INVESTIGATION ON HOMOTHETIC FAILURE ENVELOPES IN THE LAYER-BASED FATIGUE ANALYSIS OF CFRP

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ABSTRACT

Since the need for efficient lightweight constructions rises strongly in various sectors, there is a high demand in covering in-depth computation and design capabilities concerning the failure of composite materials. Precisely because most of the parts are used in applications exposed to cyclic loading, the fatigue of composite materials is a central focus of research. The present study mainly focuses on the fatigue lifetime estimation of carbon fibre reinforced plastics (CFRP) with the use of a residual strength-based ply-wise model. The aim of the study is to calculate fatigue life and residual strength of multi-directional laminates at any number of load cycles with the use of input data from uni-directional plies only. The approach is based on repetitive calculations of the current stress states within each ply and subsequent analysis of the material stressing effort. The present paper mainly deals with the design of a homothetic fatigue failure envelope under multiaxial stress states. One of the major problems within residual strength-based analyses is that a separated residual strength reduction of the in-plane strength (without a connection due to loads in the same failure mode) will lead to delusive failure envelopes in specific cases. Therefore, a model that considers the multilateral effect for the transverse and in-plane shear strength is presented on the following pages.

1 INTRODUCTION

The layer-based fatigue life simulation takes place on the lamina level (Meso-scale) within a representative volume element (RVE), which represents the laminate properties of a structural part or component (Macro-scale). As shown in Figure 1, the general approach is based on iterative progressive calculations of the in-plane stresses of each layer in a laminate and the subsequent analysis of the material condition.

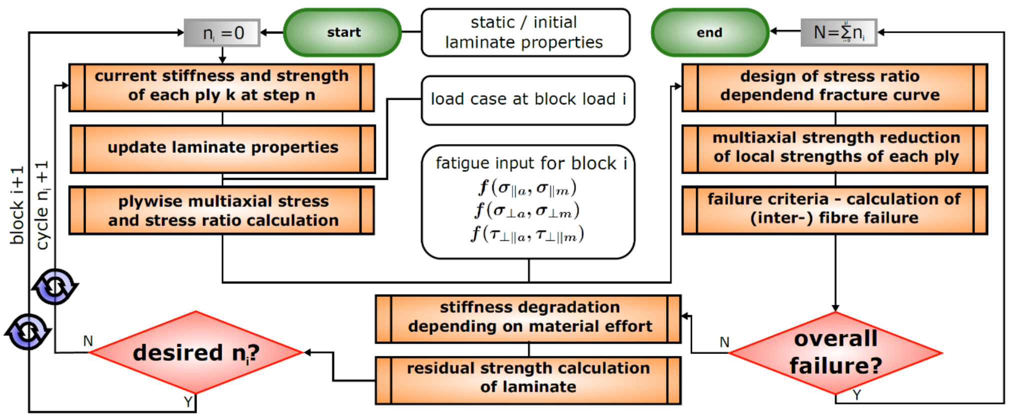

Figure 1: Shortened flowchart for the lifetime estimation of composites - LEoC [1]

As known from the classical laminate theory, in the first steps of the routine, the laminate properties are calculated using the current lamina properties. In the next step the three occurring in-plane stresses (the maximum longitudinal stress, the maximum transverse stress and the maximum in-plane shear stress within the current cycle) as well as the stress ratios for each of the three stresses are derived from the applied cyclic line forces and line moments. Regarding the stress situation, the lamina strengths are reduced according to the number of endured cycles at the actual stress amplitude level and the stiffness properties are reduced with the stress exposure factor derived by the applied failure criteria. Previous research focused on several mainly-UD laminates and balanced angle-plies under constant [2] and variable amplitude loads [1]. One of the main problems concerning a holistic approach for the lifetime and residual strength prediction within the layer-based fatigue analysis of composites, is the modelling of the failure envelope with the number of endured cycles for arbitrary load situations. An appropriate model for a downsized failure envelope is mandatory for the prediction of fatigue life within the model, in particular for multiaxial loads with variable amplitudes or varying subsequent stress states due to load direction changes. If the residual strength reduction of the transverse and the in-plane shear strengths are unrelated to each other's reduction, the resulting failure envelope for inter-fibre failure Mode A is delusive in most of the cases for multiaxial loads. Figure 2 shows the static failure envelope for Puck's failure criteria [3] in the left diagram and demonstrates the problem of unrelated strength reduction within a schematic illustration for in-plane shear dominated and transverse tension dominated loads in the right diagram.

