VISCO-ELASTIC BENDING OF SANDWICH BEAM WITH LIGHT CORE

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Summary

The formulation and analysis for visco-elastic bending of sandwich beam with light core will be described in the paper.

We will work with integral equations of Volterra 2nd type in using of special properties of resolvent operators. Applications are focused on vertical displacement and stress as the functions of time.

Besides the space- operations the time- operations are occured in the analysis. Then the both we can separate each other and at first to handle with the time- operators as numbers.

1. Formulation

We consider, that the beam of symmetrical structure and light core with thickness 2s is subjected to vertical loading $q(x,t)$ described by relation

$$q(x,t) \;=\; g(x)\;\Phi\,(t)$$

where $\Phi\,(t)$ is Heaviside function. Thickness of the beam is $2\,h$ and the relation $s \;=\; h - r$ is valid, where r denotes thickness of the external layer.

In case of light core the value of normal stress σ_x is very small in comparison with external layer, so that we can consider $\sigma_x \;=\; 0$. The shear stress τ is of main importance. We can define

$$\tau \;=\; \varphi_1(x,t)\,, \qquad \sigma_z \;=\; \varphi_2(x,t) \;-\; z\,\varphi_{1,x}$$

These relations are satisfying the equilibrium in direction z.

Physical equations for viscoelastic core can be described in the form

$$\varepsilon_z \;=\; \frac{C_1}{E_c}\left(-\;z\;\varphi_{1,x}\right), \qquad \gamma \;=\; \frac{C_2}{G_c}\;\varphi_1 \qquad ,$$

where besides the modules of elasticity the integral time operators C_1 , C_2 are occured. By integration we obtain stepwise

$$u_z \;=\; \varphi_3(x,t) \;-\; \frac{z^2}{2\,E_c}\;C_1\;\varphi_{1,x}$$

$$u_x \;=\; \varphi_4(x,t) \;+\; z\left(\frac{C_2}{G_c}\,\varphi_1 \;-\; \varphi_{3,x}\right) \;+\; \frac{z^3}{6}\,\frac{C_1}{E_c}\,\varphi_{1,xx}$$

with integration functions φ_3 , φ_4 ($\varphi_4(x,t) = 0$ can be considered).

For relatively thin external layers the transverse incompressibility can be considered and then

$$u_z = w(x,t)$$

According to expression of displacement components in core we define for external layers following relations

$$u_z = \varphi_3(x,t) - \frac{s^2}{2E_c} C_1 \varphi_{1,x}$$

$$u_x = z\left(\frac{C_2}{G_c}\varphi_1 - \varphi_{3,x}\right) + \frac{z^3}{6}\frac{C_1}{E_c}\varphi_{1,xx}$$

From which appears

$$\varphi_3 = w + \frac{s^2}{2E_c} C_1 \varphi_{1,x}$$

Deciding component of stress σ_x in external layer is given by the relation $\sigma_x = E S \varepsilon_x$, i.e.

$$\sigma_x = E S \left[\pm s \left(\frac{C_2}{G_c}\varphi_{1,x} - \varphi_{3,xx} \right) \mp \frac{s^3}{6}\frac{C_1}{E_c}\varphi_{1,xxx} - z\, w_{,xx} \right] ,$$

where S is again an integral operator.

Further is necessary to define stress component τ satisfying the conditions on surface and on cross-over of layers:

$$\tau = \frac{h \mp z}{r}\varphi_1 + \frac{1}{2}(z \mp s)(z \mp h) E S w_{,xxx}$$

Equilibrium in z direction is then satisfied at the integral form.

Basic equations are the equilibrium condition $\sigma_{x,x} + \tau_{,z} = 0$ in external layers

$$\frac{1}{E r}\varphi_1 + S\left(-\frac{s C_2}{G_c}\varphi_{1,xx} + \frac{s^3 C_1}{3E_c}\varphi_{1,xxxx} + s_0 w_{,xxx} \right) = 0$$

and the condition of integral character along to beam height is

$$\int_{-h}^{h} \tau_{,x}\, dz + q = 0, \quad \text{ie.}$$

$$2 s_0 \varphi_{1,x} - \frac{r^3}{6} E S w_{,xxxx} + q = 0 \quad , \qquad \text{where } s_0 = \frac{h+s}{2} .$$

Boundary conditions for the hinged support on both sides are of the form $\varphi_{1,x} = 0$, $\varphi_{1,xxx} = 0$, $w = 0$, $w_{,xx} = 0$

