# DYNAMICAL CHARACTERISTICS OF FIBRE COMPOSITES

Werner Hufenbach[1], Carsten Holste[1], Günther Pfeifer[2]

[1]Institut für Leichtbau und Kunststofftechnik, [2]Institut für Technische Akustik
Technische Universität Dresden, D-01062 Dresden, Germany

**SUMMARY:** Advanced lightweight applications for dynamically loaded structures increasingly have to satisfy extreme requirements concerning their dynamic behaviour. This leads to a rising need to use load-adaptedly designed material characteristics. Only in this way a structural-dynamic design following the function-integrating modern lightweight principals is possible.

With the carbon fibre-reinforced polymers (CFRP) coming up lately, damping properties can be realized, which are markedly superiour to those of light metals. Besides the existing dependence of stiffness and strength on the fibre orientation, the material damping shows a pronounced anisotropy, as well. Therefore this anisotropic damping has to be considered as an optimization variable of the structural-dynamic design.

In this paper the structural vibration of lightweight structures is presented, which is a complex function of material, damping value, eigenmodes, frequency and boundary conditions. Therefore the investigations were carried out using advanced experimental techniques and numerical simulation methods such as the Laser Scanning Vibration Interferometer (LASVI) and the Finite Element (FEM). As a practical example the structural-dynamic design of a CFRP plate with restricted eigenfrequencies and critical buckling loads is presented.

**KEYWORDS:** Anisotropic damping, Structural-dynamic design, analytical and numerical simulation

## 1 INTRODUCTION

Increasing dynamic loads in structural components and parts of technical lightweight constructions lead to a rising need to use load-adaptedly designed dynamical characteristics for vibrating elements [1, 2]. Therefore the quantity of applications of multilayered CFRP materials for vibrationally loaded structures is growing, because of the possibility to optimally design their layered composition according to the existing loads [3].

Fibre-reinforced polymers belong to the established composites, what can be seen especially in the successful use of structural-dynamically optimized components. Such hybrid materials consist of a polymer matrix, which has a fibre-reinforcement implemented in the direction of the largest stresses. With these materials, damping values are reached, that significantly exceed those of light metals.

The main task of practical applications is the optimization of the structural dynamic behaviour only by adjusting the layer orientations dependent on the fibre volume content, without the aid of further constructive measures.

For an analytical structural dynamic analysis in general plane load-bearing structures such as plates and shells are used due to the fact, that well known mathematical solution methods can be used [4]. In the following the eigenmodes and vibrations of plates are analysed dependent on the frequency.

## 2 DYNAMIC BEHAVIOUR OF ANISOTROPIC COMPOSITES

The elasticity and damping properties needed for the design of dynamically loaded CFRP lightweight structures have been determined at the ILK for many different unidirectional (UD) CFRP composites. As an example the dynamic properties for a PEEK/CF and a EP/CF composite are presented in this paper. For this investigations, an experimental resonance technique based on the fast Fourier transformation (FFT) has been applied. The measured dynamic properties build the basis for the later calculation of the composite plate.

For the design and calculation of CFRP components, a series of characteristic values has to be determined for the individual fibres and matrices as well as for the final composite dependent on the fibre orientation. In the present case of dynamic tests, the dependence on the load frequency and – for higher load levels – on the stress amplitude has to be considered additionally.

In the resonance tests, five specimens of each composite type are examined. The necessary experimental programme contains resonance tests of UD layers with a fibre orientation of $0°, 45°$ and $90°$. The evaluation of the tests is done in wide-range resonance diagrams as a concurrent analysis of the resonances with respect to frequency, order and mode.

The peak frequencies and the damping of the different resonance points can be determined exactly with a special evaluation algorithm based on the adjustable frequency range of the frequency generator and an adapted window zoom of the FFT analyzer. Finally, the dynamic moduli are calculated with a specially developed software tool [4].

