis, mainly the program packages I-DEAS and ANSYS were used and the acoustic analysis was performed using the special BE Software SYSNOISE and I-DEAS VIBRO-ACOUSTICS.

The eigenforms calculated by the FEM and the measured eigenforms by LASVI show a very good agreement (Fig. 4).

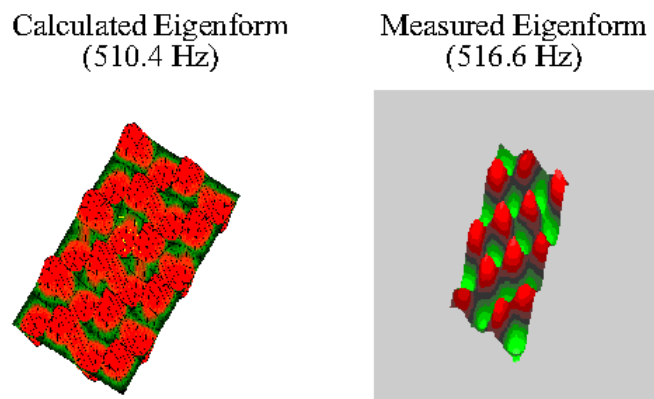


Fig. 4: Numerically and experimentally determined eigenforms

This high accuracy of the structural-dynamic FE calculations is the main basis for the following vibro-acoustic analysis using the BEM. As input data for the BE analysis, the structural FE net is needed. This net is then converted into a BE net, which has the displacement field calculated within the harmonic response analysis as a boundary condition. The acoustic quantities on the radiating surface and on specially located points (field points) are computed.

These field points are positioned inside the air volume, which surrounds the vibrating plate [6].

For the analysis of the acoustic behaviour, the values of the sound pressure and the sound pressure dB level, the integrated sound power and the effective surface velocity are of importance. In Fig. 5 the calculated sound pressure distribution on the basis of the given surface velocity field of a three-layer sandwich plate is shown.

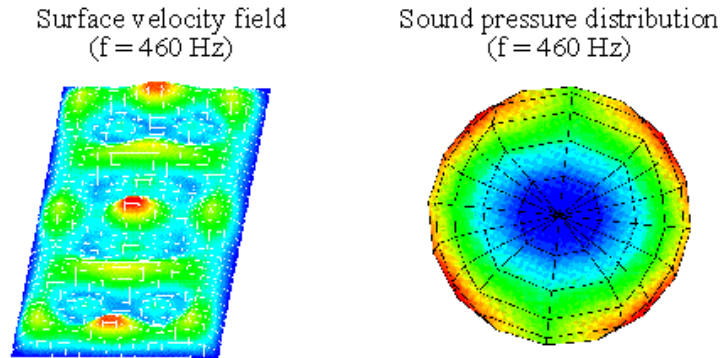


Fig. 5: Calculated surface velocity and sound pressure distribution of a three-layer sandwich plate

In order to verify the coupling of surface velocity and radiated sound power, extended FE and BE calculations for different materials and different material damping values were performed. As an example, Fig. 6 shows the frequency distribution of the averaged surface velocity and the sound power level calculated by the FEM and the BEM for a titanium plate within the frequency interval from 0 Hz to 1000 Hz.

