WHEN AN INTERFACE DEBONDING OCCURS IN A COMPOSITE

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ABSTRACT

The load level at which a debonding occurs on the constituent fiber/matrix interface of a composite subjected to any load is predicted. Only the original properties of the fiber and matrix measured from monolithic material specimens in addition to the transverse tensile strength of a unidirectional (UD) composite made from the same constituents are required. Stress concentration factors (SCFs) of the matrix in the composite are crucial for this purpose. Such an SCF cannot be defined following a classical approach. The SCFs with the perfect and debonded interfaces are derived, respectively. Letting the predicted transverse tensile strength of the UD composite with an initial perfect and later debonded interface be equal to the measured counterpart, a critical Mises stress of the matrix at which the debonding occurs is obtained. Each Mises stress of the matrix in any other composite from the same constituent system is assessed against the critical value, and the load level when the interface debonding occurs is determined accordingly.

1 INTRODUCTION

The interface between reinforcing fibers and matrix of a composite plays a key role in transferring loads on the composite into the fibers. In order to improve the overall load carrying capacity of the composites made from various fiber and matrix systems, a great many techniques have been developed to perform interface modifications. However, given a fiber and matrix system, it is still a challenge whether the interface bonding between the two constituents is strong enough and how much potential exists if an interface modification is going to carry out.

The original purpose of this work is to predict the strength of a composite with an initial perfect and later debonded interface subjected to any load using its constituent properties measured independently. SCFs of the matrix in the composite have been found to be crucially important for this purpose[1,2]. Such an SCF cannot be defined, following a classical approach, as a maximum point-wise stress divided by an overall applied one. It must be determined upon an averaged stress. The SCFs of the matrix having a perfect interface bonding with the fiber have been derived previously. The predicted transverse tensile strengths of some composites were in large discrepancy with the measured data[2]. There must have been an interface debonding in those composites to cause the discrepancy.

In this work, the transverse tensile SCF of the matrix with a debonded fiber/matrix interface is derived. By imposing that the predicted transverse tensile strength of a UD composite with an initial perfect and later debonded interface to be equal to the measured one, a critical Mises stress of the matrix is determined. Any other composite made from the same fiber and matrix system will undergo an interface debonding if a Mises stress of the matrix is equal to or greater than the critical value. Only a transverse tensile test on a UD composite made from the same fiber and matrix system under consideration is necessary to understand the interface characteristics. Off-axial tensile strengths of two UD composites are estimated with the assumptions of perfect and debonded interfaces, respectively. The predictions with the debonded interfaces correlate the best with the available experiments.

2 SUMMARY OF BRIDGING MODEL

Any composite is heterogenous by nature. Stresses should be defined upon averaged quantities with respect to its RVE (representative volume element) *V'* through

$$\sigma_i = (\int_{V'} \tilde{\sigma}_i dV)/V', \quad (1)$$

which results in $$\{\sigma_i\} = V_f\{\sigma_i^f\} + V_m\{\sigma_i^m\} \quad (2)$$

if only fiber and matrix are within *V'*. *V* is a volume fraction, with a super-/sub-script *f* or *m* referring to fiber or matrix. A stress with ~ on head represents a point-wise quantity. Eqution (1) represents a homogenization for the composite.

Suppose that there is a bridging tensor, $[A_{ij}]$, such that

$$\{\sigma_i^m\} = [A_{ij}]\{\sigma_j^f\}, \quad (3)$$

which together with Eqs. (2) results in

$$\{\sigma_i^f\} = (V_f[I] + V_m[A_{ij}])^{-1}\{\sigma_j\} = [B_{ij}]\{\sigma_j\}, \quad (4)$$

$$\{\sigma_i^m\} = [A_{ij}][B_{ij}]\{\sigma_j\}. \quad (5)$$

Further, a compliance tensor of the composite is given by[3]

$$[S_{ij}] = (V_f[S_{ij}^f] + V_m[S_{ij}^m][A_{ij}])(V_f[I] + V_m[A_{ij}])^{-1}. \quad (6)$$

On the other hand, it is found from Eq. (6) that

$$[A_{ij}] = V_f([S_{ij}] - [S_{ij}^m])^{-1}([S_{ij}^f] - [S_{ij}])/V_m. \quad (7)$$

