THE MECHANICS OF INTERFACE FRACTURE IN LAYERED COMPOSITE MATERIALS: (1) BRITTLE HOMOGENEOUS AND BI-MATERIAL INTERFACES

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ABSTRACT

A powerful analytical methodology is discovered and is used to partition the total energy release rate of one dimensional (1D) interface fractures on brittle homogeneous or bi-material interfaces into its mode I and II components based on the classical and shear-deformable plate theories and 2D elasticity. Previously unsolvable problems are solved and confusions explained.

1 INTRODUCTION

Layered composite materials find many advanced applications in science, engineering and technology sectors. A major concern is the interface fracture during service, which leads to tremendous research effort on the mechanics of interface fracture in layered composite materials under various service conditions. Four well known traditional fundamental concepts on the topic are energy release rate (ERR), stress intensity factors (SIFs), virtual crack closure technique (VCCT) and J-integral, all of which are maturely understood. One less well known fundamental concept is the mixed mode partition (MMP). It, however, plays a key role in studying the mechanics of interface fracture since interface fracture toughness is not a purely intrinsic material property, but instead also depends on the mode mixity. There has been a great deal of controversies and confusions on MMP in last few decades until recently when a powerful orthogonal pure-mode partition theory (OPPT) [1-8] is discovered based on a fundamental understanding of the mechanics of interface fracture. This paper aims to give a brief summary on OPPT.

2 ORTHOGONAL PURE-MODE PARTITION THEORY (OPPT)

Although interface fracture generally occurs as mixed-mode fracture with all three opening, shearing and tearing actions (i.e. mode I, II and III), 1D interface fractures have received much more attention as it is simpler and still captures the essential fracture mechanics. The expression '1D interface fracture' means that a fracture propagates in one direction with mode I and mode II actions only. Examples of 1D interface fracture are shown in Fig. 1 including through-width delamination in double cantilever straight and curved beams and blisters in layered plates and shells. A central task in studying 1D interface fracture is to partition the total energy release rate (ERR) of a mixed-mode fracture into its individual mode I and II ERR components. The total ERR G of a 1D mixed-mode fracture on a brittle homogeneous or bi-material interface can be expressed in the following form.

$$G = \{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\}[C]\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\}^T \tag{1}$$

It is a quadratic form non-negative definite in terms of the crack tip bending moments per unit width M_{1B}, M_{2B}, axial forces per unit width N_{1B}, N_{2B} and shear forces per unit width P_{1B}, P_{2B} as shown in Fig. 2 with subscripts 1 and 2 denoting the layers above and below the crack and with B denoting the crack tip. The coefficient matric $[C]$ depends on material properties, layups of the layered materials and location of the interface fracture.

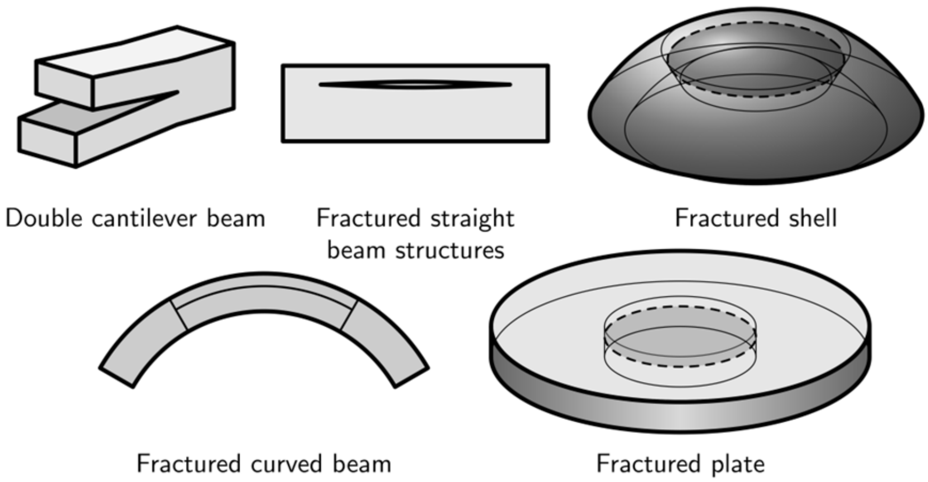

Figure 1: Examples of 1D interface fracture.

