A CRITICAL ASSESSMENT ON THE PREDICTABILITY OF 12 MICROMECHANICS MODELS FOR STIFFNESS AND STRENGTH OF UD COMPOSITES

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ABSTRACT

Any micromechanics model developed to predict the stiffness (elastic properties) of a composite is also applicable to the prediction of a composite strength, as long as the homogenized stresses in the matrix are converted into true values by virtue of its stress concentration factors (SCFs) obtained recently. In this paper, the predictability of 12 most well known micromechanics models for stiffnesses and strengths of unidirectional (UD) composites is assessed. The measured elastic and strength data of the 9 UD composites adopted in three world-wide failure exercises (WWFEs) are used as a benchmark in this assessment. It is shown that Bridging Model is among the most accurate in the prediction of both the stiffnesses and the strengths.

1 INTRODUCTION

Fiber reinforced composites, serving as the fourth class of structural materials in addition to metals, ceramics, and polymers, have been used in almost every branch of industry. Different from the latter three materials, the composites are anisotropic. Their mechanical properties are difficult or at least expensive to understand. Establishment of mathematical models to link the overall behaviors of the composites with their constituent structures and properties is an objective of micromechanics. It is expected that the effective properties of a composite can be predicted from its constituent information, and sample testing will be performed only at the final stage for validation of a "virtual" design.

So far,Numerous micromechanics models have been developed to predict elastic properties of the composites only from those of the constituent fiber and matrix materials. But very few of them have been applied to estimate failure and strength behaviors of the composites just based on the original constituent data with a reasonable accuracy[1]. Here, the original constituent properties stand for those measured independently using the monolithic material specimens. Very recently, we have found the homogenized stresses in the matrix of a composite determined by a micromechanics theory must be converted into true values before a failure assessment can be made if only the original property parameters are available [2-5]. The conversion is done by multiplying the homogenized quantities with the respective stress concentration factors (SCFs) of the matrix due to introduction of the fiber. Such an SCF cannot be defined following a classical approach, and the explicit formula of it has been derived elsewhere[2-5]. In this way, each micromechanics model is also applicable to the prediction of a composite failure and strength behaviour.

Then, there may arise a question. Which micromechanics theory will result in a better prediction for a composite strength? One purpose of this paper is to compare the predicability of 12 most well known micromechanical models for the stiffness (elastic property) and mainly strength of a UD composite, only using information of the fiber and matrix properties and the fiber volume fraction. The previous comparisons, e.g. Refs. [6-8], were made only for the stiffness predictions by different models. Seldom have been done for the strength predictions. The models considered in this paper are Bridging Model[9], Mori-Tanaka method[10], rule of mixture method, Chamis' model[11], modified rule of mixture method[12], Halpin-Tsai formulae[13], Hill-Hashin-Christensen-Lo model[14,15], self-consistent method[16], generalized self-consistent method[17], generalized method of cells(GMC)[18], finite volume method (FVM)[19], and finite element approach(FEA). The measured stiffness and strength data of all the 9 independent UD composites adopted in the hree WWFEs are used as benchmark to judge the accuracy of each model's predictions. An accuracy ranking is made based on the overall correlation errors of all he models' predictions with the experiments. It has been shown among the 12 models considered, Bridging Model exhibits overall the best accuracy both in stiffness and in strength predictions for the composites.

Another purpose of this paper is to explore what accuracy can be achievable in strength prediction by a current micromechanis model. To predict a composite stiffness, only elastic properties of the fiber and matrix together with the fiber volume fraction are required. In order to estimate a composite strength, one must know additionally ultimate strengths of the constituents. Moreover, the influence of other factors, if any, such as thermal residual stress, matrix plasticity, interface debonding, void content, etc. on the prediction of a strength is much more significant than on that of a stiffness. Any deviation in determining each of the constituent properties and the other factors will cause a correlation error between a predicted and measured composite data. In view of these, it is natural that the overall prediction accuracy for a strength be one half less than that for a stiffness of the same composite. We will show in the paper that such an accuracy is indeed achievable.