A favourable formulation of the problem can be obtained on base of certain formal differential operations by adopting a special function $\omega(x,t)$ and giving

$$\varphi_1 = s_0 S \omega_{,xxx} , \qquad w = -\frac{\omega}{E r} + \frac{s S C_2}{G_c}\omega_{,xx} - \frac{s^3 S C_1}{3E_c}\omega_{,xxxx}$$

The first basic equation is satisfied identically and the second one is transformed to form

$$L\, \omega = -q$$

where

$$L = \frac{1}{6} S \frac{d^4}{dx^4} \left[12 s_0^2 + r^2 - \frac{r^3 s E S}{G_c} \frac{d^2}{dx^2} \left(C_2 - \frac{s^2 G_c C_1}{3 E_c} \frac{d^2}{dx^2} \right) \right]$$

The boundary conditions applicable to hinged support are then

$$\frac{d^{2k}}{dx^{2k}} \omega = 0 \qquad k = 0,1,2,3$$

2. Solving of the problem

At solution of the problem we apply an expansion of the functions $g(x)$ and $\omega(x,t)$ into Fourier series with coefficients g_m, ω_m. By substitution into the basic equation and after some arrangements we obtain

$$\omega_m = \frac{6 \frac{l^4}{m^4 \pi^4} g_m}{12 s_0^2 + r^2 + \frac{r^3 s}{G_c} E S \frac{m^2 \pi^2}{l^2} \left(C_2 + \frac{s^2}{3} \frac{G_c}{E_c} \frac{m^2 \pi^2}{l^2} C_1 \right)} \frac{1}{S \Phi(t)}$$

As an example the analysis of the beam in a middle ($x = \frac{l}{2}$) will be given. After introducing of non-dimensional quantities

$$\lambda = \frac{r}{h}, \quad \rho = \frac{h}{l}, \quad \kappa = \frac{G_c}{G}$$

$$\alpha_m = m^2 \pi^2 \frac{2(1+\mu)}{\kappa} \lambda (1-\lambda) \rho^2, \ \beta_m = \frac{m^2 \pi^2}{6(1+\mu_c)} (1-\lambda)^2 \rho^2, \quad \gamma = \frac{\lambda^2}{12 \left(1 - \frac{\lambda}{2}\right)^2}$$

we get eg. for uniform loading with intensity g , ie.

$$g(x) = \frac{4}{\pi} g \sum_m \frac{1}{m} \sin \frac{m \pi x}{l} \qquad (m = 1, 3, 5\ldots)$$

$$w = -\frac{24 \, l g \gamma}{\pi^5 E \lambda^3 \rho^3} \sum_m \frac{1}{m^5} \frac{1 + \alpha_m S (C_2 + \beta_m C_1)}{1 + \gamma \left[1 + \alpha_m S (C_2 + \beta_m C_1)\right]} \frac{\Phi(t)}{S \Phi(t)} \sin \frac{m\pi}{2}$$

$$\sigma_x = \pm \frac{24 \, g \, \gamma}{\pi^3 \lambda^3 \rho^2} \sum_m \frac{1}{m^3} \frac{1 + \frac{\lambda}{2} \alpha_m S (C_2 + \beta_m C_1)}{1 + \gamma \left[1 + \alpha_m S (C_2 + \beta_m C_1)\right]} \frac{\Phi(t)}{S \Phi(t)} \sin \frac{m\pi}{2}$$

The integral operators can be determined from rheological models (eg. Poynting- Thomson) for core C_1, C_2 and S for external layers. In this case the both vertical displacement w and stress σ_x above are time (viscoelastic) depending.

For product of operators applies eg.

$$S C = \int_\tau^t S (t-\alpha) \, C(\alpha-\tau) \, d\alpha = \int_\tau^t C (t-\alpha) \, S(\alpha-\tau) \, d\alpha \quad .$$

The calculation is simplified, when the exponential operators responding to mechanical models are used. For multiplication of operators applies

$$E^*(\alpha)\ E^*(\beta) \quad = \quad \frac{E^*(\alpha) - \ E^*(\beta)}{\alpha - \beta} \qquad \text{(kde} \quad E^*(\alpha)\,\phi \ = \ \int e^{\alpha(t-\tau)}\ \phi(\tau)\,d\,\tau \quad \text{apod.)}$$

from which follows the relation

$$\frac{1}{1+\kappa\,E^*(-\alpha)} \quad = \quad 1 - \ \kappa\ E^*(-\kappa - \alpha)$$

