Stress-strain response and fracture behaviour of plain weave ceramic matrix composites under uni-axial tension, compression or shear

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ABSTRACT

This paper presents a detailed study of stress-strain response and fracture behaviour of plain weave CMCs under uni-axial tension, compression or shear. A unit cell model of matrix and woven fibre tows is employed to minimise the computational cost. Three dimension structural solid elements have been used to generate the mesh in HyperMesh. The model is then exported to a commercial finite element package, Abaqus. A Python script has been developed to impose the periodic boundary conditions of the unit cell model. The translational symmetry transformation has been employed to derive the equations for the periodic boundary conditions. This makes it possible to use a single set of boundary conditions to apply uni-axial or a combination of arbitrary macroscopic stresses or strains simultaneously.

For the fibre tows, non-linear orthotropic material properties have been used to consider the effects of strain induced damage and fibre ruptures on the stress strain response. For the ceramic matrix, linear elastic properties and critical failure strains have been employed to define the Young's moduli and shear moduli. These material inputs are defined by a user defined subroutine (USDFLD) using Fortran in Abaqus. The non-linear material properties have been discretised into multi-linear stress strain curves. A set of field variables, which control the values of material inputs, are updated at each increment to model the material nonlinearity.

Three loading cases of uni-axial tension, compression or shear have been analysed, respectively. A comparison between the predicted results and experimental data for the uni-axial tension has been made, and a good correlation between them is found. Failure behaviour of the material under uni-axial compression or shear have also been predicted. The unit cell model developed by the current work is computational efficient and accurate to predict the uni-axial behaviour of textile composites.

1 INTRODUCTION

Ceramic matrix composites (CMCs), with superior material properties e.g. low density and good antioxidant, mechanical and thermal properties at high temperatures, have a wide range of applications in aerospace[1-2].

In early studies on CMCs, two-dimensional woven SiC/SiC composite materials were idealised by longitudinal fiber tows, transverse fiber tows, matrix and cracks. Based on this assumption, the unit cell model of a two-dimensional woven structure was constructed [3]. Later on, an exact set of functions for the interface layer was proposed to develop a three-dimensional finite element model of micromechanics unit cell by Kuhn[4]. In recent years, the equivalent elastic modulus of the plain weave sic CMCs was predicted by the establishment of unite cell model of fiber and fiber tows on microscopic and mesoscopic scales respectively[5]. Then, the mechanical properties of 2D C/C composites were evaluated on the laminate scale considering multiple types of material damage and the non-linear response resulting from non-elastic strain [6].

Although great efforts have been made to investigate mechanical behavior of CMCs in recent years, it is difficult to obtain the critical mechanical properties such as the material stiffness through mechanical test comprehensively, due to the high cost of material manufacturing and the immaturity of the test method. Therefore, the numerical simulation is of great significance to predict their mechanical behavior.

2 FORMULATION OF THE FINITE ELEMENT MODEL

For a periodic material, a unit cell can be used to form the whole macroscopic material by the translational symmetry transformation. A unit cell model of CMCs can minimise the computational cost. The precise selection of a unit cell facilitates the accurate analysis of CMCs properties. The determination of a unit cell mainly includes the determination of its overall shape, boundary, and dimensions of the fiber tows.

2.1 Finite model for unit cell

The selection of the unit cell is shown in Fig. 1. The unit cell contains a complete fiber tow and two half-fiber tows in the length and width direction respectively. The dimensions of the unit cell are obtained by observational data of the electron scanning microscope. SEM image is shown in Fig. 2. The geometry parameters of the unit cell obtained are shown in Table 1.