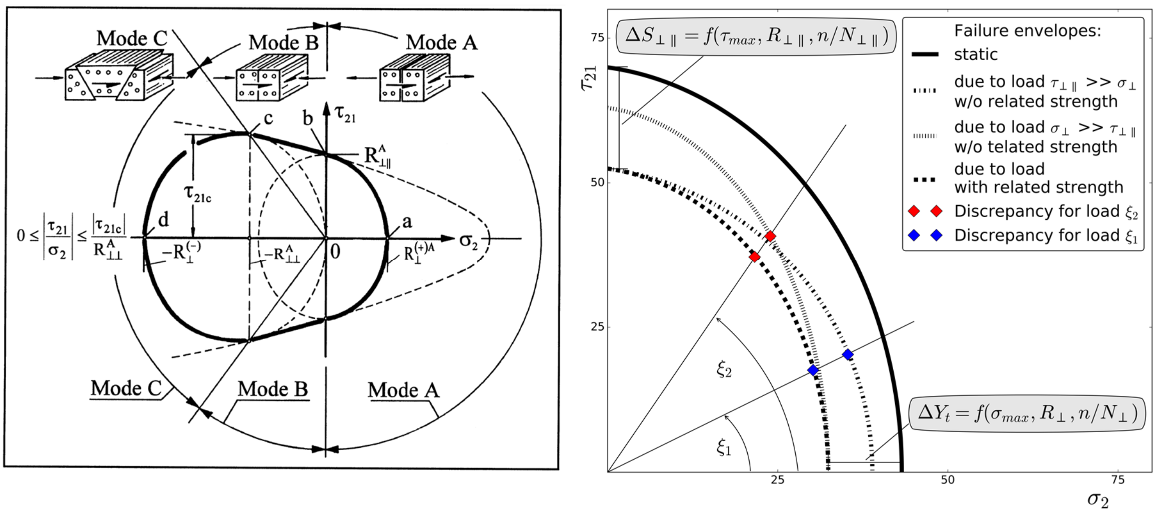

Figure 2: $\boldsymbol{\sigma_2}, \boldsymbol{\tau_{21}}$ - failure envelope (also referred to as $\boldsymbol{\sigma_\perp}, \boldsymbol{\tau_{\perp\parallel}}$ - envelope) for Puck's failure criterion [3] (left) and a schematic envelope with related and unrelated strength reduction in Mode A (right)

2 MODEL

Within the flowchart in Figure 1, the present paper mainly deals with the design of the stress ratio depending fracture envelope and the multiaxial strength reduction of the local strengths of each ply. To take into account multiaxial loads on laminates and to focus on off-axis stresses, a model is presented, which calculates the residual strength based on the biaxiality ratio of the in-plane shear and transverse stress. Gude et al. have already presented the idea of using the SN-curves for in-plane shear and transverse tension/compression for the shrinkage of the inter-fibre failure envelope and compared it to experimental data in [2]. In the present paper, the basic idea of a reduced failure envelope based on the SN-curve parameter is considered for implementation in the layer-based fatigue model for the multiaxial strength reduction. Therefore, a model for a homothetic downsized failure envelope for residual strength-based implementations is developed in the present study. The homothetic downsizing means, that for first analyses the inclines of the static failure envelope are held constant with the number of endured cycles within the fatigue simulations. The model is based on calculating the maximum number of cycles for a specific biaxiality ratio λ with the use of an appropriate failure criteria and the respective SN-curves to consider arbitrary stress ratios. The biaxiality angle ξ, as shown in the right diagram of Figure 2, is here defined as the arctangent of the biaxiality ratio with:

$$\xi = tan^{-1}\left(\frac{\tau_{\perp\parallel,max}}{\sigma_{\perp,max}}\right) = tan^{-1}(\lambda) \tag{1}$$