In other words, any micromechanics model corresponds to a bridging tensor. Thus, the determination of internal stresses through Eqs. (4) and (5) in the fiber and matrix of a composite is equivalent to that of the elastic properties of the same composite. Several comparisons have shown that Bridging Model is among the most efficient micromechanics models[4,5]. Non-zero bridging tensor elements are

$$A_{11} = E^m / E_{11}^f, \quad (8.1)$$

$$A_{22} = A_{33} = A_{44} = 0.3 + 0.7\frac{E^m}{E_{22}^f}, \quad (8.2)$$

$$A_{55} = A_{66} = 0.3 + 0.7\frac{G^m}{G_{12}^f}, \quad (8.3)$$

$$A_{12} = A_{13} = \frac{\nu^m E_{11}^f - E^m \nu_{12}^f}{E^m - E_{11}^f}(A_{11} - A_{22}). \quad (8.4)$$

All of the other A_{ij}'s not shown above are zero. E_{11}^f, E_{22}^f, and G_{12}^f are longitudinal, transverse, and in-plane shear moduli of the fiber, respectively. ν_{12}^f is its longitudinal Poisson's ratio. E^m, G^m, and ν^m are Young's and shear moduli and Poisson's ratio of the matrix.

3 SCFS OF THE MATRIX UNDER PERFECT INTERFACE BONDING

Consider an E-Glass/LY556 UD composite used in WWFEs which is only subjected to a transverse tensile load, σ_{22}^0. The non-zero internal stresses by Bridging Model are easily found to be

$$\sigma_{11}^f = -0.082\sigma_{22}^0, \ \sigma_{22}^f = 1.342\sigma_{22}^0, \ \sigma_{11}^m = 0.134\sigma_{22}^0, \ \sigma_{22}^m = 0.442\sigma_{22}^0. \quad (9)$$

Under a transverse tension, a composite failure is attained firstly by a matrix failure. Thus, the transverse tensile strength of the composite is given by $\sigma_{22}^{u,t}$ =Y_m/0.422, where Y_m is the allowable transverse tensile stress of the matrix in the composite. A straightforward choice is to set Y_m=$\sigma_{u,t}^m$ =80MPa[6], where $\sigma_{u,t}^m$ is the original tensile strength of the pure matrix. If so, one gets $\sigma_{22}^{u,t}$ =181MPa, which is more than 5.2 times greater than *Y* (=35MPa[6]), the measured transverse tensile strength of the composite.

This is not a special case, but valid for almost every composite. In other words, the internal stresses by Eqs. (4) and (5) are homogenized quantities. They must be converted into "true" values before a failure assessment can be made against the original strengths of the fiber and matrix. As point-wise stresses of the fiber are uniform[7], its homogenized and true stresses are the same. The true stresses of the matrix can be obtained by multiplying its homogenized ones by respective SCFs. This is because a matrix plate with a hole generates an SCF when subjected to an in-plane tension. If the hole is filled with a fiber of different properties, an SCF occurs as well.

The most critical issue is that such an SCF can not be defined, following a classical approach, as a maximum point-wise stress of the matrix divided by the overall applied one. Otherwise, the resulting SCF would be infinite when there is a crack on the fiber and matrix interface, due to the fact that the point-wise stresses of the matrix at the crack tip are singular. As such, an averaged stress must be used for the new definition. A classical SCF of a plate with a hole is defined as a point-wise (something like zero-dimensional) stress divided by the overall applied quantity, which is in fact a surface-averaged (two-dimensional) stress, the new definition for an SCF of the matrix must be made by a line-averaged (one-dimensional) stress divided by a volume-averaged (three- dimensional) one, as the three is the maximum attainable in the denominator. The new definition for an SCF of the matrix subjected to a transverse load (tension or compression) is given by[2]

$$K_{22}(\varphi)=\frac{1}{\left|\vec{R}_{\varphi}^{b}-\vec{R}_{\varphi}^{a}\right|}\int_{\left|\vec{R}_{\varphi}^{a}\right|}^{\left|\vec{R}_{\varphi}^{b}\right|}\frac{\tilde{\sigma}_{22}^{m}}{(\sigma_{22}^{m})_{BM}}d\left|\vec{R}_{\varphi}\right| \tag{10}$$

where $\tilde{\sigma}_{22}^{m}$ is a point-wise stress of the matrix determined on a CCA (concentric cylinder assemblage, **Fig. 1** with $b\to\infty$) model along the loading direction, $(\sigma_{22}^{m})_{BM}$ is given by Bridging Model, φ is the inclined angle of the outward normal to the failure surface with an external load (**Fig. 1**), and $\vec{R}_{\varphi}^{a}$ and $\vec{R}_{\varphi}^{b}$ are the vectors of $\vec{R}_{\varphi}$ at the surfaces of the fiber and matrix cylinders within a RVE, respectively. The latter requirement implies