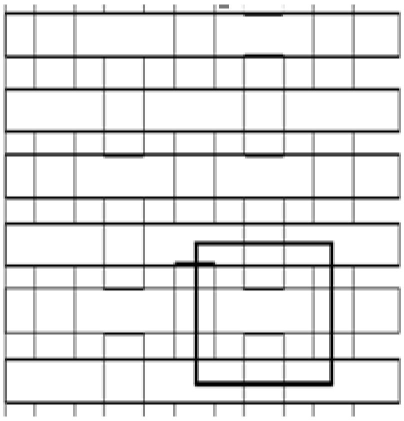


Figure 1: The selection of a unit cell

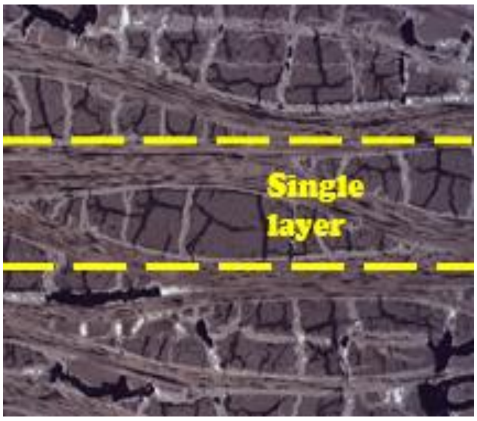


Figure 2: The SEM image of a plain weave CMC

Table1：The geometry parameters of unit cell

Length/mm	Width/mm	Thickness/mm	a/mm	b/mm
3.488	3.488	0.33	0.872	0.0825

The section of fiber tows are assumed to be elliptical with a major axis as 2a and a minor axis as 2b, shown in Fig. 3. Therefore, the dimension of the unit cell in the length, width and thickness direction are respectively 4a, 4a and 4b. The direction of weft tow is roughly composed of three elliptic curves shown by the dotted line in Fig. 3[7].

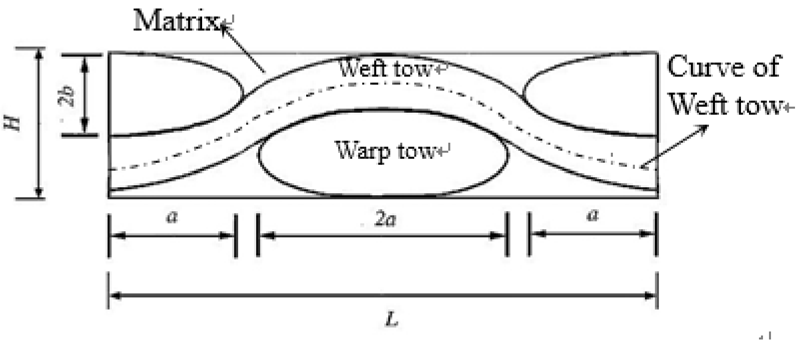


Figure 3: The section and direction of fiber tows

In this paper, the unit cell model of matrix and plain woven fiber tows is employed, as shown in Fig. 4. Three dimension structural solid elements have been used to generate the mesh in HyperMesh.

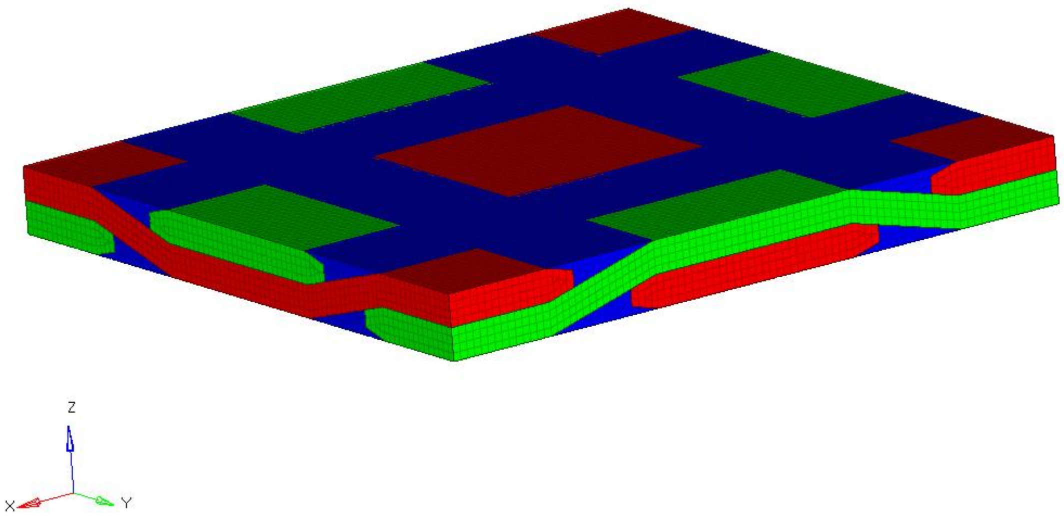


Figure 4: Finite element model of a unit cell for a plain weave ceramic matrix composite