The homothetic method is presented here, for the coupling of the in-plane shear and transverse tension strength reduction for residual strength based fatigue life models. The model is based on the equations for inter-fibre failure of Puck's failure, which lead to Puck's failure envelope as shown before in Figure 2. In the following the method is shown for inter-fibre failure Mode A:

$$f_{e,A} = \sqrt{\frac{\tau_{\perp\parallel,max}^2}{S_{\perp\parallel}^2(n)} + \left(1 - p_{\perp\parallel}^{+}\frac{Y_t(n)}{S_{\perp\parallel}(n)}\right)^2 \left(\frac{\tau_{\perp\parallel,max}}{Y_t(n)\cdot tan\,\xi}\right)^2} + p_{\perp\parallel}^{+}\frac{\tau_{\perp\parallel,max}}{S_{\perp\parallel}(n)\cdot tan\,\xi} \tag{2}$$

With the maximum in-plane shear stress $\tau_{\perp\parallel,max}$ within the cyclic load, the residual in-plane shear strength $S_{\perp\parallel}(n)$, the residual transverse tension strength $Y_t(n)$ and the inclination $p_{\perp\parallel}^{+}$ of the $\sigma_{\perp}, \tau_{\perp\parallel}$ - failure envelope at $\sigma_{\perp} = 0$ for the range $\sigma_{\perp} > 0$. The inclination is derived from a parameter fit of static experimental data at $\xi = 0°, 60°$ and $90°$. It is important to notice that the lowercase n always stands for the number of cycles in progressive analyses such as residual strength calculations, while the capital letter N is used for the maximum allowed number of cycles in SN-curves. As shortly mentioned before, the homothetic downsizing means that the static inclines of the failure envelope for Mode A and B are held constant within the whole fatigue life simulations as follows:

$$p_{\perp\parallel}^{+} = const., \qquad p_{\perp\parallel}^{-} = const., \qquad p_{\perp\perp}^{-}(n) = \frac{p_{\perp\parallel}^{-}}{S_{\perp\parallel}(n)}\cdot R_{\perp\perp}^{A} \neq const. \tag{3}$$

In the rightmost equation, $R_{\perp\perp}^{A}$ is the fracture resistance of the action plane against its fracture due to a transverse/transverse shear stressing. Since the inclination $p_{\perp\perp}^{-}$ of the τ_{nt}, σ_n - failure curve at $\sigma_n = 0$ for the range $\sigma_{\perp} < 0$ depends on the residual in-plane shear strength, it will depend strongly on the chosen residual strength model and its parameter. Because of the assumption of a homothetic function, only two SN-curves are needed as experimental input data for the use of the model. These are on the one hand the pure transverse tension SN-curve with $\xi = 0°$ and on the other hand the pure in-plane shear SN-curve at $\xi = 90°$. When searching for the maximum number of cycles for a biaxial load with $\tau_{\perp\parallel,max}$ and a specific biaxiality ratio λ, the equation 2 for inter-fibre failure Modus A ($0° < \xi < 90°$) can be rearranged and solved with the quadratic formula as follows:

$$\tau_{\perp\parallel,A,max}(N_1,N_2) = -\frac{a_{\perp\parallel,max}(N_1,N_2)}{2} + \sqrt{\frac{a_{\perp\parallel,max}(N_1,N_2)^2}{4} - b_{\perp\parallel,max}(N_1,N_2)} \tag{4}$$