The laminate properties of the studied composites determined from resonance bending tests are summarized in Tab. 1. The here given fibre volume content was determined in standard glow tests.

Tab. 1: Dynamic material properties of the studied laminates

| | material | **PEEK/CF** | **EP/CF** |
|---|---|---|---|
| components | fibre | IM 7 | HTA 5331 |
| | matrix | peek | epoxy resin |
| structure | composite | UD | UD |
| fibre volume content | $\psi$ [%] | 62±1.0 | 60±1.0 |
| Young's modulus (axial) | $E'_{\parallel}$ [GPa] | 113.13 | 85.45 |
| Young's modulus (transv.) | $E'_{\perp}$ [GPa] | 11.20 | 6.49 |
| dyn. shear modulus | $G'_{\#}$ [GPa] | 4.39 | 2.26 |
| Poisson's ratio | $\nu_{\parallel\perp}$ | 0.24 | 0.32 |
| axial damping | $d_{\parallel}$ [%] | 0.124 | 0.193 |
| transverse damping | $d_{\perp}$ [%] | 0.218 | 1.552 |
| shear damping | $d_{\#}$ [%] | 0.566 | 2.414 |
| density | $\varrho$ [kg/m$^3$] | 1450 | 1500 |

The dynamic properties of the composites are visualized in Fig. 1. The polar diagrams show the dynamic Young's moduli and the material dampings of the composites PEEK/CF and EP/CF dependent on the fibre orientation.

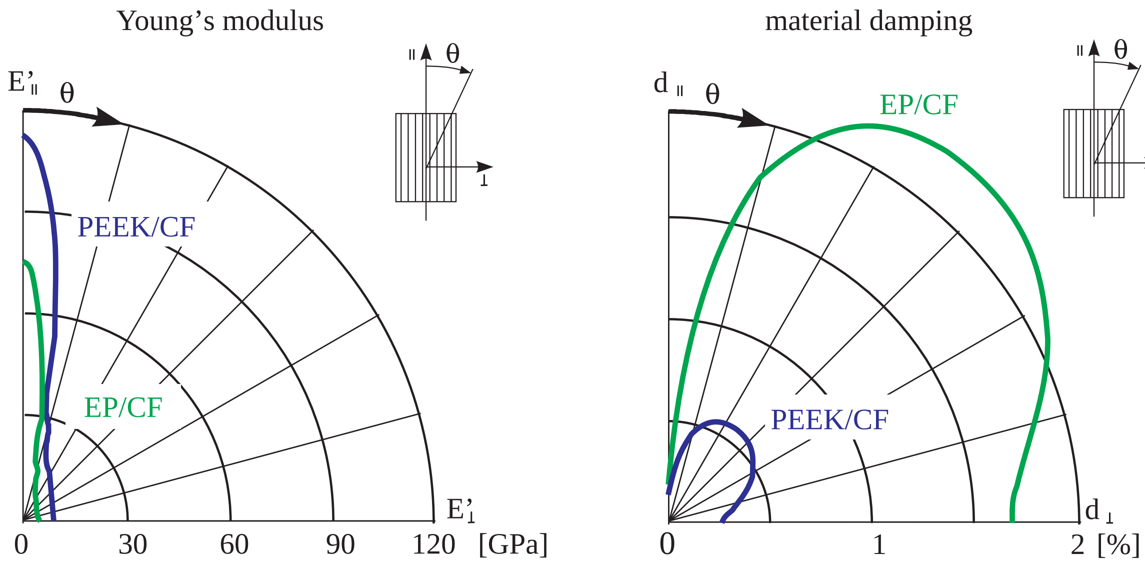


Fig. 1: Anisotropic material properties of CFRP

In the polar diagrams, the antagonism of the dynamic stiffness $E'$ and the damping $d$ of the UD structure is illustrated very clearly. For $\theta = 0°$ (on-axis loads), the damping is always at a minimum and Young's modulus is at a maximum. Comparing the damping behaviour of the composites with reaction resins PEEK and EP it can be seen from the polar diagram, that they have a fundamentally similar damping behaviour with a maximum at a fibre orientation between 25° and 55°, although their damping values are different.