2.2 Periodic boundary conditions

In this paper, the mechanical behaviour of a plain weave CMCs have been analysed by the unit cell model. Therefore, periodic boundary conditions have to be correctly applied, which influences the accuracy of the predicted mechanical properties of macroscopic structures. For a periodic continuous material, the boundary conditions of adjacent unit cells should satisfy continuity conditions of stress and displacement. The translational symmetry transformation has been employed to derive the equations for the periodic boundary conditions. This makes it possible to use a single set of boundary conditions to apply uni-axial or a combination of arbitrary macroscopic stresses or strains simultaneously. Under translational symmetry transformations, relative displacements are of the same nature as strains, so they are transformed in the same way as strains[8]. This results in the relation between the macroscopic strains and the relative displacements[9] as equation (1):

$$\begin{aligned} u'-u &= (x'-x)\varepsilon_x^0 + (y'-y)\gamma_{xy}^0 + (z'-z)\gamma_{xz}^0 \\ v'-v &= (y'-y)\varepsilon_y^0 + (z'-z)\gamma_{yz}^0 \\ w'-w &= (z'-z)\varepsilon_z^0 \end{aligned} \tag{1}$$

where x, y and z are the coordinates of point P, u, v and w are the displacements at point P, x', y' and z' are the coordinates of the image of point P, u', v' and w' are the displacements at the image of point P, and ε_x^0, ε_y^0, ε_z^0, γ_{xy}^0, γ_{xz}^0 and γ_{yz}^0 are the macroscopic strains[8-10].

The three rigid body translations are eliminated by constraining the displacements at any point as equation (2), and the rotations of the x-axis about the y and z-axes and that of the y-axis about the x-axis are constrained as equation[10] (3).

$$u = v = w = 0 \tag{2}$$

$$\frac{\partial w}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial w}{\partial y} = 0 \text{ at } x = y = z = 0 \tag{3}$$

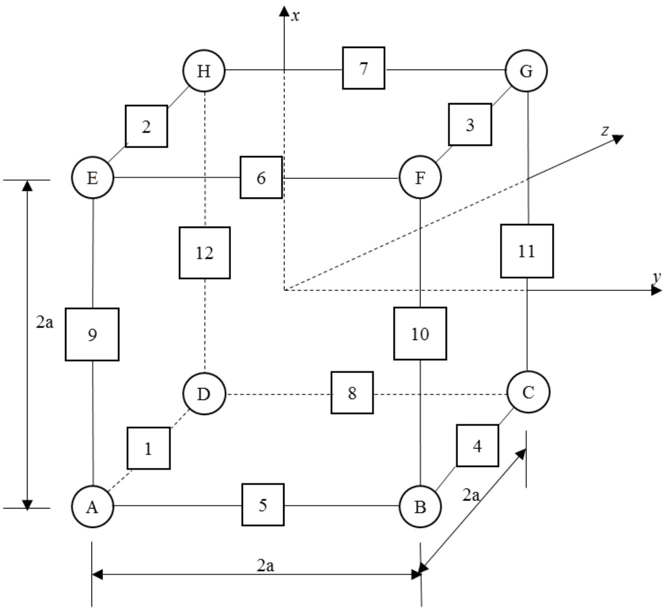


Figure 5： A cubic unit cell

As shown in Fig. 5, 2a represents the length of the cube unit cell. Equations (4)-(6) are applied to three pairs of faces but excluding the points on the edges shared by two adjacent faces.

$$(u_{x=\mathrm{a}} - u_{x=-\mathrm{a}})_{y,z} = 2\mathrm{a}\varepsilon_x^0$$
$$(\mathrm{v}_{x=\mathrm{a}} - v_{x=-\mathrm{a}})_{y,z} = 0$$
$$(\mathrm{w}_{x=\mathrm{a}} - w_{x=-\mathrm{a}})_{y,z} = 0$$

$$\text{abbreviated as } U_M - U_N = F_{MN} \tag{4}$$

$$u_{\mathrm{y=a}} - u_{y=-\mathrm{a}})_{\mathrm{x,z}} = 2\mathrm{a}\gamma_{xy}^0$$
$$(\mathrm{v}_{y=\mathrm{a}} - v_{y=-\mathrm{a}})_{\mathrm{x,z}} = 2\mathrm{a}\varepsilon_y^0$$
$$(\mathrm{w}_{y=\mathrm{a}} - w_{y=-\mathrm{a}})_{\mathrm{x,z}} = 0$$

$$\text{abbreviated as } U_P - U_Q = F_{PQ} \tag{5}$$

$$(u_{z=\mathrm{a}} - u_{z=-\mathrm{a}})_{\mathrm{x,y}} = 2\mathrm{a}\gamma_{xz}^{0}$$
$$(\mathrm{v}_{z=\mathrm{a}} - v_{z=-\mathrm{a}})_{\mathrm{x,y}} = 2\mathrm{a}\gamma_{yz}^{0}$$
$$(\mathrm{w}_{z=\mathrm{a}} - w_{z=-\mathrm{a}})_{\mathrm{x,y}} = 2\mathrm{a}\varepsilon_{z}^{0}$$