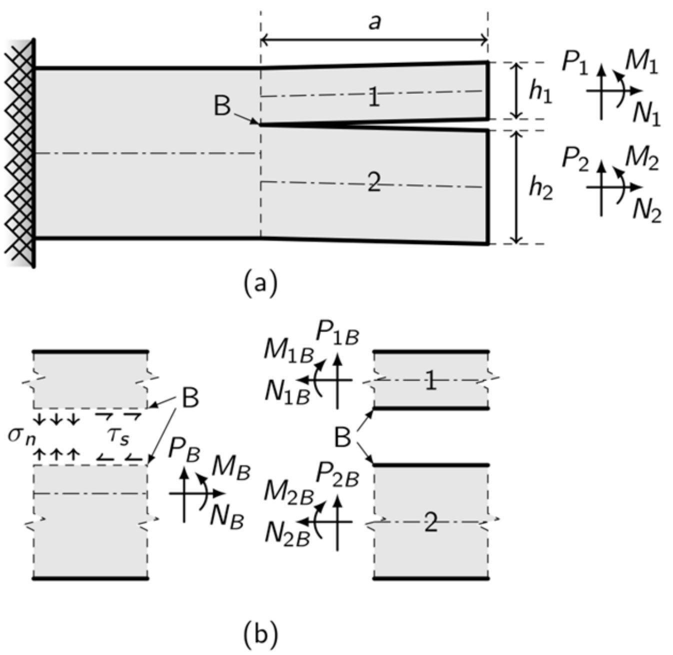

Figure 2: A representative 1D interface fracture model. (a) General description. (b) Details local to the crack tip.

By using the VCCT, the individual mode I and II ERR components, that is, G_I and G_{II} can be written in the following equations:

$$G_I = \lim_{\Delta a \to 0} \frac{F_n d_n}{2\Delta a} \tag{2}$$

$$G_{II} = \lim_{\Delta a \to 0} \frac{F_s d_s}{2\Delta a} \tag{3}$$

where Δa is the crack extension size, F_n and d_n are the crack tip opening force and relative opening displacement, respectively whilst F_s and d_s are the crack tip shearing force and relative shearing displacement, respectively. Obviously, the determination of F_n, d_n, F_s and d_s play the key role in the calculation of G_I and G_{II}. In the case of rigid interface fractures considered here both F_n and F_s have finite values due to the stress singularity at the crack tip. However, both the relative crack opening and shearing displacements right at the crack tip B are negligible. In the VCCT, i.e. Eqs. (2) and (3), the relative crack tip opening displacement d_n and shearing displacement d_s are measured at the location immediately behind the crack tip B by assuming crack extension similarity. The traditional analytical approach aims to obtain F_n, d_n, F_s and d_s by solving differential equations with a very limited capability leaving some important problems unsolved and confusions.

A powerful orthogonal pure-mode partition theory (OPPT) is discovered recently. The OPPT expresses the G_I in Eq. (2) and G_{II} in Eq. (3) in the following general equations:

$$G_I = c_I \left(M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\beta_2} - \frac{N_{2B}}{\beta_3} - \frac{P_{1B}}{\beta_4} - \frac{P_{2B}}{\beta_5} \right) \left(M_{1B} - \frac{M_{2B}}{\beta_1'} - \frac{N_{1B}}{\beta_2'} - \frac{N_{2B}}{\beta_3'} - \frac{P_{1B}}{\beta_4'} - \frac{P_{2B}}{\beta_5'} \right) \tag{4}$$

$$G_{II} = c_{II} \left(M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} - \frac{N_{2B}}{\theta_3} - \frac{P_{1B}}{\theta_4} - \frac{P_{2B}}{\theta_5} \right) \left(M_{1B} - \frac{M_{2B}}{\theta_1'} - \frac{N_{1B}}{\theta_2'} - \frac{N_{2B}}{\theta_3'} - \frac{P_{1B}}{\theta_4'} - \frac{P_{2B}}{\theta_5'} \right) \tag{5}$$

where c_I and c_{II} are two constants which are material property, layup and thickness ratio dependent. The first and second brackets of Eq. (4) represent F_n and d_n in Eq. (2), respectively. When either of them equals to zero, mode I ERR G_I becomes zero leading to a pure mode II fracture mode. The first bracket becomes zero in the following loading conditions:

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad \beta_1 \quad 0 \quad 0 \quad 0 \quad 0\} \tag{6}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad \beta_2 \quad 0 \quad 0 \quad 0\} \tag{7}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad 0 \quad \beta_3 \quad 0 \quad 0\} \tag{8}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad 0 \quad 0 \quad \beta_4 \quad 0\} \tag{9}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad 0 \quad 0 \quad 0 \quad \beta_5\} \tag{10}$$

which are called force type pure mode II modes. The second bracket becomes zero in the following loading conditions:

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad \beta_1' \quad 0 \quad 0 \quad 0 \quad 0\} \tag{11}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad \beta_2' \quad 0 \quad 0 \quad 0\} \tag{12}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad 0 \quad \beta_3' \quad 0 \quad 0\} \tag{13}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad 0 \quad 0 \quad \beta_4' \quad 0\} \tag{14}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad 0 \quad 0 \quad 0 \quad \beta_5'\} \tag{15}$$

which are called displacement type pure mode II modes. Similarly, the first and second brackets of Eq. (5) represent d_s and F_s in Eq. (2), respectively. When either of them equals to zero, mode II ERR G_{II} becomes zero leading to pure mode I. The first bracket becomes zero in the following loading conditions:

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad \theta_1 \quad 0 \quad 0 \quad 0 \quad 0\} \tag{16}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad \theta_2 \quad 0 \quad 0 \quad 0\} \tag{17}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad 0 \quad \theta_3 \quad 0 \quad 0\} \tag{18}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad 0 \quad 0 \quad \theta_4 \quad 0\} \tag{19}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad 0 \quad 0 \quad 0 \quad \theta_5\} \tag{20}$$

which are called displacement type pure mode I modes. The second bracket becomes zero in the following loading conditions:

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad \theta_1' \quad 0 \quad 0 \quad 0 \quad 0\} \tag{21}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad \theta_2' \quad 0 \quad 0 \quad 0\} \tag{22}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad 0 \quad \theta_3' \quad 0 \quad 0\} \tag{23}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad 0 \quad 0 \quad \theta_4' \quad 0\} \tag{24}$$

$$\{M_{1B} \quad M_{2B} \quad N_{1B} \quad N_{2B} \quad P_{1B} \quad P_{2B}\} = \{1 \quad 0 \quad 0 \quad 0 \quad 0 \quad \theta_5'\} \tag{25}$$

which are called force type pure mode I modes. All β_i, β_i', θ_i and θ_i' (with $i = 1,2,3,4,5$) are material property, layup and thickness ratio dependent. It is extremely difficult if not impossible to find their analytical expressions using the traditional approach. This is particularly true to find the analytical expressions for β_i and θ_i' as they are related to the interface stresses with singularity at the crack tip. In the powerful OPPT, the two most fundamental displacement type pure modes Eq. (11) and Eq. (16) , i.e. β_1' pure mode II mode and θ_1 pure mode I mode are determined first either analytically or numerically or experimentally. Then, all other pure modes given in the above equations can be determined by using the powerful orthogonality principle. That is, for examples, using the following equations to find β_1 and β_2 when θ_1 is known.

$$\{1 \quad \beta_1 \quad 0 \quad 0 \quad 0 \quad 0\}[C]\{1 \quad \theta_1 \quad 0 \quad 0 \quad 0 \quad 0\}^T = 0 \tag{26}$$

$$\{1 \quad 0 \quad \beta_2 \quad 0 \quad 0 \quad 0\}[C]\{1 \quad \theta_1 \quad 0 \quad 0 \quad 0 \quad 0\}^T = 0 \tag{27}$$

and then using the following equations to find θ_2 and θ_3.

$$\{1 \quad \beta_1 \quad 0 \quad 0 \quad 0 \quad 0\}[C]\{1 \quad 0 \quad \theta_2 \quad 0 \quad 0 \quad 0\}^T = 0 \tag{28}$$

$$\{1 \quad \beta_1 \quad 0 \quad 0 \quad 0 \quad 0\}[C]\{1 \quad 0 \quad 0 \quad \theta_3 \quad 0 \quad 0\}^T = 0 \tag{29}$$

Similarly, for examples, using the following equations to find θ_1' and θ_2' when β_1' is known.