2 EVALUATION OF INTERNAL STRESSES

A composite is heterogeneous by nature. Stresses and strains should be defined upon averaged quantities with respect to its RVE (representative volume element) V' through

$$\sigma_i = (\int_{V'} \tilde{\sigma}_i dV) / V', \ \varepsilon_i = (\int_{V'} \tilde{\varepsilon}_i dV) / V' \tag{1}$$

where a stress or strain with ~ on head represents a point-wise quantity. Equations (1) represent a homogenization on the composite. Even though point-wise stresses and strains, $\tilde{\sigma}_i$ and $\tilde{\varepsilon}_i$, are obtainable by, e.g., an FEA, the homogenization of Eqs. (1) must be carried out before the effective properties of the composite can be determined. When only fiber and matrix are contained in V', he above integrations read

$$\{\sigma_i\} = V_f\{\sigma_i^f\} + V_m\{\sigma_i^m\} \tag{2.1}$$

$$\{\varepsilon_i\} = V_f\{\varepsilon_i^f\} + V_m\{\varepsilon_i^m\} \tag{2.2}$$

where V is a volume fraction with $V_f+V_m=1$. A super-/sub-script f or m refers to fiber or matrix, whereas a quantity without any suffix is related to the composite.

Suppose that there is a bridging tensor, $[A_{ij}]$, such that

$$\{\sigma_i^m\} = [A_{ij}]\{\sigma_j^f\}, \tag{3}$$

which together with Eq. (2.1) results in

$$\{\sigma_i^f\} = (V_f[I] + V_m[A_{ij}])^{-1}\{\sigma_j\} = [B_{ij}]\{\sigma_j\}, \tag{4}$$

$$\{\sigma_i^m\} = [A_{ij}][B_{ij}]\{\sigma_j\}. \tag{5}$$

From Eq. (2.2) together with constitutive relationships of the fiber, matrix, and the composite, the compliance tensor of the composite is given by[22]

$$[S_{ij}] = (V_f[S_{ij}^f] + V_m[S_{ij}^m][A_{ij}])(V_f[I] + V_m[A_{ij}])^{-1}. \tag{6}$$

On the other hand, any homogenization based micromechanics model corresponds to a bridging tensor, which can be used to determine the internal stresses of the fiber and matrix through Eqs. (4) and (5). In fact, from Eq. (6), the bridging tensor is given by

$$[A_{ij}] = V_f([S_{ij}] - [S_{ij}^m])^{-1}([S_{ij}^f] - [S_{ij}])/V_m. \tag{7}$$

Thus,determination of the internal stresses in the fiber and matrix of a composite is equivalent to that of the elastic properties of the same composite. The accuracy of a micromechanics model in evaluating the internal stresses can be assessed by comparing the model's prediction and the measurement on elastic properties of the composite.

3 ASSESSMENT ON STIFFNESS PREDICTIONS

Hinton et al. organized the three WWFEs to judge efficiency of the current strength theories for composites. A total number of 9 independent material systems were used. Mechanical properties of the fibers and matrix , fiber volume fractions and ultimate strengths of the 9 UD composites were provided[20-22], and are shown in Table 1. Measured effective properties and strength of the composites by the exercise organizers[20-22], used as a benchmark to assess the predictability of the12 models summarized before, are listed in Table 2. Predictions for the effective five elastic moduli of each of the 9 UD composites by the 12 models are made. The overall averaged errors by the 12 models and a ranking for the stiffness prediction accuracy are presented in Table 3. It is seen that the Bridging Model's prediction for the stiffnesses of the 9 composites is the most accurate, with an overall correlation error of 10.38%. The second highest accuracy is achieved by the finite volume method, with an overall correlation error of 12.83%, which shows an accuracy lost of 24% in comparison with the Bridging Model's prediction.