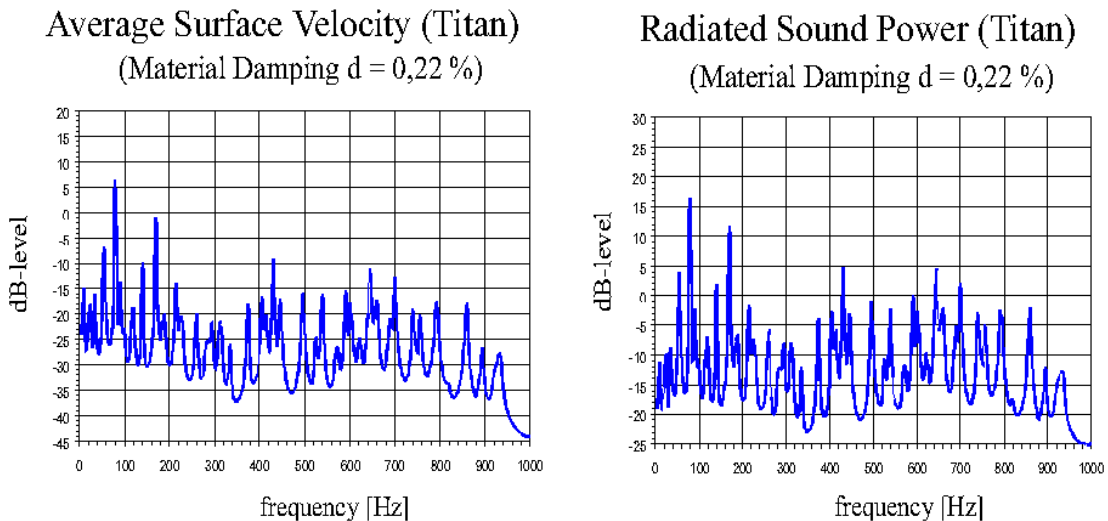


Fig. 6: Frequency distribution of the averaged surface velocity and the sound power level

According to the results of the LASVI measurements and the acoustic investigations within the reverberant chamber, the numerical calculations using the FE and BE techniques clearly reveal the coupling of the surface velocity and the radiated sound power. These two physical quantities have the same frequency behaviour, but different absolute dB values due to the different reference values. Therefore, the coupling of the structural vibration with the acoustic radiation results in the coupling of the physical parameters surface velocity and radiated sound power. This fundamental vibro-acoustic coupling is the basis for the following analytical model.

MODELLING OF THE SOUND RADIATION

The vibro-acoustic behaviour of acoustically excited structures is characterised by the coupling of the structural vibration with the surrounding air volume. This complex process of excitation, structural response and radiation can only be analysed if the vibration is described as a time-dependent solid-borne sound field. Here, bending vibrations of plates represent a relatively simple model for the creation and propagation of solid-borne sound.

In the following, rectangular plates with small thickness and deflection as well as a homogeneous mass distribution are analysed. For the description of the plate dimensions, a cartesian x, y, z coordinate system is used. The x and y coordinates open the plane, in which the edges l_x and l_y of the plate are situated, the thickness h and the deflection $w(x, y, t)$ are chosen to be parallel to the z coordinate (Fig. 7).

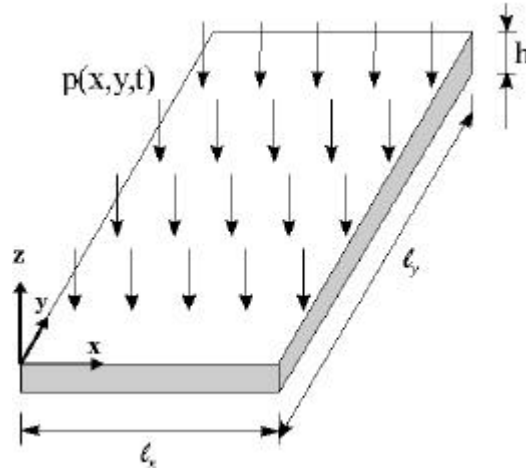


Fig. 7: Plate under an exterior pressure field $p(x, y, t)$

An exterior pressure field normal to the surface $p(x, y, t)$ like

$$p(x, y, t) = p(x, y)e^{i\omega t} \quad (6)$$

is assumed to create a time-dependent bending wave, which can be written for the deflection $w(x, y, t)$ in the form

$$w(x, y, t) = w(x, y)e^{i\omega t}. \quad (7)$$

This deflection field $w(x, y, t)$ is related to the surface velocity distribution normal to the plates' $x - y$ plane by

$$v_w(x, y, t) = i\omega w(x, y, t). \quad (8)$$

In terms of the chosen energy formulation of this solid-borne sound field within an anisotropic plate, the surface velocity $v_w(x, y, t)$ has to satisfy the following variational equation

$$\mathbf{dI}(v_w(x, y, t)) = 0 \quad (9)$$

with Π being the total energy in terms of

$$\Pi(v_w) = U(v_w) - T(v_w) + W(v_w). \quad (10)$$

Here $U(v_w)$ denotes the deformation energy functional (see (2)), $T(v_w)$ the kinetic energy (see (1)) and $W(v_w)$ is the potential energy due the exterior pressure field $p(x, y, t)$ with

$$W(v_w) = -i\mathbf{w} \int_0^{l_x} \int_0^{l_y} p(x, y, t) w(x, y, t) dy dx. \quad (11)$$