The polar diagrams of the anisotropic properties of the UD reinforced layers serve as a basis for the optimization of the layer orientation in the multi-layered compound. Optimized working points can be found for angles in a range of $10° \leq \theta \leq 30°$. Here an appropriate stiffness is combined with an already satisfactory damping.

## 3 DYNAMICAL ANALYSIS OF ANISOTROPIC FIBRE-REINFORCED COMPOSITES

The description of structural-mechanical problems by means of the linear three-dimensional theory of elasticity especially for anisotropic composite structures has proved to lead to complicated mathematical equations. Thus only for special cases analytical solutions can be found. Simplifications, such as the reduction of geometric degrees of freedom, give the possibility of solving the analytical problems without influencing the quality of the calculation results too much.

### 3.1 LAMINATE THEORY OF MULTILAYERED COMPOSITES

The starting point in the calculation of multilayered composite structures is the mechanical behaviour of the unidirectional (UD) reinforced single layer (Fig. 2).

In the fibre-oriented cartesian coordinate system $1, 2, 3$ ($\parallel$,$\perp$, $\perp$), which is normally used for the material constants, such UD layers show an orthotropic material behaviour. The calculations are done in a structure-oriented or load-adapted coordinate system $x, y, z$, in which the UD-layer composite has anisotropic and in special cases monoclinic material behaviour. The material properties of the single layers in this specific coordinate system are obtained by polar transformation.

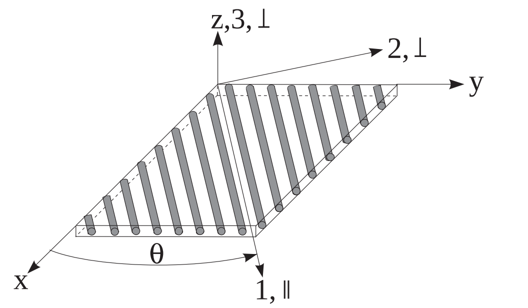


Fig. 2: Fibre-oriented and structure-oriented coordinate system

The transformed layer stiffnesses are combined to the stretching-, coupling- and bending stiffnesses $A_{ij}, B_{ij}$ and $D_{ij}$ of the multilayered composite (Fig. 3) for the coupled disk/plate-problems using the classical laminate theory.

By doing so, the position and thickness of each layer (layer index: k) has to be taken into account:

Stretching stiffness

$$A_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{Q}_{ij}^{(k)} dz = \sum_{k=1}^{N} \tilde{Q}_{ij}^{(k)} h_k$$

Coupling stiffness

$$B_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{Q}_{ij}^{(k)} \cdot z dz = \sum_{k=1}^{N} \tilde{Q}_{ij}^{(k)} \bar{z}_k h_k$$

Bending stiffness

$$D_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{Q}_{ij}^{(k)} \cdot z^2 dz = \sum_{k=1}^{N} \tilde{Q}_{ij}^{(k)} (\bar{z}_k^2 h_k + \frac{h_k^3}{12}),$$

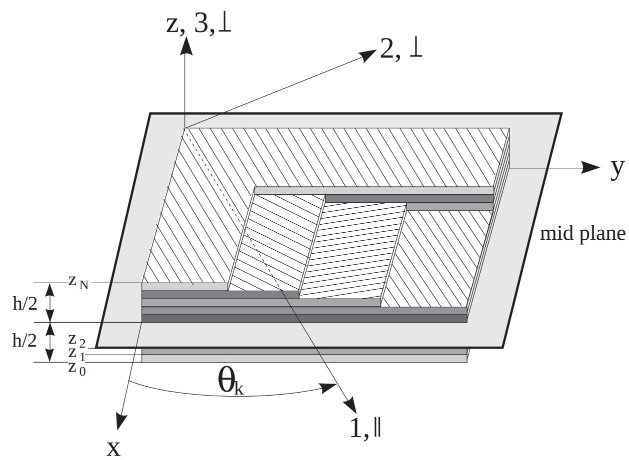


Fig. 3: Multilayered composite

with $h_k = z_k - z_{k-1}$ for each single layer thickness and $\bar{z}_k = \frac{1}{2}(z_k + z_{k-1})$ as the distance of the k-th layer to the mid plane.