4 HIGHLIGHT ON UN IAXIAL STRENGTH PREDICTION

The compliance tensor of a composite by a micromechanics model is given by

$$[S_{ij}] = \begin{bmatrix} [S_{ij}]_\sigma & 0 \\ 0 & [S_{ij}]_\tau \end{bmatrix}, \tag{8.1}$$

$$[S_{ij}]_\sigma = \begin{bmatrix} 1/E_{11} & -\nu_{12}/E_{11} & -\nu_{12}/E_{11} \\ -\nu_{12}/E_{11} & 1/E_{22} & -\nu_{23}/E_{22} \\ -\nu_{12}/E_{11} & -\nu_{23}/E_{22} & 1/E_{22} \end{bmatrix}, \ [S_{ij}]_\tau = \begin{bmatrix} 1/G_{23} & 0 & 0 \\ 0 & 1/G_{12} & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix}. \tag{8.2}$$

Substituting Eqs. (8) into Eq. (7), the bridging tensor for the composite is obtained. Under a uniaxial load condition, only a longitudinal strength is controlled by a fiber failure, whereas all of the other failures are resulted from matrix's failures. Thus, we only need to determine the following relationships

$$\sigma_{11}^f = \lambda_1 \sigma_{11}^0, \ \sigma_{22}^m = \lambda_2 \sigma_{22}^0, \ \sigma_{23}^m = \lambda_3 \sigma_{23}^0, \ \sigma_{12}^m = \lambda_4 \sigma_{12}^0. \tag{9}$$

Where λ_i's are dependent on the bridging tensor and σ_{11}^0, σ_{22}^0, σ_{23}^0, and σ_{12}^0 are external loads applied individually to the composite once at a time.

It is noted that the stresses by Eqs. (9) are homogenized quantities. They should be converted into "true" values before used in assessing a fiber or matrix failure if the original strength data of the fiber or matrix are used. The stresses in the fiber are uniform[17], implying that its homogenized and true stresses are the same. The stresses in the matrix, however, are not uniform. Each of them should be multiplied with a stress concentration factor (SCF) of the matrix owing to the introduction of the fiber. A matrix plate with a hole generates a stress concentration in its neighborhood if the plate is subjected to an external load. Similarly, when the hole is filled with a fiber of different properties the matrix sustains a stress concentration as well.

5 SUMMARY ON SCFS

The most importantly, an SCF of the matrix in a composite cannot be defined as a point-wise stress divided by an overall applied one[2], as done in a classical approach. Otherwise, the resulting SCFs would be infinite at voids and micro-cracks which occur in a composite by nature. At those defects, point-wise stresses of the matrix are singular. As a classical SCF is defined as a point-wise (something like zero-dimensional) stress divided by the overall applied quantity, which is in fact a surface-averaged (two-dimensional) stress, the new definition for an SCF of the matrix must be made by a line-averaged (one-dimensional) stress divided by a volume-averaged (three- dimensional) one.

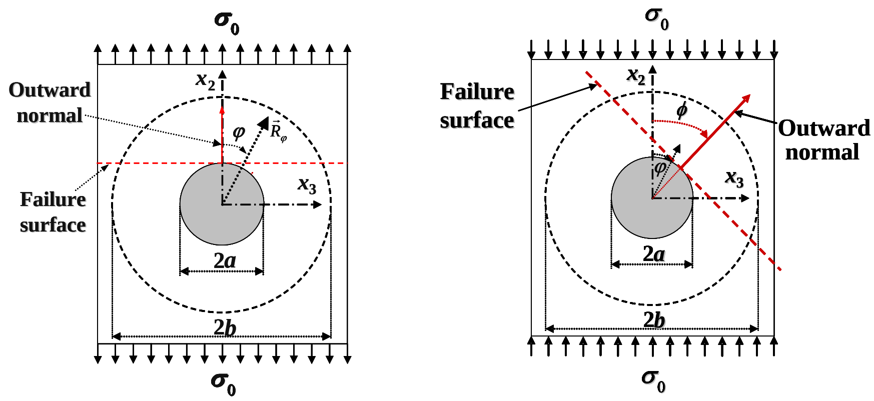
5.1 SCFS under transverse loads

Under a transverse load (**Fig. 1**), a matrix SCF is defined as[5]