$$b=a/\sqrt{V_f} \tag{11}$$

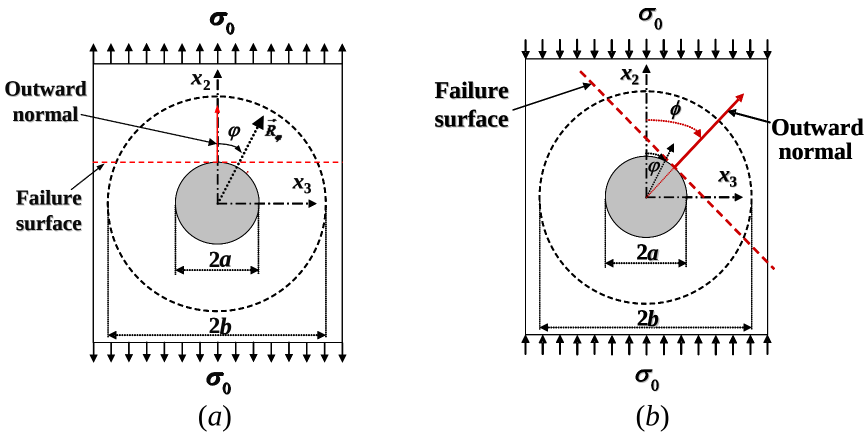


Figure 1: Schematic of a RVE used in defining SCF of matrix in a composite subjected to (*a*) a transverse tension, (*b*) a transverse compression

Explicit expressions for the SCFs of the matrix under a transverse tension, transverse compression, transverse shear, and a longitudinal shear are derived, respectively, as[1,2]

$$K_{22}^{t}=K_{22}(0)=\left[1+\frac{\sqrt{V_f}}{2}A+\frac{\sqrt{V_f}}{2}(3-V_f-\sqrt{V_f})B\right]\frac{(V_f+0.3V_m)E_{22}^{f}+0.7V_mE^{m}}{0.3E_{22}^{f}+0.7E^{m}}, \tag{12.1}$$

$$\begin{aligned}K_{22}^{c}=K_{22}(\phi)=&\left\{1-\frac{\sqrt{V_f}}{2}A\frac{\sigma_{u,c}^{m}-\sigma_{u,t}^{m}}{2\sigma_{u,c}^{m}}+\frac{B}{2(1-\sqrt{V_f})}\left[-V_f^{2}\left(1-2\left(\frac{\sigma_{u,c}^{m}-\sigma_{u,t}^{m}}{2\sigma_{u,c}^{m}}\right)^{2}\right)+\right.\right.\\&\left.\left.\frac{\left(\sigma_{u,c}^{m}+\sigma_{u,t}^{m}\right)V_f}{\sigma_{u,c}^{m}}\left(1+\frac{\sigma_{u,c}^{m}-\sigma_{u,t}^{m}}{\sigma_{u,c}^{m}}\right)-\sqrt{V_f}\left(\frac{\sigma_{u,c}^{m}-\sigma_{u,t}^{m}}{\sigma_{u,c}^{m}}+1-2\left(\frac{\sigma_{u,c}^{m}-\sigma_{u,t}^{m}}{2\sigma_{u,c}^{m}}\right)^{2}\right)\right]\right\}\times\\&\frac{(V_f+0.3V_m)E_{22}^{f}+0.7V_mE^{m}}{0.3E_{22}^{f}+0.7E^{m}},\end{aligned} \tag{12.2}$$

$$K_{23} = 2\sigma_{u,s}^{m}\sqrt{\frac{K_{22}^{t}K_{22}^{c}}{\sigma_{u,t}^{m}\sigma_{u,c}^{m}}}\,, \tag{12.3}$$

$$K_{12} = \left[1 - V_f\,\frac{G_{12}^{f} - G^{m}}{G_{12}^{f} + G^{m}}\{W(V_f) - \frac{1}{3}\}\right]\frac{(V_f + 0.3V_m)G_{12}^{f} + 0.7V_m G^{m}}{0.3G_{12}^{f} + 0.7G^{m}}\,, \tag{12.4}$$