## 3.2 EIGENFREQUENCY CALCULATION

The examination of the free vibrations and the determination of the related eigenfrequencies and mode shapes respectively are the first step in an extensive structural-dynamic analysis, which consists of a sinusoidal excitation with the period $T = 2\pi/\omega$.

The mode shape analysis of anisotropic composites is done here on the basis of the Hamiltonian principal as extremum principal of elastodynamics. The starting point of the modal analysis is the functional $\mathcal{H}$ as an integral over the kinematic potential $\mathcal{L}$ (Lagrange function)

$$\mathcal{H} = \int_0^{2\pi/\omega} \mathcal{L} dt = \int_0^{2\pi/\omega} (T - \Pi) dt \tag{1}$$

with the kinetic energy $T$ and the potential energy $\Pi = \Pi_i + \Pi_a$ of an elastic body in the x,y,z global coordinate system, which is derived from the potentials $\Pi_i$ and $\Pi_a$ of the inner and outer forces. The volume and surface forces of the free vibrations are assumed to be zero.

On the basis of the HAMILTONIAN principal the RITZ method is applied to approximately calculate the eigenfrequencies. Each of the used approximation functions has to satisfy the kinematic boundary conditions of the problem to achieve convergence of the approximate solution.

The minimization of the functional $\mathcal{H}$ leads to a system of linear equations for the determination of the unknown eigenvalues $\omega_n$ and eigenfrequencies $f_n$, which can be solved by using standard analytical methods.

### 3.3 DAMPING CALCULATION

For the calculation of the damping values the concept of the complex moduli is applied, what leads to complex material values in the material laws in the case of anisotropic multilayered composites. The complex reduced stiffnesses

$$\tilde{Q}_{ij}^{*(k)} = \tilde{Q}_{ij}^{\prime(k)} + i\tilde{Q}_{ij}^{\prime\prime(k)}. \tag{2}$$

here are obtained in the same way as their elastic equivalents. The stretching-, coupling- and bending stiffnesses are transformed to complex quantities in the same way. For the calculation of modal loss factors these quantities have to be considered adequately in the functional $\mathcal{H}$. By minimization of the functional, the solution of the complex eigenvalue equation is derived, which yields (besides the eigenvalues, eigenfrequencies and eigenvectors and mode shapes) also the modal loss factor.

For the superposition of single layers to a multilayered composite the following equation

$$D_{ij}^{*} \equiv \frac{h^3}{12} Q_{ij}^{*(C)} \tag{3}$$

is used to design laminates under bending loads, which leads to the equivalent dynamic values of the multilayered composite (index:C):
Equivalent dynamic Young's modulus

$$E^{\prime(C)} = \frac{1}{S_{11}^{\prime(C)}} \tag{4}$$

Equivalent material damping (composite damping)

$$d^{(C)} = -\frac{S_{11}^{\prime\prime(C)}}{S_{11}^{\prime(C)}}. \tag{5}$$

The calculation procedures developed above are supported by detailed experimental studies and exist at the ILK in the form of a calculation software, which increases the possibility of a fast structural-dynamic analysis of fibre-reinforced composites.

## 4 DYNAMIC COMPOSITE DESIGN

Based on the determined material data functions for the UD layer (Fig. 1) now the property profile of different multilayered composites dependent on the layer angle can be calculated. In the following a damping optimization is done for an example of a symmetric EP/CF three-layer composite (material data in Tab. 1) of two outer layers ($i = 1, 3$) with the same fibre angles $\theta_1 = \theta_3$ and an inner layer ($i = 2$) with the same thickness as the two outer layers and the fibre angle $\theta_2$.