Where the values $a_{\perp\parallel,max}(N)$ and $b_{\perp\parallel,max}(N)$ are derived from the rearranged Puck's criterion for failure mode A (equation 2) with the following equations:

$$a_{\perp\parallel,max}(N) = \frac{2\,p_{\perp\parallel}^{+}\,S_{\perp\parallel}Y_t^2(N)\,tan\,\xi}{Y_t^2(N)\,tan^2\,\xi + \left(1 - p_{\perp\parallel}^{+}\,\frac{Y_t(N)}{S_{\perp\parallel}(N)}\right)^2 S_{\perp\parallel}^2(N) - p_{\perp\parallel}^{+}\,Y_t^2(N)} \tag{5}$$

$$b_{\perp\parallel,max}(N) = \frac{S_{\perp\parallel}^2\,Y_t^2(N)\,tan^2\,\xi}{Y_t^2(N)\,tan^2\,\xi + \left(1 - p_{\perp\parallel}^{+}\,\frac{Y_t(N)}{S_{\perp\parallel}(N)}\right)^2 S_{\perp\parallel}^2(N) - p_{\perp\parallel}^{+}\,Y_t^2(N)} \tag{6}$$

The final transverse and in-plane shear strengths at the last cycle ($Y_t(n = N)$ and $S_{\perp\parallel}(n = N)$) are calculated from the respective SN-curve at a specific number of cycles with:

$$Y_t(N) = 10^{\left(\frac{log\, C_{\perp} - log\, N}{k_{\perp}}\right)}, S_{\perp\parallel}(N) = 10^{\left(\frac{log\, C_{\perp}\parallel - log\, N}{k_{\perp\parallel}}\right)} \tag{7}$$

Where in the present paper $C_{\perp}$, $C_{\perp\parallel}$ and $k_{\perp}$, $k_{\perp\parallel}$ are found via log-log regression with the base 10 logarithm of the experimental data to the basquin formula [3]:

$$N = C \cdot \sigma_{max}^{-k} \tag{8}$$

Where C is the vertical intercept and k is the negative slope. In the last step the maximum number of cycles N_{ξ} for a specific multiaxial load can be determined as follows:

$$log\, N_{\xi,A} = log\, N_1 + log\, \tau_{\perp\parallel,A,max}(N_1) \cdot \frac{log\, N_1 - log\, N_2}{log\, \tau_{\perp\parallel,A,max}(N_2) - log\, \tau_{\perp\parallel,A,max}(N_1)} \tag{9}$$

For all of the above calculations in Mode A, the condition for validity is $0° < \xi < 90°$. For $\xi° = 0$ and $\xi = 90°$ the maximum number of cycles from the respective experimentally determined SN-curve is used as N_{ξ}. In the same way the equation for inter-fibre failure Mode B is rearranged and used to calculate the maximum number of cycles $N_{\xi,B}$. N_{ξ} is then used within the residual strength calculations presented in Figure 1 for the design of the stress ratio depended fracture curve.

The normalized residual strength model (NRSM) by Stojkovic et al. [6] is used for a reduction of each of the in-plane strength values on the ply-level. The NRSM is a two-parameter model, which is capable of describing an initial loss of strength at the beginning as well as a rapid drop of strength at the end of the fatigue life. Previous studies dealt with the use of the model within the layer-based fatigue analysis for both constant [3] and variable amplitude loads [4] for data from several glass fibre-reinforced plastics (GFRP) databases. The use of the NRSM model provided precise and mostly conservative results for fatigue life and residual strength data of various material lay-ups from different data sets. It seemed as if it improved the fatigue life analysis under variable amplitude loads, and yield less conservative results than the one-parameter and linear models. The non-linear residual strength model for iterative calculations is defined as