$$A = \frac{2E_{22}^{f}E^{m}(\nu_{12}^{f})^2 + E_{11}^{f}\{E^{m}(\nu_{23}^{f} - 1) - E_{22}^{f}[2(\nu^{m})^2 + \nu^{m} - 1]\}}{E_{11}^{f}[E_{22}^{f} + E^{m}(1 - \nu_{23}^{f}) + E_{22}^{f}\nu^{m}] - 2E_{22}^{f}E^{m}(\nu_{12}^{f})^2}\,, \tag{13.1}$$

$$B = \frac{E^{m}(1 + \nu_{23}^{f}) - E_{22}^{f}(1 + \nu^{m})}{E_{22}^{f}[\nu^{m} + 4(\nu^{m})^2 - 3] - E^{m}(1 + \nu_{23}^{f})}\,, \tag{13.2}$$

$$W(V_f) = \int_0^a \frac{1}{a}\sqrt{1 - \frac{x_3^2}{a^2}}\sqrt{\frac{1}{V_f} - \frac{x_3^2}{a^2}}\,dx_3 \approx \pi\sqrt{V_f}\,[\frac{1}{4V_f} - \frac{4}{128} - \frac{2}{512}V_f - \frac{5}{4096}V_f^{\,2}]. \tag{13.3}$$

In the above, ν_{23}^{f} is the transverse Poisson's ratio of the fiber. $\sigma_{u,t}^{m}$, $\sigma_{u,c}^{m}$, and $\sigma_{u,s}^{m}$ are, respectively, the original tensile, compressive, and shear strengths of the matrix.

The true stresses of the matrix due to an external load applied on the composite are obtained through

$$\{K_i\sigma_i^{m}\}^T = \{\sigma_{11}^{m}, K_{22}\sigma_{22}^{m}, K_{33}\sigma_{33}^{m}, K_{23}\sigma_{23}^{m}, K_{12}\sigma_{13}^{m}, K_{12}\sigma_{12}^{m}\}\,, \tag{14}$$

where σ_{ij}^{m} are the homogenized stresses of the matrix obtained from Eqs. (5). Supposing that two transverse normal stress components, σ_{22}^{m} and σ_{33}^{m}, do not occur simultaneously in the matrix, as seen in most composites, the transverse SCFs, K_{22} and K_{33}, in Eqs. (14) are defined as

$$K_{22} = K_{33} = \begin{cases} K_{22}^{t}, if\ \sigma_{22}^{m}(or\ \sigma_{33}^{m}) \geq 0 \\ K_{22}^{c}, if\ \sigma_{22}^{m}(or\ \sigma_{33}^{m}) < 0 \end{cases}. \tag{15}$$

Original elastic and strength data of the 9 UD composites used in the three WWFEs[6] are listed in **Table 1**. From it, the SCFs of the matrices in the composites are evaluated as per Eqs. (12) and (13), and are shown in **Table 2**. Predicted uniaxial strengths of the 9 composites together with averaged relative error made upon Bridging Model are summarized in **Table 3**. Both the predictions with and without the SCFs incorporated are included. As can be seen from the table, with no SCFs, the overall averaged correlation error between the predicted and the measured transverse tensile, transverse compressive, transverse shear, and longitudinal shear strengths of the composites is 115.3%. Incorporation of the SCFs, that error is reduced to 22.7%, which is 5.08 times smaller! As most composite failures are resulted from matrix failures, SCFs of the matrix play a fatal role in analysis of a composite failure if only original constituent information is used.

	E-Glass LY556	E-Glass MY750	AS4 3501-6	T300 BSL914C	IM7 8511-7	T300 PR319	AS Epoxy	S2-Glass Epoxy	G400-800 5260
E_{11}^{f} (GPa)	80	74	225	230	276	230	231	87	290
E_{22}^{f} (GPa)	80	74	15	15	19	15	15	87	19
ν_{12}^{f}	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
G_{12}^{f} (GPa)	33.33	30.8	15	15	27	15	15	36.3	27
ν_{23}^{f}	0.2	0.2	0.07	0.07	0.36	0.07	0.07	0.2	0.357
E^{m}(GPa)	3.35	3.35	4.2	4	4.08	0.95	3.2	3.2	3.45
ν^{m}	0.35	0.35	0.34	0.35	0.38	0.35	0.35	0.35	0.35
V_f	0.62	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
$\sigma_{u,t}^{f}$ (MPa)	2150	2150	3350	2500	5180	2500	3500	2850	5860
$\sigma_{u,c}^{f}$ (MPa)	1450	1450	2500	2000	3200	2000	3000	2450	3200
$\sigma_{u,t}^{m}$ (MPa)	80	80	69	75	99	70	85	73	70
$\sigma_{u,c}^{m}$ (MPa)	120	120	250	150	130	130	120	120	130