$$K_{22}(\varphi)=\frac{1}{\left|\vec{R}_{\varphi}^{b}-\vec{R}_{\varphi}^{a}\right|}\int_{\left|\vec{R}_{\varphi}^{a}\right|}^{\left|\vec{R}_{\varphi}^{b}\right|}\frac{\tilde{\sigma}_{22}^{m}}{(\sigma_{22}^{m})_{BM}}\,d\left|\vec{R}_{\varphi}\right| \tag{10}$$

where $\tilde{\sigma}_{22}^{m}$ is a point-wise stress of the matrix in the loading direction determined on a CCA model, $\vec{R}_{\varphi}$ is a a vector along a line which has an inclined angle φ with the loading direction, as shown in **Fig. 1(a)**, $\vec{R}_{\varphi}^{a}$ and $\vec{R}_{\varphi}^{b}$ are the vectors of $\vec{R}_{\varphi}$ at the surfaces of the fiber and matrix cylinders, respectively. It is noted that the fiber and matrix domains in Eq. (10) must be within a representative volume element (RVE) of the composite, i.e.,

$$b=a/\sqrt{V_f} \tag{11}$$



(*a*) (*b*)

Fig. 1. Schematic of a RVE used in defining SCF of matrix in a composite subjected to (*a*) a transverse tension, (*b*) a transverse compression

V_f is the fiber volume fraction. If one recalls that the classical definition for the SCF of a plate containing a hole was made on the stress field determined from an infinite domain assumption, one can appreciate the pertinence of Eq. (10) together with Eq. (11). In the latter case, the line-averaged stress in the numerator of Eq. (10) becomes a point-wise quantity if $b \to a$. Meanwhile, the volume averaged stress in the denominator should be replaced by the surface averaged one, i.e., by σ_{22}^{0}. In this way, the SCF obtained from Eq. (10) by setting $\varphi=90^0$ equals 3, exactly the same as the classical one.

In Eq. (10), $(\sigma_{22}^{m})_{BM}$ is the transverse stress in the matrix calculated by the Bridging Model through

$$(\sigma_{22}^{m})_{BM}=\frac{a_{22}\sigma_{22}^{0}}{(V_f+V_m a_{22})} \tag{12}$$

where V_m=1-V_f. Explicit integration of Eq. (10) reads[5]

$$K_{22}(\varphi)=\Big\{1+\frac{A}{2}\sqrt{V_f}\cos 2\varphi+\frac{B}{2(1-\sqrt{V_f})}\big[V_f^2\cos 4\varphi+4V_f(\cos\varphi)^2(1-2\cos 2\varphi)+$$
$$\sqrt{V_f}(2\cos 2\varphi+\cos 4\varphi)\big]\Big\}(V_f+a_{22}V_m)/a_{22}, \tag{13.1}$$

$$A=\frac{2E_{22}^{f}E^{m}(\nu_{12}^{f})^2+E_{11}^{f}\{E^{m}(\nu_{23}^{f}-1)-E_{22}^{f}[2(\nu^{m})^2+\nu^{m}-1]\}}{E_{11}^{f}[E_{22}^{f}+E^{m}(1-\nu_{23}^{f})+E_{22}^{f}\nu^{m}]-2E_{22}^{f}E^{m}(\nu_{12}^{f})^2}, \tag{13.2}$$

$$B = \frac{E^m(1+\nu_{23}^f) - E_{22}^f(1+\nu^m)}{E_{22}^f[\nu^m + 4(\nu^m)^2 - 3] - E^m(1+\nu_{23}^f)} . \tag{13.3}$$

In Eq. (13.1), φ is the inclined angle between the outward normal to the failure surface and the loading direction. Under a transverse tension, one has φ=0 (**Fig. 1(a)**). However, under a transverse compression, φ=ϕ (**Fig. 1(b)**) determined by virtue of Mohr's theory as[5]

$$\phi = \frac{\pi}{4} + \frac{1}{2}\arcsin\frac{\sigma_{u,c}^m - \sigma_{u,t}^m}{2\sigma_{u,c}^m} . \tag{14}$$