If we consider for the beam core Poynting-Thomsonův model (standard solid) characterized by relations $E_c - E_x / K_1$ and $G_c - \ G_x / K_2$ respectively, we obtain

$$C_1 \ = \ 1 + \frac{E_c}{K_1}\ E^*\left(-\frac{E_x}{K_1}\right) \qquad \text{resp.} \qquad C_2 \ = \ 1 + \frac{G_c}{K_2}\ E^*\left(-\frac{G_x}{K_2}\right)$$

from which

$$E_c\,\varepsilon_z \ = \ o_z \ + \frac{E_c}{K_1}\int_0^t e^{-\frac{E_x}{K_1}(t-\tau)}\ \sigma_z\ d\tau \quad , \qquad G_c\,\gamma \quad = \quad \tau \ + \frac{G_c}{K_2}\int_0^t e^{-\frac{G_x}{K_2}(t-\alpha)}\ \tau\ d\,\alpha$$

and vice versa

$$C_1^{-1} \ = \ 1 - \frac{E_c}{K_1}\ E^*\left(-\frac{E_c + E_x}{K_1}\right) \quad , \qquad C_2^{-1} \ = \ 1 - \frac{G_c}{K_2}\ E^*\left(-\frac{G_c + G_x}{K_2}\right)$$

from which

$$E_c^{-1}\,\sigma_z \ = \ \varepsilon_z - \frac{E_c}{K_1}\int_0^t e^{-\frac{E_c + E_x}{K_1}(t-\tau)}\ \varepsilon_z\,d\tau \quad , \qquad G_c^{-1}\tau \ = \ \gamma - \frac{G_c}{K_2}\int_0^t e^{-\frac{G_c + G_x}{K_2}(t-\alpha)}\ \gamma\ d\,\alpha$$

Further we define the integral operators S , S^{-1} for external layers. If we choose a model with the structural equation $E \ - \ (E \ / \ K)$ we get

$$S \ = \ 1 - \frac{E}{K}\ S^* \qquad \text{ie.} \qquad \sigma_x \ = \ E\,S\ \varepsilon_x \ = \ E\left(I - \frac{E}{K}\,S^*\right)\varepsilon_x \qquad\qquad s$$

with operator $S^* \ = \ E^*\left(-\frac{2E}{K}\right)$ and kernel $\left|S^*\right| \ = \ e^{-\frac{2E}{K}(t-\tau)}$.

Then it is valid

$$\frac{\sigma_x}{E} \ = \ \varepsilon_x - \frac{E}{K}\int_0^t e^{-\frac{2E}{K}(t-\tau)}\ \varepsilon_x\ d\tau \qquad \text{ie.} \quad S \ = \ I - \frac{E}{K}\ E^*\left(-\frac{2E}{K}\right) \ .$$

Inverted operator is $S^{-1} \ = \ I \ + \frac{E}{K}\ E^*\left(-\frac{E}{K}\right)$ with responding relation

$$E\,\varepsilon_x \ = \ \sigma_x \ + \frac{E}{K}\int_0^t e^{-\frac{E}{K}(t-\tau)}\ \sigma_x\ d\tau$$

By application of time operators to Heaviside function at time interval t considered we get

$$S\ \Phi(t) \quad = \quad \Phi(t) \ \text{-}\ \frac{E}{K}\int_0^t e^{-2\frac{E}{K}(t-\tau)}\Phi(\tau)\,d\,\tau \quad = \quad 1 - \frac{1}{2}(1 - e^{-2\frac{E}{K}t})$$

$$S^{-1}\,\Phi(t) \quad = \quad \Phi(t) + \frac{E}{K}\int_0^t e^{-\frac{E}{K}(t-\tau)}\Phi(\tau)\,d\,\tau \quad = \quad 1 + 1 - e^{-\frac{E}{K}t}$$

$$C_1 \Phi(t) \quad = \quad \Phi(t) + \frac{E_c}{K_1}\int_0^t e^{-\frac{E_x}{K_1}(t-\tau)}\Phi(\tau)\,d\tau \quad = \quad 1 + 1 - e^{-\frac{E_c}{K_1}t}$$

$$C_2 \Phi(t) \quad = \quad \Phi(t) + \frac{G_c}{K_2}\int_0^t e^{-\frac{G_x}{K_2}(t-\tau)}\Phi(\tau)\,d\tau \quad = \quad 1 + 1 - e^{-\frac{G_c}{K_2}t}$$

The parameters of integral operators can be determined from mechanical testing. In this case the both vertical displacement and w and stress σ_x are time depending.