$$S_{r,n} = S_{r,n-1} - \{\Delta S_{r,n-1} - \Delta S_{r,n}\} \tag{10}$$

where $\Delta S_{r,n}$ and $\Delta S_{r,n-1}$ are the residual strengths at the current and the last load step respectively and are defined as

$$\Delta S_{r,n} = (S_{st} - \sigma_{max,i})\left(1 - \left(\frac{n_i}{N_{\xi,i}}\right)^{\alpha_{\xi}}\right)^{\beta_{\xi}} \tag{11}$$

and

$$\Delta S_{r,n-1} = (S_{st} - \sigma_{max,i})\left(1 - \left(\frac{n_{i-1}}{N_{\xi,i}}\right)^{\alpha_{\xi}}\right)^{\beta_{\xi}} \tag{12}$$

where n_i is the number of cycles endured, N_i is the maximum allowed number of cycles for the maximum stress $\sigma_{max,i}$ and the parameters α_{ξ} and β_{ξ} define the shape of the degradation function.

3 MATERIAL

Hollow tube specimens are used in order to apply any desired load combination of transverse and in-plane shear stresses within the experiments. The geometry of the specimen is shown in Figure 4. The CFRP laminates are made from carbon fibre rovings Tenax®-E HTS45 E23 with 12K and 800 tex and the matrix material is a hot curing epoxy system based on Araldite® LY556, Hardener HY 917 and Accelerator DY 070 with a mix ratio of 100:90:1 parts by weight. All of the specimen are made by filament winding and are cured in oven for 60 minutes at 90°C and post-cured for 135 minutes at 140°C. The fatigue tests are carried out on a servohydraulic test machine INSTRON 8802 with a nominal force of 100 kN and a nominal torque of 1 kNm for combined multiaxial loads. To guarantee a uniform distribution of force across the surface, the tubes are clamped over their entire circumference with a length of 50-60 mm from both ends using a system based on hydraulic collets. While testing, aluminium inlets with a length of 70 mm are inserted on both sides of the specimen to overcome a buckling load due to the clamping pressure.

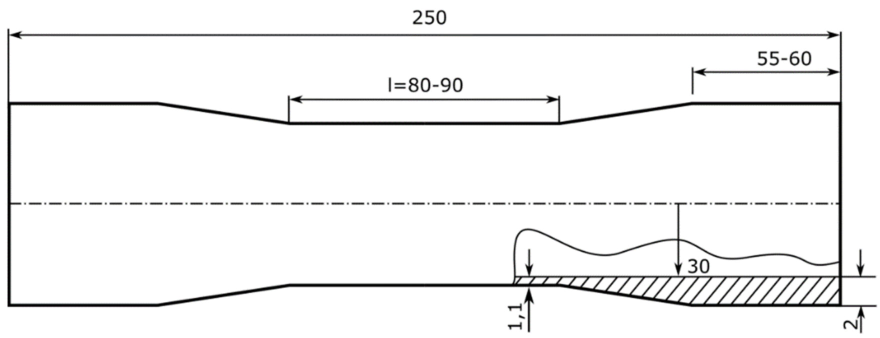

Figure 3: Geometry of CFRP-specimen for combined tension-torsion loads

As the objective of the research is to model the fatigue of multi-directional (MD) laminates under complex multiaxial loads with input data from uni-directional (UD) specimen only, solely the experimentally determined input data in Table 1 is considered for the simulations.

Stiffness properties		SN-curve parameter		Strength parameter	
$E_{\parallel}$	133.40 GPa	C_0	8.96E+26	$p_{\perp\parallel}^{+}$	0.163
$E_{\perp}$	8.19 GPa	k_0	16.2	$p_{\perp\parallel}^{-}$	0.319
$G_{\perp\parallel}$	4.08 GPa	C_{90}	6.67E+40	α_0	1.000
$\nu_{\perp\parallel}$	0.275	k_{90}	21.6	β_0	1.000
Y_t	43.38 MPa			α_{90}	0.320
$S_{\perp\parallel}$	69.96 MPa			β_{90}	0.170