Thus, the transverse tensile and compressive SCFs of the matrix in the composite, K_{22}^t and K_{22}^c, are resulted from Eqs. (13) as

$$K_{22}^t = K_{22}(0), \quad K_{22}^c = K_{22}(\phi). \tag{15}$$

The transverse shear SCF of the matrix is obtained also through Mohr's theory as[5]

$$K_{23} = 2\sigma_{u,s}^m \sqrt{\frac{K_{22}^t K_{22}^c}{\sigma_{u,t}^m \sigma_{u,c}^m}} . \tag{16}$$

In Eqs. (14) and (16), $\sigma_{u,t}^m$, $\sigma_{u,c}^m$, and $\sigma_{u,s}^m$ are the original tensile, compressive, and shear strengths of the monolithic matrix, respectively.

5.2 SCF under longitudinal shear

Under a longitudinal shear, the failure surface of a UD composite is shown in **Fig. 2**[23,24]. Following Eq. (10), a longitudinal shear SCF of the matrix is given by **(Fig. 3)**

$$K_{12}(\varphi) = \frac{1}{\sqrt{2}[\sqrt{b^2 - (a\sin\varphi)^2} - a\cos\varphi]} \int_0^{\sqrt{2}[\sqrt{b^2-(a\sin\varphi)^2} - a\cos\varphi]} \frac{\tilde{\sigma}_{12}^m}{(\sigma_{12}^m)_{BM}} dS , \tag{17}$$

where $\tilde{\sigma}_{12}^m$ and $(\sigma_{12}^m)_{BM}$ are obtained on a CCA model[17] and by the Bridging Model, respectively, as

$$\tilde{\sigma}_{12}^m = \sigma_{12}^0 \left[1 - a^2 \frac{(G_{12}^f - G^m)(x_2^2 - x_3^2)}{(G_{12}^f + G^m)(x_2^2 + x_3^2)^2} \right], \tag{18}$$

$$(\sigma_{12}^m)_{BM} = \frac{a_{33}\sigma_{12}^0}{(V_f + V_m a_{33})}, \tag{19}$$

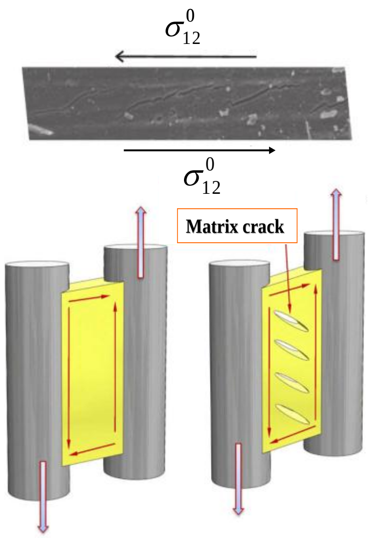


Fig. 2. Failures of the matrix in a UD composite under a longitudinal shear[23,24]