abbreviated as $U_R - U_S = F_{RS}$ (6)

$$\begin{aligned} &U_2 - U_1 = F_{MN}, U_3 - U_1 = F_{MN} + F_{PQ}, U_4 - U_1 = F_{PQ} \\ &U_6 - U_5 = F_{MN}, U_7 - U_5 = F_{MN} + F_{RS}, U_8 - U_5 = F_{RS} \\ &U_{10} - U_9 = F_{PQ}, U_{11} - U_9 = F_{PQ} + F_{RS}, U_{12} - U_9 = F_{RS} \end{aligned} \tag{7}$$

$$\begin{aligned} &U_E - U_A = F_{MN}, U_F - U_A = F_{MN} + F_{PQ}, U_B - U_A = F_{PQ}, U_D - U_A = F_{RS} \\ &U_H - U_A = F_{MN} + F_{RS}, U_G - U_A = F_{MN} + F_{PQ} + F_{RS}, U_C - U_A = F_{PQ} + F_{RS} \end{aligned} \tag{8}$$

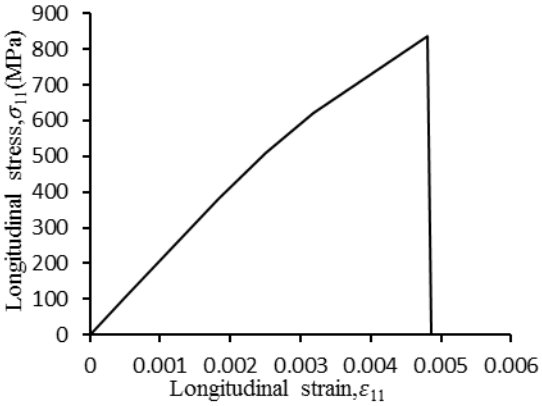
Equation (7) is applied to the edges but excluding the ends of the edges. Equation (8) is applied to the vertices

Applying periodic boundary conditions requires a one-to-one correspondence between the nodes on corresponding faces and edges of the unit cell. The model established in HyperMesh can generate periodic mesh and satisfy this requirement. The model is then exported to a commercial finite element package, Abaqus. A Python script has been developed to impose the periodic boundary conditions of the unit cell model, using Equations (4)-(8).

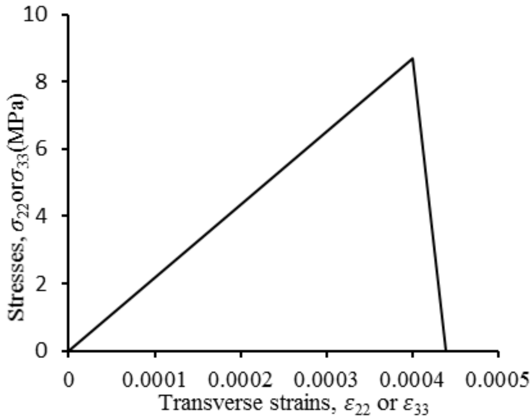
2.3 USDFLD implementation in Abaqus

The properties of fiber tows and matrix can be described by the user defined subroutine (USDFLD) using Fortran in Abaqus. For the fiber tows, non-linear orthotropic material properties have been used to consider the effects of strain induced damage and fibre ruptures on the stress strain response[11-13]. For the ceramic matrix, linear elastic properties and critical failure strains have been employed to define the Young's moduli and shear moduli. These material inputs are defined by USDFLD. The non-linear material properties have been discretised into multi-linear stress strain curves. A set of field variables, which control the values of material inputs, are updated at each increment to model the material nonlinearity.