Besides this variational formulation, also physically meaningful initial and boundary conditions have to be defined in order to fully describe the surface velocity field in the plate. The solution of this elasto-dynamic field problem for a general excitation $p(x, y, t)$ can only be approximated by an appropriate set of functions defined on the whole domain of the elastic body. A reasonable solution can be found by substituting the general surface velocity field $v_w(x, y)$ by

$$v_s(x, y) = \sum_{n=1}^{M^R \cdot N^R} c_f v_f(x, y), \quad (12)$$

with $v_f(x, y)$ being the eigenfunctions of the problem and $M^R \cdot N^R$ as the order of the *RITZ* solution, which is used to determine the eigenfunctions of the problem. Substituting the approximation (12) in the variational equation (9) finally yields

$$v_s(x, y) = \sum_{f=1}^{M^R \cdot N^R} \frac{i\mathbf{w} v_f(x, y)}{\mathbf{h}(\mathbf{w}_f^2 - \mathbf{w}^2) \Lambda_f} \int_0^{l_x} \int_0^{l_y} v_f(x, y) p(x, y) dy dx, \quad (13)$$

with Λ_f as the norm of the eigenfunctions v_f [7]. This expansion gives the possibility to calculate the structural-dynamic response for a general acoustic excitation $p(x, y)$ by using the eigenfrequencies and eigenforms of the structure.

The described solution method for the structural-dynamic analysis of plates with isotropic material behaviour is implemented into a specially developed program system and is the basis for the analysis of the radiation process. The surface velocity field (13) causes the radiation of sound waves into the air volume. In order to describe these sound waves, an idealized model is built.

The rectangular plate is assumed to be embedded in a rigid wall of infinite extent, what creates two semi-infinite spaces. In the negative half-space, a constant sound pressure p_0 due to a diffuse sound field exists. A radiation in this half-space is not calculated. The positive half-space is thought to be the air volume, in which the sound waves propagate. The sound waves are described by the physical parameters of the sound particle velocity normal to the rigid wall $v_{z/air}(x, y, z, t)$ and the sound pressure $p(x, y, z, t)$.

The radiation of sound waves is caused by the coupling of the solid-borne sound field v_s with the air in the positive half-space. An analytically continuous description of the sound particle velocity on the surface of the wall is therefore only possible in the form of a Fourier integral expansion

$$v_{z/air}(x, y, z = 0) = v_s(x, y) = \frac{1}{4\mathbf{p}^2} \int_{-\infty-\infty}^{+\infty+\infty} \mathcal{V}(k_x, k_y) e^{-ik_x x} e^{-ik_y y} dk_x dk_y \quad (14)$$

and the Fourier transform $\mathcal{V}(k_x, k_y)$

$$\mathcal{V}(k_x, k_y) = \int_{-\infty-\infty}^{+\infty+\infty} v_s(x, y) e^{ik_x x} e^{ik_y y} dx dy. \quad (15)$$

Inside the full positive half-space $z \geq 0$, the sound particle velocity is assumed to propagate as a plane wave in the z -direction due to the plane sound source and the infinite air volume

$$v_{z/air}(x, y, z, t) = v_{z/air}(x, y) e^{-ik_z z} e^{i\omega t} \quad (16)$$

with

$$v_{z/air}(x, y) = v_{z/air}(x, y, z = 0) = v_s(x, y) = \frac{1}{4\mathbf{p}^2} \int_{-\infty-\infty}^{+\infty+\infty} \mathcal{V}(k_x, k_y) e^{-ik_x x} e^{-ik_y y} dk_x dk_y. \quad (17)$$

The calculation of the sound pressure is then performed using the basic relation between the sound particle velocity and the sound pressure

$$v_{z/air}(x, y, z, t) = \frac{-1}{i\mathbf{w}\mathbf{r}_0} \frac{\partial p}{\partial z} \quad (18)$$

as a plane wave of the form

$$p(x, y, z, t) = \frac{\mathbf{w}\mathbf{r}_0}{k_z} v_{z/air}(x, y) e^{-ik_z z} e^{i\omega t}. \quad (19)$$