$$\{1 \quad \theta_1' \quad 0 \quad 0 \quad 0 \quad 0\}[C]\{1 \quad \beta_1' \quad 0 \quad 0 \quad 0 \quad 0\}^T = 0 \tag{30}$$

$$\{1 \quad 0 \quad \theta_2' \quad 0 \quad 0 \quad 0\}[C]\{1 \quad \beta_1' \quad 0 \quad 0 \quad 0 \quad 0\}^T = 0 \tag{31}$$

and then using the following equations to find β_2' and β_3'.

$$\{1 \quad \theta_1' \quad 0 \quad 0 \quad 0 \quad 0\}[C]\{1 \quad 0 \quad \beta_2' \quad 0 \quad 0 \quad 0\}^T = 0 \tag{32}$$

$$\{1 \quad \theta_1' \quad 0 \quad 0 \quad 0 \quad 0\}[C]\{1 \quad 0 \quad 0 \quad \beta_3' \quad 0 \quad 0\}^T = 0 \tag{33}$$

(θ_i, β_i) and (θ_i', β_i') (with $i = 1,2,3,4,5$) are called double set orthogonal pure modes. The orthogonality exists between any pairs in each set. The double set orthogonal pure modes are material property, layup and thickness ratio dependent. More importantly, they depend on the choice of mechanical theories.

When Euler beam or classical plate/shell theory are used Eqs. (4) and (5) become

$$G_{IE} = c_{IE}\left(M_{1B} - \frac{M_{2B}}{\beta_{1-E}} - \frac{N_{1B}}{\beta_{2-E}} - \frac{N_{2B}}{\beta_{3-E}}\right)\left(M_{1B} - \frac{M_{2B}}{\beta_{1-E}'} - \frac{N_{1B}}{\beta_{2-E}'} - \frac{N_{2B}}{\beta_{3-E}'}\right) \tag{34}$$

$$G_{IIE} = c_{IIE}\left(M_{1B} - \frac{M_{2B}}{\theta_{1-E}} - \frac{N_{1B}}{\theta_{2-E}} - \frac{N_{2B}}{\theta_{3-E}}\right)\left(M_{1B} - \frac{M_{2B}}{\theta_{1-E}'} - \frac{N_{1B}}{\theta_{2-E}'} - \frac{N_{2B}}{\theta_{3-E}'}\right) \tag{35}$$

for both brittle homogeneous and bi-material interfaces where the subscript *E* denotes Euler beam or classical plate/shell theory. The two sets of orthogonal pure modes are different from each other and the through thickness shearing effects disappear. It was used to be claimed that mixed mode partition was unsolvable based on Euler beam or classical plate/shell theory. The powerful OPPT solves it. In the case of thin layer delamination Eqs. (34) and (35) reduce to

$$G_{IE} = c_{IE}\left(M_{1B} - \frac{N_{1B}}{\beta_{2-E}}\right)\left(M_{1B} - \frac{N_{1B}}{\beta_{2-E}'}\right) \tag{36}$$

$$G_{IIE} = c_{IIE}\left(M_{1B} - \frac{N_{1B}}{\theta_{2-E}}\right)\left(M_{1B} - \frac{N_{1B}}{\theta_{2-E}'}\right) \tag{37}$$

When Timoshenko beam or first order shear deformable plate/shell theory are used Eqs. (4) and (5) become

$$G_{IT} = c_{IT}\left(M_{1B} - \frac{M_{2B}}{\beta_{1-T}} - \frac{N_{1B}}{\beta_{2-T}} - \frac{N_{2B}}{\beta_{3-T}} - \frac{P_{1B}}{\beta_{4-T}} - \frac{P_{2B}}{\beta_{5-T}}\right)^2 \tag{38}$$

$$G_{IIT} = c_{IIT}\left(M_{1B} - \frac{M_{2B}}{\theta_{1-T}} - \frac{N_{1B}}{\theta_{2-T}} - \frac{N_{2B}}{\theta_{3-T}}\right)^2 \tag{39}$$

for both brittle homogeneous and bi-material interfaces where the subscript *T* denotes Timoshenko beam or first order shear deformable plate/shell theory. The two sets of orthogonal pure modes coincide at the first set of orthogonal pure modes (θ_i, β_i) and the through thickness shearing effects only produce mode I ERR. It was thought that Eqs. (34) and (35) were equal to Eqs. (38) and (39), respectively in the absence of thoroughness shear forces. The powerful OPPT clears the confusion. In the case of thin layer delamination Eqs. (38) and (39) reduce to

$$G_{IT} = c_{IT}\left(M_{1B} - \frac{N_{1B}}{\beta_{2-T}} - \frac{P_{1B}}{\beta_{4-T}} \right)^2 \tag{40}$$

$$G_{IIT} = c_{IIT}\left(M_{1B} - \frac{N_{1B}}{\theta_{2-T}} \right)^2 \tag{41}$$