Table 1: Experimental data from uni-directional radial-filament-wound hollow tubes used for the simulation of the multi-directional material

The stiffness and strength values in the first column of Table 1 are determined by quasi-static tests. The cyclic tests for $\tau_{\perp\parallel}$ are performed at a frequency of 3 Hz, for $\sigma_{\perp}$ at a frequency of 8 Hz. The multiaxial loaded UD materials with $\xi = 60°$ and $\xi = 30°$ are tested at a frequency of 5 Hz and the multiaxial loaded multi-directional material with $\xi = 48.8°$ is tested at a frequency of 3 Hz. The frequency is chosen in order to limit the temperature increase to a maximum of $\Delta T = 10\ °C$. The surface temperature of the specimen is constantly monitored by a pyrometer sensor with a temperature resolution of 0.1 °C and selectively monitored by a thermographic infrared camera. The static inclines are determined with quasi-static experimental data at $\xi = 0°$, $\xi = 60$ and $\xi = 90°$. A guideline for the determination of the static inclination parameter is presented by Puck et. Al [7]. The residual strength parameters α_0 , β_0, α_{90} and β_{90} are estimated with experimental data from quasi-static tests at $\xi = 0°$ and $\xi = 90°$ after a pure in-plane shear load with constant amplitude and stress ratio R=0.1 at 25000 $(n\ /\ \overline{N}_{\perp\parallel} = 0.25)$ and 75000 $(n\ /\ \overline{N}_{\perp\parallel} = 0.75)$ life cycles. The estimation of the residual strength parameters is shown in the right diagram of Figure 5. To be more conservative in the predictions, the linear residual strength reduction by Broutman and Sahu [8] with $\alpha_0 = \beta_0 = 1$ is chosen for the transverse tension strength, because of the higher standard deviation at the 75%-life tests. Since the standard deviation for the residual strength data for in-plane shear strength is very low, the non-linear NRSM model is chosen here. In the left diagram of Figure 5, the resultant fracture curve with the use of the homothetic failure envelope model for a pure pulsating in-plane shear load is illustrated and compared to experimental for $\xi = 60°$.

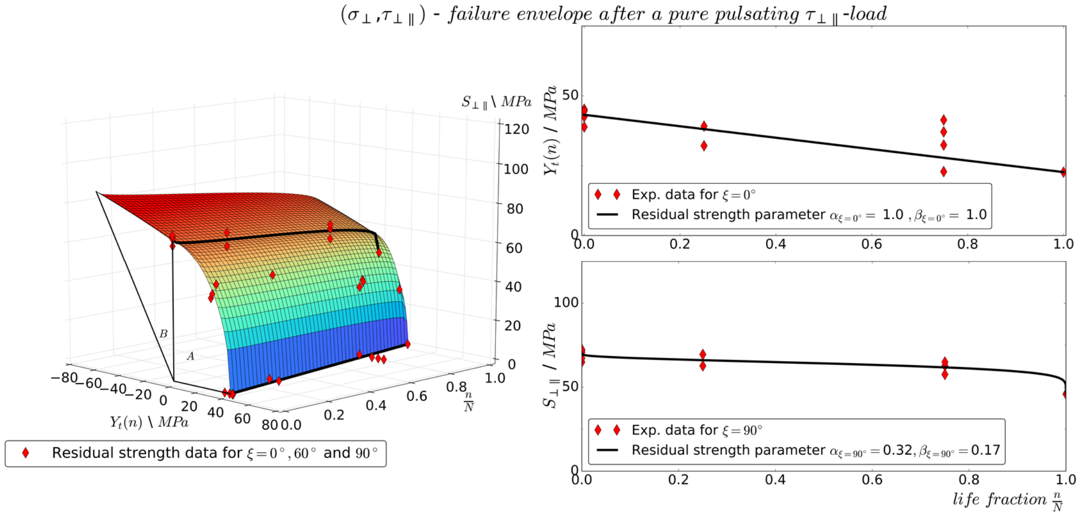

Figure 4: Homothetic failure envelope after a pure pulsating in-plane shear load with stress ratio R=0.1 (left) and the estimation of the residual strength parameters with residual strength data for $\boldsymbol{\tau}_{\perp\parallel}$ and $\boldsymbol{\sigma}_{\perp}$ after 25% and 75% of fatigue life (right).