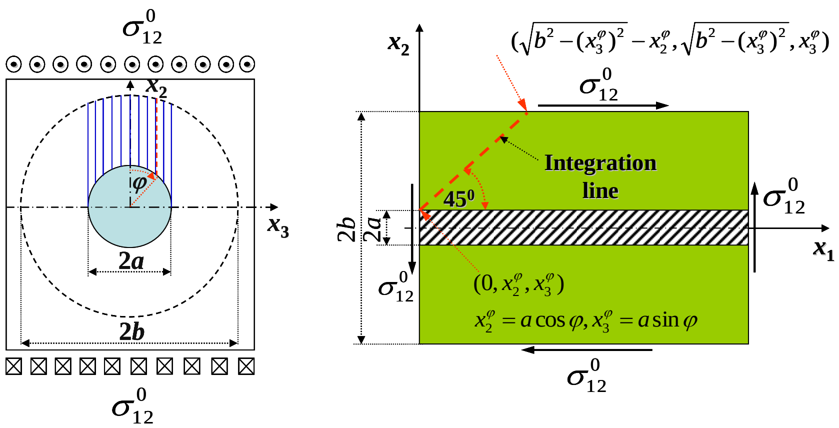


Fig. 3. Schematic definition for the SCF of matrix under a longitudinal shear

Substituting Eqs. (18) and (19) into Eq. (17) gives rise to

$$K_{12}(\varphi) = \left[1 - \frac{(G_{12}^f - G^m)\{\sqrt{V_f}\cos\varphi - V_f\sqrt{1-V_f(\sin\varphi)^2}\}}{(G_{12}^f + G^m)\{\sqrt{(1-V_f(\sin\varphi)^2)} - \sqrt{V_f}\cos\varphi\}}\right]\frac{(V_f + a_{33}V_m)}{a_{33}} \qquad (20)$$

Under a transverse load, the matrix transverse stress component in a CCA model is uniform along the thickness (where in the x_1-axial) direction, see Ref. [17]. On the other hand, the longitudinal shear load results in non-uniform shear stress in the thickness (here in the x_3-axial) direction, as seen from Eq. (18) and **Fig. 3**. Thus, an average on Eq. (20) along the thickness direction is necessary, resulting in

$$K_{12} = \frac{1}{2a}\int_{-a}^{a} K_{12}(x_3)(dx_3) = \frac{1}{\pi a}\int_{-\pi/2}^{\pi/2} K_{12}(\varphi)\cos(\varphi)(ad\varphi)$$

$$= \left[1 - V_f\frac{G_{12}^f - G^m}{G_{12}^f + G^m}\{W(V_f) - \frac{1}{3}\}\right]\frac{(V_f + a_{33}V_m)}{a_{33}}, \qquad (21.1)$$

.

$$W(V_f) = \int_0^a \frac{1}{a}\sqrt{1-\frac{x_3^2}{a^2}}\sqrt{\frac{1}{V_f}-\frac{x_3^2}{a^2}}dx_3 \approx \pi\sqrt{V_f}\left[\frac{1}{4V_f} - \frac{4}{128} - \frac{2}{512}V_f - \frac{5}{4096}V_f^{\,2}\right]. \qquad (21.2)$$

5.3 Longitudinal normal SCF

When a longitudinal normal load is applied to a CCA model, the resulting stresses in the matrix are uniform[17]. Thus,no stress concentration occurs in the matrix under a longitudinal load. In other words, the longitudinal normal SCF of the matrix in a composite equals one.

6 ASSESSMENT ON STRENGTH PREDICTIONS

From Eqs. (9), the longitudinal tensile, longitudinal compressive, transverse tensile, transverse compressive, transverse shear, and longitudinal shear strengths of a UD composite are estimated per any micromechanics model, respectively, as

$$\sigma_{11}^{u,t} = \sigma_{u,t}^f/\lambda_1,\ \sigma_{11}^{u,c} = \sigma_{u,c}^f/\lambda_1,\ \sigma_{22}^{u,t} = \sigma_{u,t}^m/(K_{22}^t\lambda_2),\ \sigma_{22}^{u,c} = \sigma_{u,c}^m/(K_{22}^c\lambda_2),$$

$$\sigma_{23}^u = \sigma_{u,s}^m/(K_{23}\lambda_3),\ \text{and}\ \sigma_{12}^u = \sigma_{u,s}^m/(K_{12}\lambda_4). \qquad (22)$$

It is possible that the volume-averaged stresses in the denominators of Eq. (10) and (17) be replaced, respectively, by σ_{22}^m and σ_{12}^m calculated through Eqs. (9). According to the two different methods in defining the denominator stresses, one through the Bridging Model and another through Eqs. (9), the resulting SCFs are called the first class (shorted to the 1st-SCFs) and the second class (shorted to the 2nd-SCFs) SCFs, respectively. It can be easily shown that the 2nd-SCFs are given by

$$K_{22}^{II,t}=\frac{\lambda_2^{BM}}{\lambda_2}K_{22}^t,\ \ K_{22}^{II,c}=\frac{\lambda_2^{BM}}{\lambda_2}K_{22}^c,\ \ K_{23}^{II}=\frac{\lambda_2^{BM}}{\lambda_2}K_{23},\ \ K_{12}^{II}=\frac{\lambda_4^{BM}}{\lambda_4}K_{12}, \tag{23.1}$$

$$\lambda_2^{BM}=\frac{a_{22}}{(V_f+V_m a_{22})},\ \ \lambda_4^{BM}=\frac{a_{66}}{(V_f+V_m a_{66})}. \tag{23.2}$$