$\sigma_{u,s}^{m}$ (MPa)	54	54	50	70	57	41	50	52	57
X (MPa)	1140	1280	1950	1500	2560	1378	1990	1700	2750
X' (MPa)	570	800	1480	900	1590	950	1500	1150	1700
Y (MPa)	35	40	48	27	73	40	38	63	75
Y' (MPa)	114	145	200	200	185	125	150	180	210
S_{23} (MPa)	50	50	55	_	57	45	50	40	57
S_{12} (MPa)	72	73	79	80	90	97	70	72	90

Table 1: Original properties of the 9 UD composites used in WWFEs[6]

	E-Glass LY556	E-Glass MY750	AS4 3501-6	T300 BSL914C	IM7 8511-7	T300 PR319	AS Epoxy	S2-Glass Epoxy	G400-800 5260
K_{22}^{t}	3.339	3.253	2.098	2.143	2.327	3.123	2.339	3.317	2.464
K_{22}^{c}	2.249	2.181	1.469	1.57	1.761	2.035	1.743	2.172	1.732
K_{23}	3.02	2.936	1.337	2.421	2.034	2.167	1.999	2.982	2.469
K_{12}	1.52	1.491	1.424	1.43	1.475	1.51	1.449	1.5	1.483

Table 2: SCFs of the matrices in the 9 UD composites with perfect interface bonding

	E-Glass LY556	E-Glass MY750	AS4 3501-6	T300 BSL914C	IM7 8551-7	T300 PR319	AS carbon Epoxy	S2-Glass Epoxy
$\sigma_{11}^{u,t}$ (MPa)	1367	1329	2035	1517	3139	1504	2119	1752
$\sigma_{11}^{u,c}$ (MPa)	922	896	1519	1214	1939	1203	1817	1506
With SCFs of perfect interfaces								
$\sigma_{22}^{u,t}$ (MPa)	54.2	54.3	52.9	57.1	73.7	48	63.1	49.3
$\sigma_{22}^{u,c}$ (MPa)	120.7	121.6	273.9	156	127.9	136.9	119.5	123.9
σ_{23}^{u} (MPa)	40.5	40.6	60.2	47.2	48.5	40.5	43.4	39.1
σ_{12}^{u} (MPa)	81	80.7	70.5	99.2	84	62.3	72	78.3
Without any SCFs								
$\sigma_{22}^{u,t}$ (MPa)	181	176.5	111.1	122	171.8	149.8	147.7	163.7
$\sigma_{22}^{u,c}$ (MPa)	271.6	265.1	402.6	245	225.5	277.9	207.9	268.9
σ_{23}^{u} (MPa)	122.3	119.4	80.7	114	99	87.9	86.8	116.5
σ_{12}^{u} (MPa)	123.1	120.2	100.1	141.9	123.5	94.1	104.4	117.5

Table 3: Predicted uniaxial strengths and relative errors for the 9 UD composites

4 TRANSVERSE TENSILE SCF WITH A DEBONDED INTERFACE

Table 3 indicates that the averaged relative error in prediction of transverse tensile strengths of the composites is 39.3%, much greater than the overall averaged error, 22.7%. This was mainly attributed to the interface cracks in some of the composite systems[8], which significantly reduced the overall load carrying ability of the composites. Thus, a transverse tensile SCF of the matrix after an interface crack needs to be derived.

Suppose that a stable crack with a central angle of 2ψ occurs on the fiber and matrix interface of a CCA model under a transverse tension, as shown in **Fig. 2(a)**. Toya obtained the stress fields in the fiber and matrix[9], and the stress component of the matrix in the loading direction is given by[9]

$$\tilde{\sigma}_{22}^{m} = \sigma_{22}^{0}\,\mathrm{Re}[(\frac{a^2}{z^2} - \bar{z})M'(z) - \frac{a^2}{z^2} \times (\overline{M}(\frac{a^2}{z}) + M(z)) + 2M(z)]/2\,, \tag{16.1}$$

$$M(z) = A_1 - \frac{k}{z^2} - ((A_1 - 0.5)z + B_1 + \frac{C}{z} + \frac{D}{z^2})\chi(z)\,, \tag{16.2}$$