4 RESULTS

In the following, the results for uniaxial loaded UD laminates, multiaxial loaded UD laminates and multiaxial loaded MD laminates are presented. In the left diagram of Figure 5, the simulation results for the uniaxial loaded UD specimen are compared to the experimental data for the verification of the implemented model.

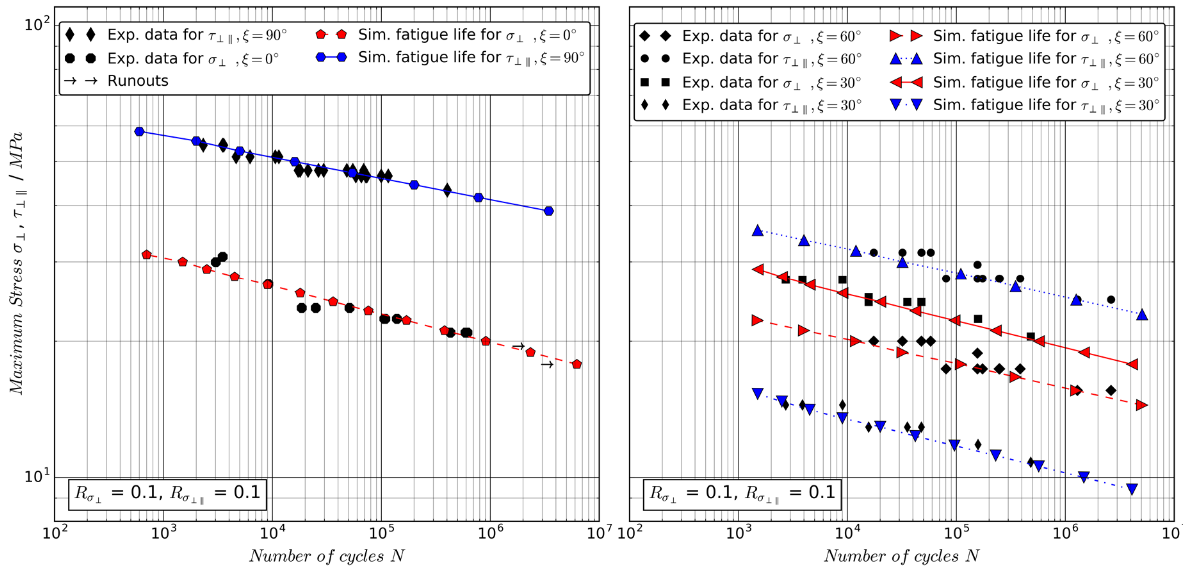

Figure 5: Verification for the average predicted life of input data $\xi = 0°$ and $\xi = 90°$ (Left) and Validation for $\xi = 30°$and $\xi = 60°$ (Right)

It is obvious, that the simulations can replicate the fatigue life data for a in-plane shear with $\xi = 90°$ and transverse stress with $\xi = 0°$. However, much more interesting are the results in the right diagram of Figure 5. As it is clearly shown in the diagram, the model seems to very accurately predict the fatigue life curves for the in-plane shear dominated multiaxial loads with a biaxiality angle of $\xi = 60°$ as well as the transverse tension dominated multiaxial loads with a biaxialty angle of $\xi = 30°$. Compared to the experimental data, the predictions tend to the conservative side.