Three kinds of predictions for the uniaxial strengths of all the 9 UD composites have been made in terms of the 12 models. The first and the second predictions are carried out with the 1st-SCFs and the 2nd-SCFs incorporated, whereas the third predictions are made with no SCFs taken into account (i.e., setting $K_{22}^t=K_{22}^c=K_{23}=K_{12}\equiv 1$ in Eqs. (22). It is noted that the first and second kinds of predictions by Bridging Model are the same. Three kinds of predictions' averaged errors by the 12 models and a ranking for the strength prediction accuracy are presented in Table 4.

	E-Glass LY556	**E-Glass MY750**	**AS4 3501-6**	**T300 BSL914C**	**IM7 8551-7**	**T300 PR319**	**AS carbon Epoxy**	**S2-Glass Epoxy**	**G40-800 5260**
E_{11}^f (GPa)	80	74	225	230	276	230	231	87	290
E_{22}^f (GPa)	80	74	15	15	19	15	15	87	19
μ_{12}^f	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
G_{12}^f (GPa)	33.33	30.8	15	15	27	15	15	36.3	27
μ_{23}^f	0.2	0.2	0.07	0.07	0.36	0.07	0.07	0.2	0.357
E_{11}^m (GPa)	3.35	3.35	4.2	4.0	4.08	0.95	3.2	3.2	3.45
E_{22}^m (GPa)	3.35	3.35	4.2	4.0	4.08	0.95	3.2	3.2	3.45
μ_{12}^m	0.35	0.35	0.34	0.35	0.38	0.35	0.35	0.35	0.35
G_{12}^m (GPa)	1.24	1.24	1.567	1.481	1.478	0.352	1.185	1.185	1.28
μ_{23}^m	0.35	0.35	0.34	0.35	0.38	0.35	0.35	0.35	0.35
$\sigma_{u,t}^f$ (MPa)	2150	2150	3350	2500	5180	2500	3500	2850	5860
$\sigma_{u,c}^f$ (MPa)	1450	1450	2500	2000	3200	2000	3000	2450	3200
$\sigma_{u,t}^m$ (MPa)	80	80	69	75	99	70	85	73	70
$\sigma_{u,c}^m$ (MPa)	120	120	250	150	130	130	120	120	130
$\sigma_{u,s}^m$ (MPa)	54	54	50	70	57	41	50	52	57
V_f	0.62	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6

Table 1

				Measured					
E_{11} (GPa)	53.5	45.6	126	138	165	129	140	52	173
E_{22} (GPa)	17.7	16.2	11	11	8.4	5.6	10	19	10
μ_{12}	0.278	0.278	0.28	0.28	0.34	0.318	0.3	0.3	0.33
G_{12} (GPa)	5.83	5.83	6.6	5.5	5.6	1.33	6	6.7	6.94
G_{23} (GPa)	6.32	5.79	3.93	3.93	2.8	1.86	3.35	6.7	3.56
$\sigma_{11}^{\mu,t}$ (MPa)	1140	1280	1950	1500	2560	1378	1990	1700	2750
$\sigma_{11}^{\mu,c}$ (MPa)	570	800	1480	900	1590	950	1500	1150	1700

$\sigma_{22}^{\mu,t}$ (MPa)	35	40	48	27	73	40	38	63	75
$\sigma_{22}^{\mu,c}$ (MPa)	114	145	200	200	185	125	150	180	210
σ_{23}^{μ} (MPa)	50	50	55	-	57	45	50	40	57
σ_{12}^{μ} (MPa)	72	73	79	80	90	97	70	72	90