When 2D elasticity theory is used Eqs. (4) and (5) become

$$G_{I2D} = c_{I2D}\left(M_{1B} - \frac{M_{2B}}{\beta_{1-2D}} - \frac{N_{1B}}{\beta_{2-2D}} - \frac{N_{2B}}{\beta_{3-2D}} - \frac{P_{1B}}{\beta_{4-2D}} - \frac{P_{2B}}{\beta_{5-2D}} \right)^2 \tag{42}$$

$$G_{II2D} = c_{II2D}\left(M_{1B} - \frac{M_{2B}}{\theta_{1-2D}} - \frac{N_{1B}}{\theta_{2-2D}} - \frac{N_{2B}}{\theta_{3-2D}} - \frac{P_{1B}}{\theta_{4-2D}} - \frac{P_{2B}}{\theta_{5-2D}} \right)^2 \tag{43}$$

for brittle homogeneous interfaces where the subscript *2D* denotes 2D elasticity theory. The two sets of orthogonal pure modes coincide at each other and the through thickness shearing effects produce both mode I and II ERRs. In the case of thin layer delamination Eqs. (42) and (43) reduce to

$$G_{I2D} = c_{I2D}\left(M_{1B} - \frac{N_{1B}}{\beta_{2-2D}} - \frac{P_{1B}}{\beta_{4-2D}} \right)^2 \tag{44}$$

$$G_{II2D} = c_{II2D}\left(M_{1B} - \frac{N_{1B}}{\theta_{2-2D}} \right)^2 \tag{45}$$

Note that the through thickness shearing effects only produce mode I ERR for thin layer delamination. For brittle bi-material interfaces, Eqs. (4) and (5) remain the same, that is,

$$G_{I2D} = c_{I2D}\left(M_{1B} - \frac{M_{2B}}{\beta_{1-2D}} - \frac{N_{1B}}{\beta_{2-2D}} - \frac{N_{2B}}{\beta_{3-2D}} - \frac{P_{1B}}{\beta_{4-2D}} - \frac{P_{2B}}{\beta_{5-2D}} \right) \left(M_{1B} - \frac{M_{2B}}{\beta'_{1-2D}} - \frac{N_{1B}}{\beta'_{2-2D}} - \frac{N_{2B}}{\beta'_{3-2D}} - \frac{P_{1B}}{\beta'_{4-2D}} - \frac{P_{2B}}{\beta'_{5-2D}} \right) \tag{46}$$

$$G_{II2D} = c_{II2D}\left(M_{1B} - \frac{M_{2B}}{\theta_{1-2D}} - \frac{N_{1B}}{\theta_{2-2D}} - \frac{N_{2B}}{\theta_{3-2D}} - \frac{P_{1B}}{\theta_{4-2D}} - \frac{P_{2B}}{\theta_{5-2D}} \right) \left(M_{1B} - \frac{M_{2B}}{\theta'_{1-2D}} - \frac{N_{1B}}{\theta'_{2-2D}} - \frac{N_{2B}}{\theta'_{3-2D}} - \frac{P_{1B}}{\theta'_{4-2D}} - \frac{P_{2B}}{\theta'_{5-2D}} \right) \tag{47}$$

The two sets of orthogonal pure modes are not only different from each other but are also crack extension size-dependent. In the case of thin layer delamination Eqs. (46) and (47) reduce to

$$G_{I2D} = c_{I2D}\left(M_{1B} - \frac{N_{1B}}{\beta_{2-2D}} - \frac{P_{1B}}{\beta_{4-2D}} \right) \left(M_{1B} - \frac{N_{1B}}{\beta'_{2-2D}} - \frac{P_{1B}}{\beta'_{4-2D}} \right) \tag{47}$$

$$G_{II2D} = c_{II2D}\left(M_{1B} - \frac{N_{1B}}{\theta_{2-2D}} - \frac{P_{1B}}{\theta_{4-2D}} \right) \left(M_{1B} - \frac{N_{1B}}{\theta'_{2-2D}} \right) \tag{48}$$