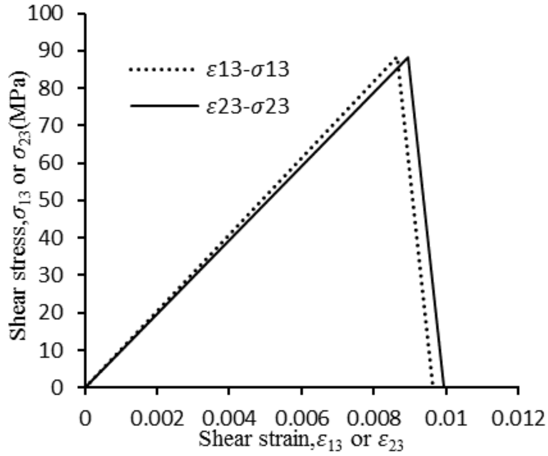
The fiber tows are assumed to be an orthotropic material and the matrix is assumed to be an isotropic material. Therefore, six field variables and six state variables are used to control the material inputs and monitor the damage state of the fiber tows. The local coordinate systems are established for the fiber tows in Abaqus. One field variable and one state variable are used to control the material inputs and monitor the damage state of matrix. A plain weave ceramic matrix composite have been modeled. The stress-strain curves of a uni-directional tow[11], shown in Fig. 6 are adopted as material input here by USDFLD. As shown in Fig. 6, longitudinal stress–strain curve has some nonlinearity, and it has been discretised into multi-linear curves. Other curves are considered as linear elastic.



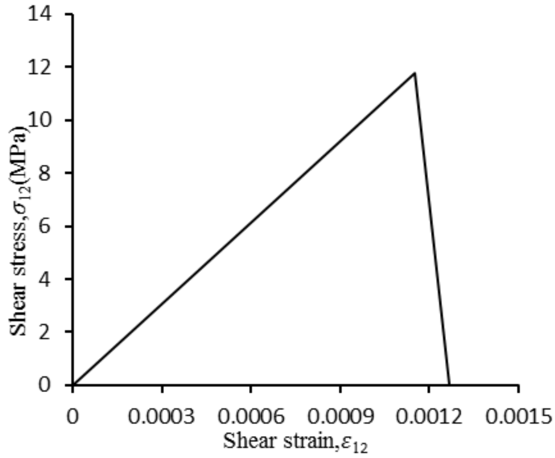
(a) Longitudinal stress–strain curve



(b) Transverse stress–strain curve



(c) Shear stress–strain curve



(d) Shear stress–strain curve

Figure 6: Material properties of CMC tow

2.4 Applying load

The macroscopic strains ε_x^0, ε_y^0, ε_z^0, γ_{xy}^0, γ_{xz}^0 and γ_{yz}^0 in section 2.2 can be considered as independent degrees of freedom to the system[9]. As periodic boundary condition has been imposed to the unit cell model, strains can be applied through the extra degrees of freedom as loads.

In this paper, three loading cases of uni-axial tension, compression or shear have been imposed to the unit cell model, respectively.

3 RESULTS

Fig.7 shows a comparison between the predicted tensile stress-strain response and the experimental data for a plain weave ceramic matrix composite. Fig. 8 plots the strain contour of the unit cell model under compression. Fig. 9 illustrates the strain contour of the unit cell under shear.

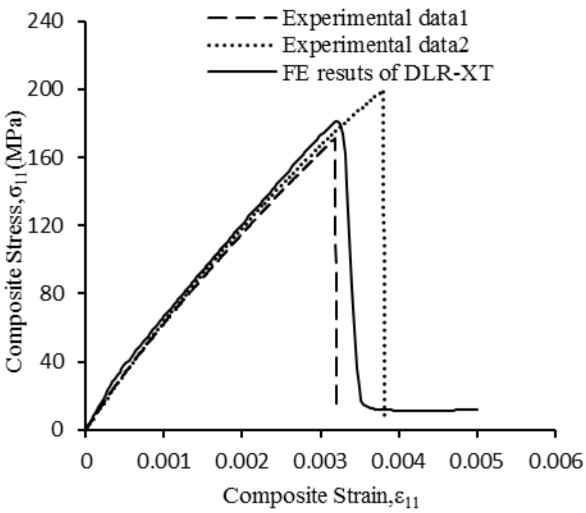


Figure 7: Comparison of predicted tensile stress-strain response with experimental data for a plain weave ceramic matrix composite