In Fig. 4 the equivalent dynamic Young's modulus $E'^{(C)}$ and the composite damping $d^{(C)}$ of this composite are shown dependent on the fibre angles $\theta_1$ und $\theta_2$.

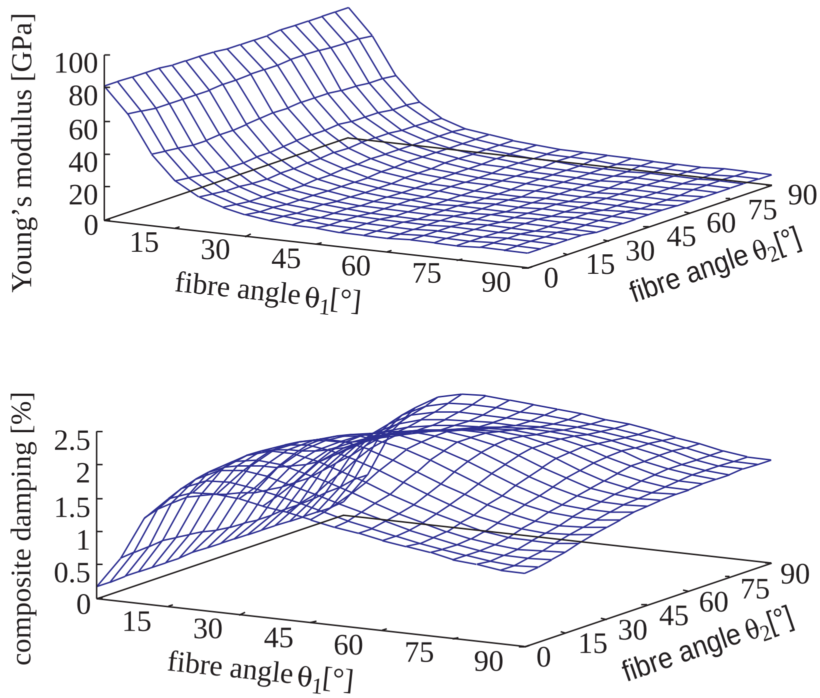


Fig. 4: Dynamic composite values (constants)

Fig. 4 gives a good overview of the property potential of the multilayered composite, that has to be designed. The Young's modulus shows the expected significant decrease with an increasing fibre angle $\theta_1$, whereas the fibre angle $\theta_2$ of the inner layer has only a minor influence. In contrast to this behaviour, the damping depends substantially on both fibre angles. The maximum damping value of 2,2192 % occurs at angles $\theta_1/\theta_2 = 35°/35°$.

## 5 STRUCTURAL VIBRATION OF PLATES

In the following, rectangular plates with small thickness and deflection are analysed. For the description of the plate dimensions a cartesian $x, y, z$ coordinate system is used. The $x$ and $y$ coordinates open the plane, in which the edges $a$ and $b$ of the plate are situated, the thickness and the deflection are chosen to be parallel to the z coordinate. Using the Kirchhoff bending theory, the equation of motion for such plates yields:

$$B\Delta\Delta w + \varrho h \frac{\partial^2 w}{\partial t^2} = p(x, y, t) \tag{6}$$

with $B$ being the bending stiffness of the plate and the well known Laplace-operator $\Delta$. Besides this equation of motion also physically meaningful initial and boundary conditions have to be defined in order to get a resolvable mechanical problem for the description of bending waves in plates. In the first place, the three classical boundary conditions [1] for the edges $x = 0, y$ or $x = a, y$ will be discussed:

[1]Within the classical theory, fictitious resultant forces have to be introduced in the form of $\bar{Q}_x = Q_x + \frac{\partial}{\partial y} M_{xy}$.