3. Mechanical testing

Mechanical tests have been performed on a symmetrical sandwich beam with bearing layers from composite, reinforced by glass fibres and polyester matrix [2]. Further, the cube testing samples were loaded by constant load in a set-up developed and manufactured in Klokner Institute (Fig.1). Vertical displacements were measured by LVDTs and strains by strain gauges. Testing samples were loaded by 30 and 60 N and unloaded at least for 100 h. The tests show a good agreement with calculated values (Fig.2).

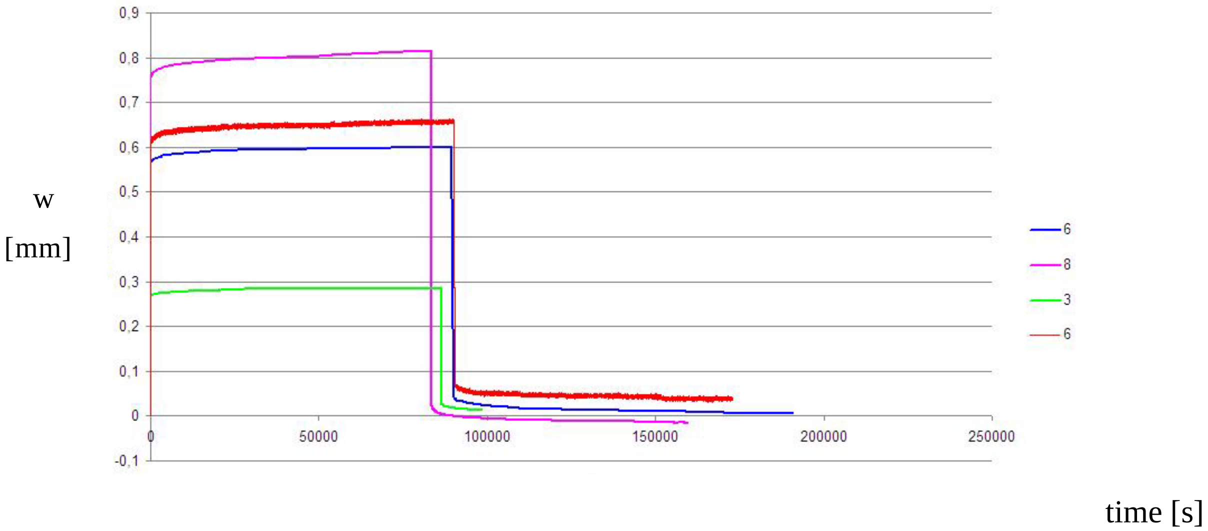

Fig.1 Experimental evaluating of creep for the symmetrical sandwich beam

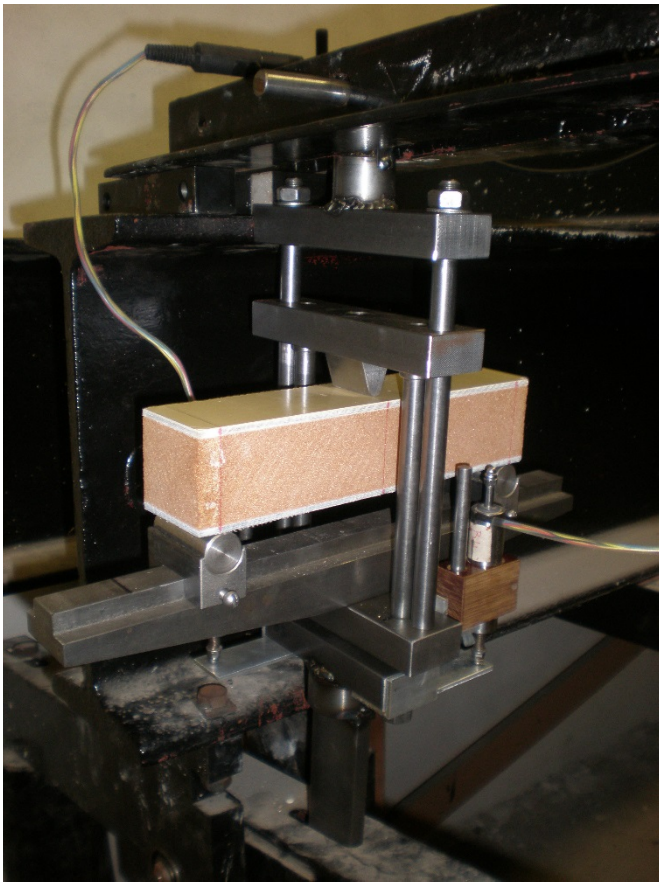

Fig.2 Experimental set-up for evaluating of creep of the symmetrical sandwich beam

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