Table 2

Model	*N*	Average error	Error ratio	Ranking	Model	*N*	Average error	Error ratio	Ranking
Bridging model	45	10.38%	1.0	**1**	Halpin-Tsai formula	45	19.24%	1.85	**7**
FAM	45	12.83%	1.24	**2**	Modified rule of mixture model	45	19.35%	1.86	**8**
FEM	45	13.08%	1.26	**3**	Mori-Tanaka model	45	19.59%	1.89	**9**
Chamis model	45	14.09%	1.36	**4**	generalized self-consistent model	45	19.66%	1.89	**10**
GMC	45	15.07%	1.45	**5**	Self-consistent model	45	21.86%	2.11	**11**
Hill-Hashin- C-L model	33	17.22%	1.66	**6**	Rule of mixtur model	45	28.4%	2.74	**12**

Table 3

Model	*N*	Average error	Error ratio	Ranking	Model	*N*	Average error	Error ratio	Ranking
Bridging model	53	21.1%	1.0	**1**	Halpin-Tsai formula	53	30.1%	1.43	**6**
FAM	53	23.0%	1.09	**2**	Generalized self-consistent model	53	30.2%	1.43	**8**
FEM	53	23.1%	1.09	**3**	Mori-Tanaka model	53	30.2%	1.43	**8**
Chamis model	53	25.4%	1.20	**4**	Modified rule of mixture model	53	30.7%	1.45	**10**
GMC	53	25.4%	1.20	**4**	Self-consistent model	53	32.7%	1.54	**11**
Hill-Hashin- C-L model	18	30.1%	1.43	**6**	Rule of mixture model	53	44.5%	2.11	**12**

Table 4.a with the 1st-SCFs

Model	*N*	Average error	Error ratio	Ranking	Model	*N*	Average error	Error ratio	Ranking
Hill-Hashin- C-L model	18	19.8%	1.0	**1**	Modified rule of mixture model	53	27.3%	1.387	**7**
Bridging model	53	21.1%	1.07	**2**	GMC	53	28.9%	1.46	**8**
Halpin-Tsai formula	53	26.4%	1.33	**3**	FAM	53	29.7%	1.50	**9**
Generalized self-consistent model	53	26.4%	1.33	**3**	FEM	53	29.7%	1.50	**9**
Mori-Tanaka model	53	26.4%	1.33	**3**	Chamis model	53	31.6%	1.60	**11**
Self-consistent model	53	26.9%	1.36	**6**	Rule of mixtur model	53	31.8%	1.61	**12**

Table 4.b with the 2st-SCFs

Model	*N*	Averag	Error	Rank	Model	*N*	Averag	Error	Rank

		e error	ratio	ing			e error	ratio	ing
Rule of mixture model	35	30.4%	1.0	**1**	Hill-Hashin- C-L model	12	65.7%	2.16	**7**
Modified rule of mixture model	35	43.5%	1.43	**2**	GMC	35	68.7%	2.26	**8**
Mori-Tanaka model	35	46.4%	1.53	**3**	FAM	35	77.1%	2.54	**9**
Halpin-Tsai formula	35	46.4%	1.53	**3**	FEM	35	77.2%	2.54	**10**
Generalized self-co nsistent model	35	46.4%	1.53	**3**	Bridging model	35	115.2%	3.79	**11**
Chamis model	35	61.9%	2.04	**6**	Self-consistent model	35	128.6%	4.23	**12**

Table 4.c with no SCFs

7 CONCLUSIONS

The following conclusions can be drawn from this study.

1. Compared with other models, bridging Model shows the best accuracy in stiffness predictions for UD composites.
2. Incorporated with the SCFs, the prediction accuracy for a composite strength is improved significantly, the Bridging Model's prediction for the strengths of the 9 composites is more accurate than the others'.
3. Without considering any SCF, all of the models show overall poor accuracy in the prediction of composite strengths.

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