$$\begin{array}{lll} \text{free edge} & M_x = 0 & \bar{Q}_x = 0, \\ \text{simply supported edge} & w = 0 & M_x = 0, \\ \text{fully clamped edge} & w = 0 & \dfrac{\partial w}{\partial x} = 0. \end{array}$$

Generally the distribution of the deflection $w$ for the time $t = 0$ is given as a initial condition:

$$w(x, y, 0) \quad = \quad W(x, y) \tag{7}$$

where $W(x, y)$ are special functions of the local coordinates. As an adequate solution method for such kind of elastodynamic problems the RITZ-method is used, working with an appropriate set of functions defined on the whole domain of the elastic body. Neglecting the influence of the strains in the midsurface for the case of pure bending loads, the deflection can be approximated using

$$W(x, y) \quad = \quad \sum_{m=1}^{M} \sum_{n=1}^{N} f_{mn}, \, _m(x), \, _n(y) \tag{8}$$

The functions $, \, _m(x), , \, _n(y)$ satisfy the above boundary conditions, if they are set to be equal to the correponding solutions for the shear-rigid beam. The combination of the above boundary conditions produces nine different bearing cases, the distinction of which is only necessary for the case of anisotropic material behaviour. For plates with isotropic material the combinations formally give $6^2 = 36$ different constellations for the boundary conditions, which are reduced to 21 due to symmetry properties [5].

The described solution method for the structural dynamic analysis of plates with isotropic and anisotropic material behaviour is implemented into a specially developed program system [6], which above all offers the possibility to calculate the

- eigenmodes,
- eigenfrequencies,
- and for anisotropic structures the modal loss factors.

In the following chapter these analytically derived results are compared with the results of experimental determinations carried out using sophisticated investigation techniques.

## 6 EXPERIMENTAL INVESTIGATIONS OF PLATES

An advanced measuring system for the analysis of vibrating structures is the Laser Scanning Vibration Interferometer (LASVI). Compared with conventional techniques using electromechanical sensors, the mass and eigenfrequencies of which have negative influence on the accuracy of the measurements, the Laser Scanning method allows a contactless and highly exact determination of vibrational quantities and conditions.

The basic principle of the Laser Scanning method is the Laser Interferometry. As a laser source an HeNe-Laser is used, which emits coherent light of a wavelength of $\lambda = 0.316$ $\mu$m. This laser beam is splitted by a prism $S_1$ into two different parts, the measuring and the reference beam (Fig. 5).