Finally, it is worth noting that the existence of double sets of orthogonal pure modes in the Euler beam or classical plate/shell partition theory is due to its global nature while the existence of double sets of orthogonal pure modes in 2D elasticity partition theory for brittle bi-material interfaces is due to the material mismatches resulting in phase difference in the variation of interface stresses and relative separations. When the extension size Δa is big enough, both the three partition theories will unify at the classical partition theory in the absence of the through thickness shear forces. That is why both the shear deformable and 2D elasticity partition theories are called local partition theory while the classical partition theory is called global partition theory. Validity of each theory has been assessed in a series of studies. Details of the orthogonal pure modes (θ_i, β_i) and (θ_i', β_i') in each theory can be found in the work [1-8].

3 SOME COMPARISONS WITH FEM RESULTS

Fig. 3 shows the comparison between the present analytical predictions based on 2D elasticity partition theory with the marked FEM results for a bi-material interface. The details of the material properties and FEM can be found in the work [6]. Excellent agreement is observed.

Fig. 4 shows the comparison between the present analytical predictions based on 2D elasticity partition theory with FEM results for homogeneous interface fracture under general loading conditions. The details of the material properties and the FEM simulation can be found in the work [7]. Excellent agreement is observed again.

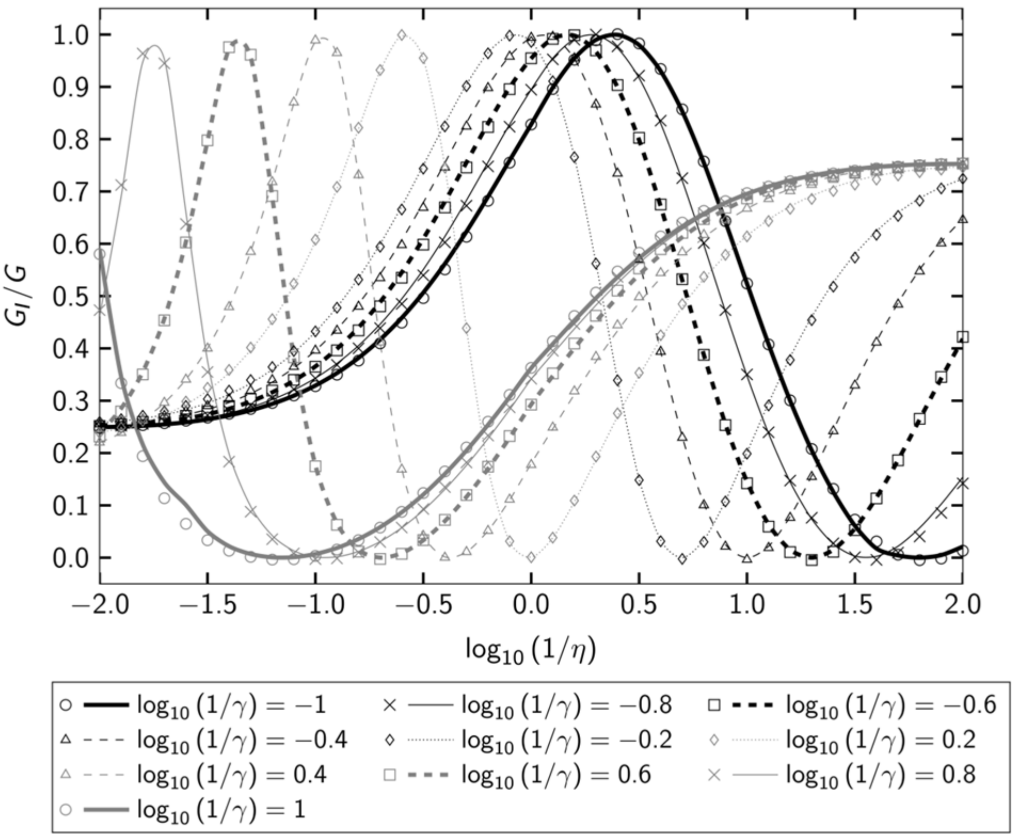

Figure 3: Comparison of the present analytical theory and FEM data for the ERR partition G_I/G at crack extension size $\Delta a = 0.05$ with $N_{2B}/M_{1B} = 10$ and various thickness ratios. $\gamma = h_2/h_1$ at a bi-material interface.