With these analytical equations for $v_{z/air}(x, y, z, t)$ and $p(x, y, z, t)$, the radiated sound field is completely determined and the sound power, which is transmitted through a plane $z = z_0$ in the positive half-space, can be calculated. Assuming a zero energy loss in the reverberant chamber, the radiated sound power inside the receiver hall can be calculated solving the integral

$$P = \frac{1}{2} \operatorname{Re} \left\{ \int_{-\infty-\infty}^{+\infty+\infty} p(x, y, z = z_0, t) v_{z/air}^*(x, y, z = z_0, t) dy dx \right\}. \quad (20)$$

Inserting equations (16) and (19) transforms the integral into the Fourier integral and yields

$$P = \frac{\mathbf{w}\mathbf{r}_0}{8\mathbf{p}^2} \int_{-\infty-\infty}^{+\infty+\infty} \frac{\mathcal{V}(k_x, k_y) \mathcal{V}^*(k_x, k_y)}{\sqrt{k_0^2 - k_x^2 - k_y^2}} dk_x dk_y. \quad (21)$$

This equation includes the Fourier transform $\mathcal{V}(k_x, k_y)$ of the solid-borne sound field, which is related to the surface velocity distribution (13) in the vibrating plate. Therefore, the developed analytical vibro-acoustic model shows the close coupling of the surface velocity with the radiated sound power.

Exemplarily, Fig. 8 presents the results, derived from this vibro-acoustic model and from the structural-dynamic and acoustic measurements. In Fig. 8a, the first resonance peak of the averaged surface velocity for a rectangular aluminium plate ($l_x = 860$ mm, $l_y = 560$ mm, $h = 1.97$ mm) is shown, which on the one hand is calculated on the basis of the excited surface velocity distribution $v_s(x, y)$ (13) by

$$\bar{v}_s = \sqrt{\frac{1}{l_x l_y} \int_0^{l_x} \int_0^{l_y} \mathbf{r} h v_s(x, y) v_s^*(x, y) dy dx} \quad (22)$$

and on the other hand is directly measured by the LASVI technique. A comparison of the experimentally determined and analytically calculated (21) sound power is given in Fig. 8b. For both the average surface velocity and the radiated sound power a good agreement of the theoretical and experimental data has been achieved.

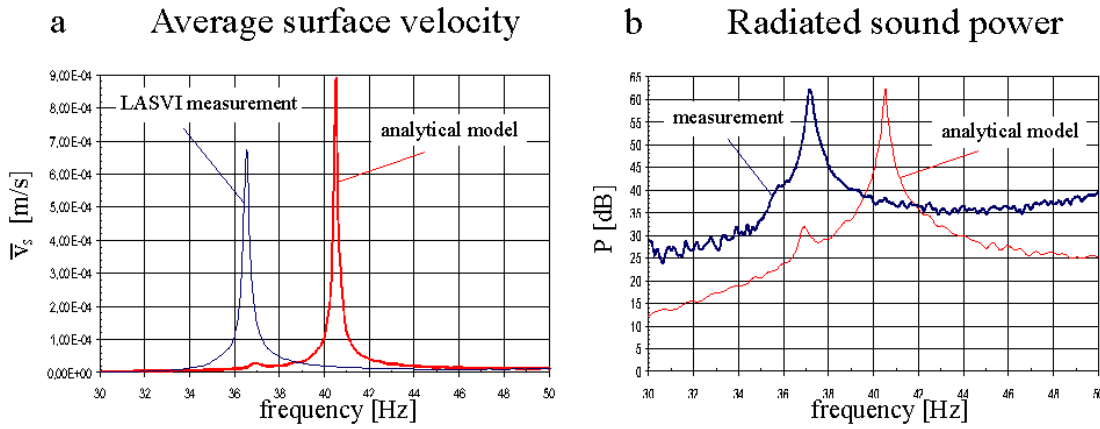


Fig. 8: Comparison of calculated and measured structural-dynamic and acoustic quantities (material: aluminium alloy, $h = 1.97$ mm)

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