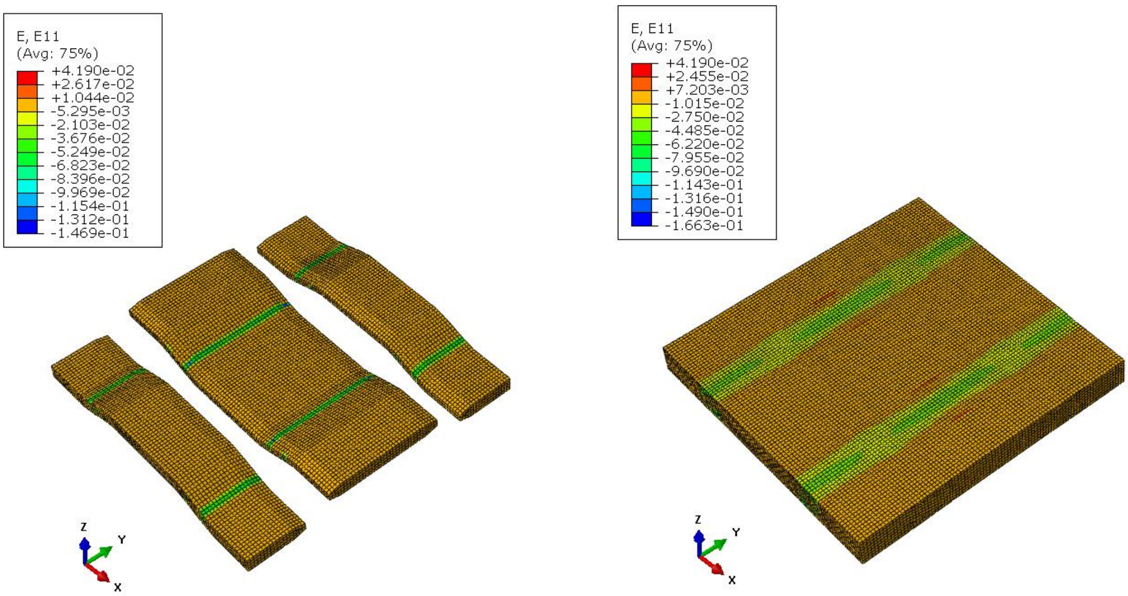


Figure 8: Strain contour of the unit cell model under compression

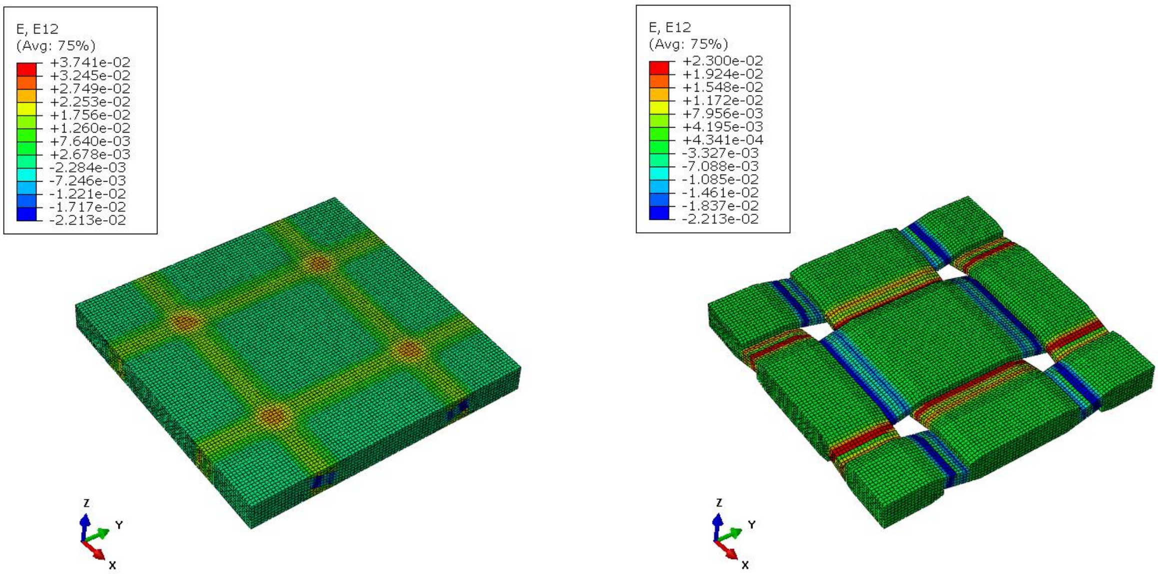


Figure 9: Strain contour of the unit cell model under the in-plane shear

4 CONCLUSIONS

This paper presents a detailed study of stress-strain response and fracture behavior of plain weave CMCs under uni-axial tension, compression or shear. The main conclusions are as follows.

(1) The prediction has a good correlation with the test data in the case of uni-axial tension.
(2) The unit cell model can be used to predict the stress-strain response of plain weave CMCs.
(3) The periodic boundary conditions has been correctly set up.
(4) The maximum strains occur in the vicinity of inclined tows.

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