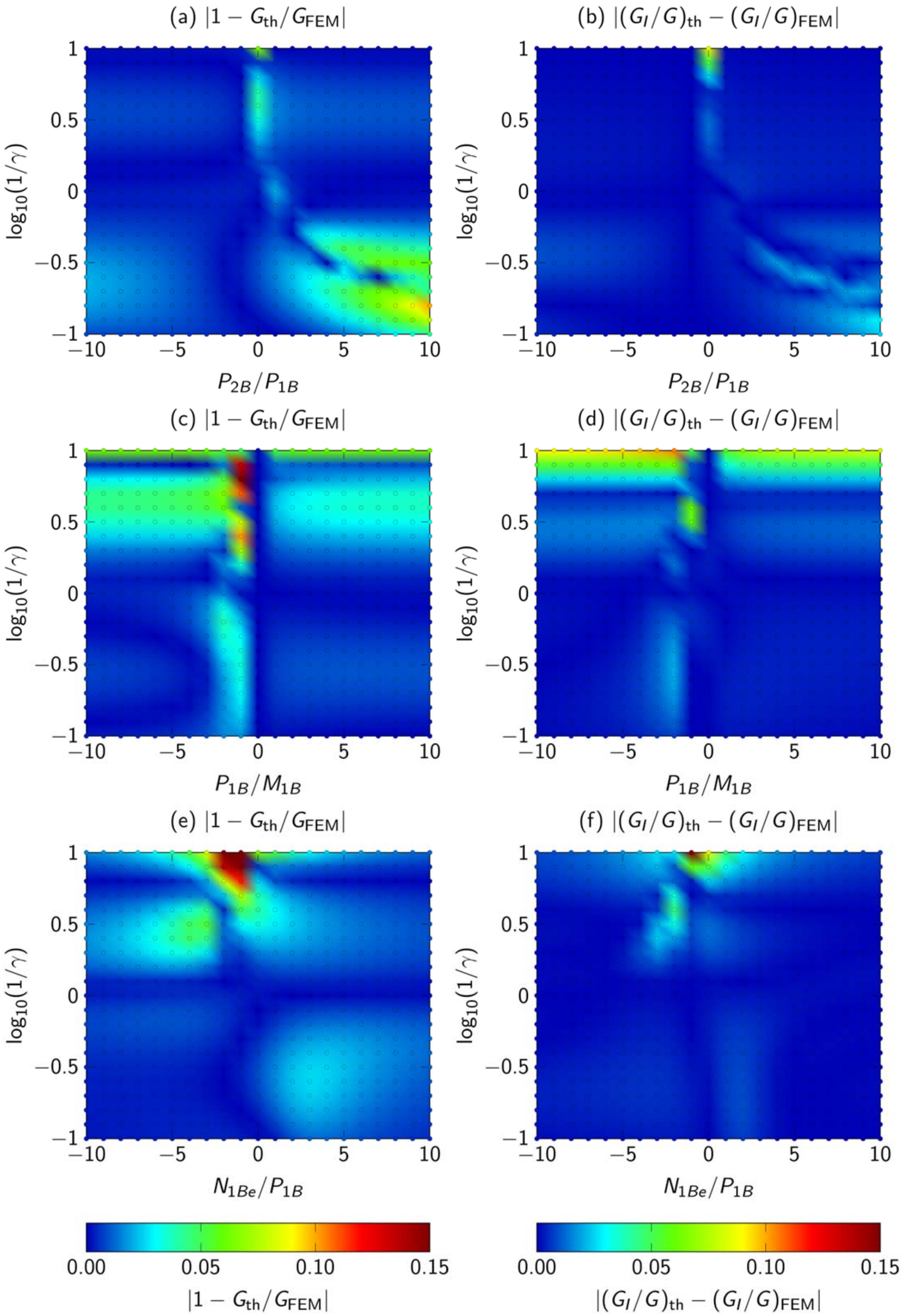

Figure 4: Comparison of the present analytical theory and the 2D FEM for the total ERR G and the ERR partition G_I/G for variable γ and loading conditions P_{2B}/P_{1B}, P_{1B}/M_{1B} and N_{1Be}/P_{1B}.

4 CONCLUSIONS

The powerful OPPT reveals some fundamental mechanics of interface fractures, and solves previously unsolvable problems and clears previous confusions. The analytical theories provide valuable means for studying interface fractures on macroscopic, microscopic and nano-scales.

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