In Figure 6, the fatigue life predictions for the multiaxial loaded balanced angle-ply laminates with a stacking sequence of $[\pm 70]_4$ are compared to first experimental data. Since the multiaxial loads have proportional stress ratios $R_{21} = R_2 = 0.1$, the local stress ratios in each ply are proportional stress ratios $R_{\perp\parallel} = R_{\perp} = 0.1$ as well. As a matter of fact, in the positive orientated ply (+70°) the stress ratio for the in-plane shear is $R_{\perp\parallel} = 10$, but assuming a symmetric Constant Life Diagram for $\tau_{\perp\parallel}$ the SN-curve parameter for $R_{\perp\parallel} = 10$ and $R_{\perp\parallel} = 0.1$ stay the same. Thus the stress ratios in both plies can be called proportional. In contrast to that, the local biaxiality ratios within each ply do vary from the global biaxiality ratios ($\xi_{gl.} \neq \xi_{+70} \neq \xi_{-70}$) for the MD laminate. The global biaxiality ratio of the multiaxial load in Figure 6 is $\xi_{gl.} = 48.8°$, whereas the local biaxiality ratios inside the laminate are $\xi_{+70} = 34.5°$ and $\xi_{-70} = 9.3°$. As can be seen in Figure 6, the model gives acceptable fatigue life predictions and especially yields predictions on the conservative side.

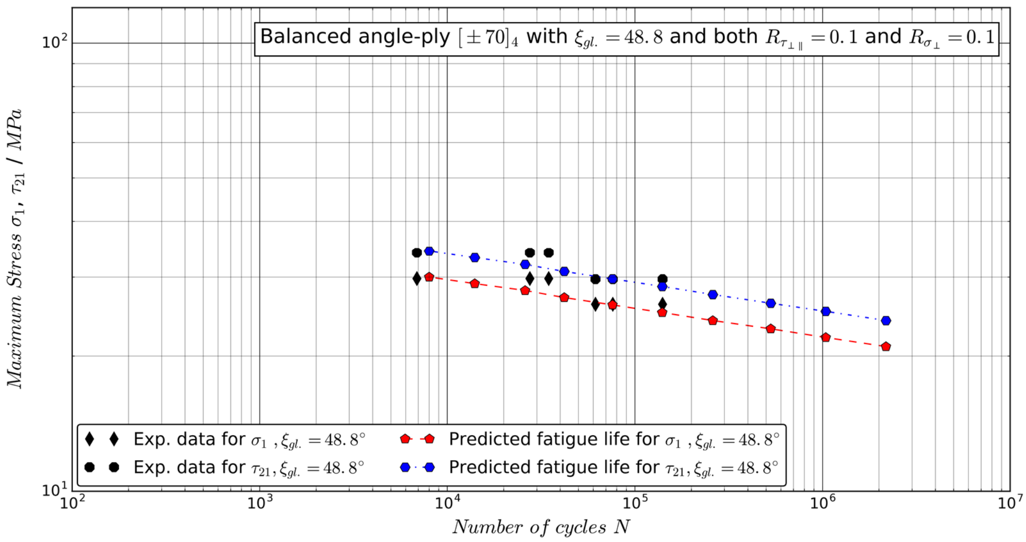

Figure 6: Balanced angle-ply tubes with stacking sequence $[\pm 70]_4$ under biaxial loads with a global biaxiality angle of $\xi_{gl.} = 48.8°$ and proportional stress ratios $R_{\perp\parallel} = R_{\perp} = 0.1$.

5 CONCLUSIONS

A model for a homothetic downsizing of the failure envelope is presented and used within the layer-based fatigue life estimation of composites. Residual strength formulations for the in-plane shear and transverse tension strengths are linked via Puck's failure criterion and used as the boundary conditions for the failure envelope modelling. The predictions of the model for multiaxial loaded uni- and multi-directional laminates with different biaxiality ratios show the potential of the model for further investigations on specific multiaxial loads and multi-directional lay-ups regarding load direction changes and multiaxial variable amplitudes.

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