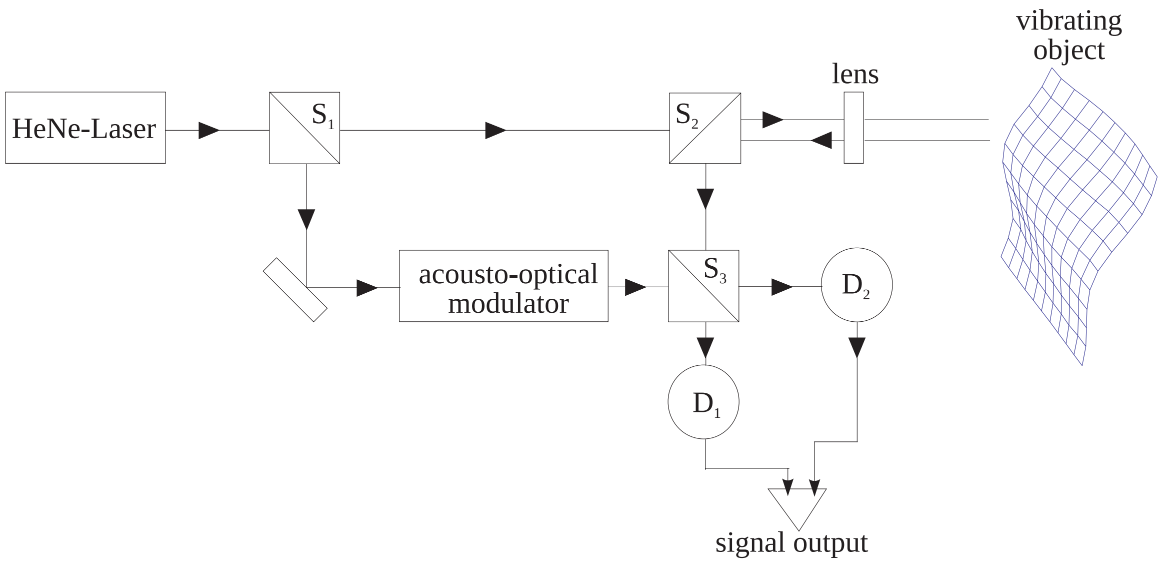


Fig. 5: Working principle of the LASVI

The measuring beam passes another prism $S_2$ and is then focused on the scanned object by a lens, where it is finally reflected. One part of the reflected light is absorbed by the lens and directed through prism $S_2$ to prism $S_3$, where the measuring and the reference beam interfere. With the help of the detectors $D_1$ and $D_2$ the intensity modulation of the light, caused by the frequency shift between the interfering light beams, is transfered into an electrical signal.

The vibration of the investigated object causes a frequency shift (Doppler effect) of the reflected light, which is proportional to the surface velocity satisfying the quation

$$f_0 = \frac{2\bar{v}}{\lambda}.$$

Having e. g. a velocity of the vibrating object of $\bar{v} = 1$ m/s, the resulting Doppler frequency shift is 3.17 MHz.

The identification of the eigenmodes is done by a scanning system, which allows the vibrational modes to be detected on different places of the surface without any change of the measuring head. In order to compare the analytical results with the eigenmodes measured by the LASVI, the 7th eigenmode of a unidirectionally (UD-) fibre reinforced composite plate (EP/CF, $240 \times 125 \times 1,25$ mm) is shown in Fig. 6.

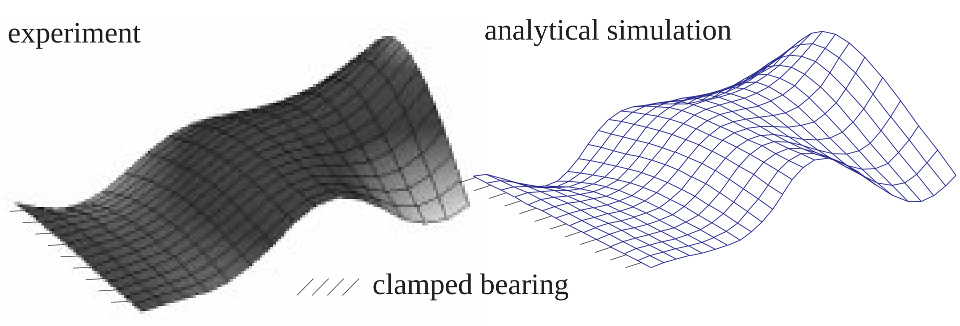


Fig. 6: 7th eigenmode of UD composite plate clamped at one edge

Even for eigenmodes of higher order an excellent agreement of experiment and theory has been found, what proves the high accuracy and practical orientation of the developed analytical calculation method. The calculated eigenfrequencies are also supported by the experiments; the maximum difference is below 7 % (Tab. 2).

Tab. 2: Analytical and experimental results for the eigenfrequencies

| **UD composite plate** (EP/CF, $240 \times 125 \times 1.25$ mm) | | | | |
|---|---|---|---|---|
| Mode | Eigenfrequency [Hz] | | | |
| | $\theta = 45°$ | | $\theta = 60°$ | |
| | analytical | experimental | analytical | experimental |
| 4 | 151.9 | 161.6 | 136.9 | 144.4 |
| 5 | 203.3 | 217.3 | 159.5 | 169.4 |
| 6 | 277.4 | 290.6 | 256.6 | 270.0 |
| 7 | 360.8 | 373.8 | 297.3 | 318.3 |
| 8 | 413.1 | 442.4 | 399.4 | 423.1 |
| 9 | 460.5 | 479.6 | 463.1 | 495.6 |
| 10 | 556.0 | 576.5 | 543.9 | 573.8 |
| 11 | 641.4 | 663.1 | 560.7 | 593.8 |

## 7 OPTIMIZATION

Fibre-reinforced composites give the possibility to specially design the material behaviour. Important parameters are fibre volume content, fibre angle, layer thickness und layer composition. Especially this large number of design variables as compared to conventional isotropic materials allows a great variety of additional design possibilities of the composite. In order to achieve shorter development times and simultaneously increase the efficiency of technical composite design, a successful design solution for such a complex property profile can only be achieved with the help of computational methods [6].

As an example the structural-dynamic optimization of an anisotropic composite plate is presented in the following:

### EP/CF-PLATE WITH RESTRICTIONS

For practical thin layered structures, besides the damping and eigenfrequency behaviour, also a sufficient stability against operational loads caused by boundary forces has to be realized. The damping of a symmetric three-layer composite ($125 \times 125 \times 1.5$ mm) with a thickness of 0.5 mm for each layer is optimized here with the help of a restriction oriented transformation using specific calculated fibre angles $\theta_1$ and $\theta_2$. Therefore the eigenfrequencies and critical buckling loads are restricted. The optimum solutions are then presented in the form of a so-called functional-sufficient surface in the three dimensional parametric space (Fig. 7). This optimization-method gives the possibility to clearly show the mutual influence of the different target function parameters.

It has been shown, that for high buckling loads ($N_x > 2$ N/mm) the choosen eigenfrequencies have no restrictive effect because the buckling restriction represents the stronger demand. Only for lower buckling loads, a restrictive effect starting with higher frequencies can be observed.

Some of the main aspects for a structural-dynamic design of anisotropic composite structures are layed out here by means of an optimization using the special technique of target consideration. The applied structural models are intentionally kept simple with respect to the optimization strategy and number of independent variables, in order to clearly point out the dominating influence of each layer angle on the structural dynamic behaviour.

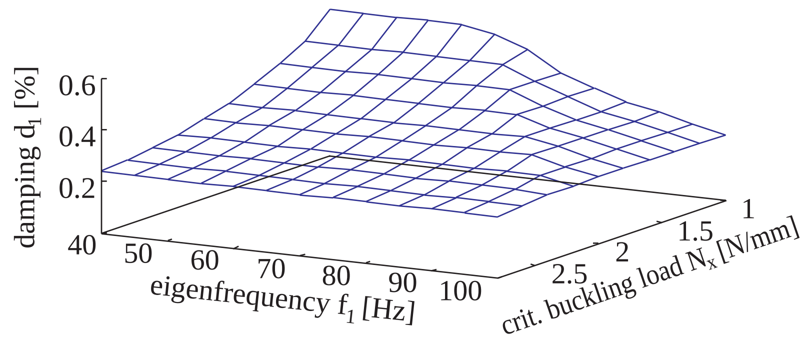


Fig. 7: Functional-sufficient surface of a EP/CF-composite plate

## 8 CONCLUSIONS

With simulation and optimization strategies of the damping behaviour of anisotropic fibre-reinforced composites new application fields for dynamically highly loaded lightweight structures can be accessed. The experimental and theoretical examinations presented here show clearly, that the design possibilities of the material damping of fibre-reinforced composites open up a large variety of options for the specific control of the vibration behaviour of dynamically highly loaded structures.

Finally it has to be noticed, that the practical optimization of multilayered fibre-reinforced composites concerning stiffness, strength and damping can only be done sufficiently using powerful optimization procedures, because of the large quantity of constructive degrees of freedom caused by the different layer parameters.

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