$$A_1=\frac{1-(\cos\psi+2\lambda\sin\psi)\exp[2\lambda(\pi-\psi)]+(1-k)(1+4\lambda^2)\sin^2\psi}{\frac{4}{k}-2-2(\cos\psi+2\lambda\sin\psi)\exp[2\lambda(\pi-\psi)]}, \quad (16.3)$$

$$B_1=a(\cos\psi+2\lambda\sin\psi)(0.5-A_1), \quad (16.4)$$

$$C=(k-1)(\cos\psi-2\lambda\sin\psi)a^2\exp[2\lambda(\psi-\pi)], \quad (16.5)$$

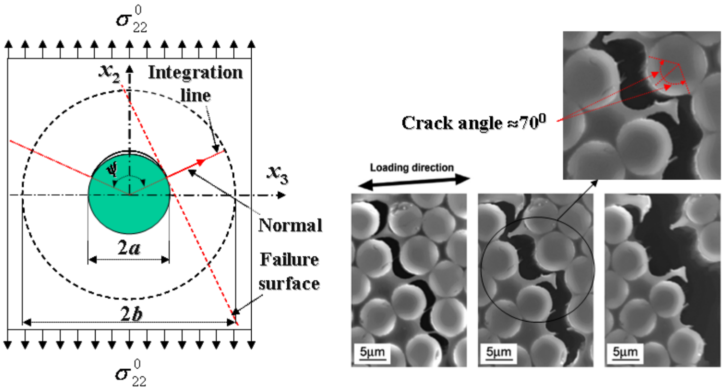
$$D=(1-k)a^3\exp[2\lambda(\psi-\pi)], \quad (16.6)$$

$$\chi(z)=(z-ae^{i\psi})^{-0.5+i\lambda}(z-ae^{-i\psi})^{-0.5-i\lambda}, \quad (16.7)$$

$$\kappa_1=\frac{3-\nu^m}{1+\nu^m},\ \kappa_2=\frac{3-\nu^f}{1+\nu^f},\ \mu_1=\frac{E^m(\kappa_1-1)\nu^m}{[1-(\nu_m)^2](3-\kappa_1)},\ \mu_2=\frac{E^f(\kappa_2-1)\nu^f}{[1-(\nu^f)^2](3-\kappa_2)}, \quad (16.8)$$

$$k=\frac{\mu_1(1+\kappa_2)}{(1+\nu)(\mu_1+\kappa_1\mu_2)},\ \xi=\frac{\mu_2+\kappa_2\mu_1}{\mu_1+\kappa_1\mu_2},\ \lambda=-\frac{\ln(\xi)}{2\pi}. \quad (16.9)$$

$z=x_2+ix_3$ is a complex number, $M'=dM/dz$, and a variable with "–" on head represents its conjugate.



(a) (b)

Figure 2: (a) Schematic failure of a transverse tensile-loaded composite with an interface crack, (b) failure locus of a composite after interface cracks[8]

Toya's solution is valid only for a plane stress state problem with an isotropic assumption for both the fiber and the matrix. In the case of a composite, a plane strain condition is applicable and a transversely isotropic material has to be considered. When such a case occurs, the strain-stress relationships of the material become

$$\tilde{\varepsilon}_{22}=\frac{1-\nu_{12}\nu_{21}}{E_{22}}\tilde{\sigma}_{22}-\frac{\nu_{32}-\nu_{12}\nu_{21}}{E_{33}}\tilde{\sigma}_{33} \text{ and } \tilde{\varepsilon}_{33}=\frac{1-\nu_{13}\nu_{31}}{E_{33}}\tilde{\sigma}_{33}-\frac{\nu_{23}-\nu_{12}\nu_{21}}{E_{22}}\tilde{\sigma}_{22}.$$

Compared with those of an isotropic material under a plane stress condition, i.e.,

$$\tilde{\varepsilon}_{22}=\frac{1}{E}(\tilde{\sigma}_{22}-\nu\tilde{\sigma}_{33}) \text{ and } \tilde{\varepsilon}_{33}=\frac{1}{E}(\tilde{\sigma}_{33}-\nu\tilde{\sigma}_{22}),$$

it is seen that only the elastic modulus and Poisson's ratio need to be replaced by

$$E=\frac{E_{22}}{1-\nu_{12}\nu_{21}} \text{ and } \nu=\frac{\nu_{23}+\nu_{12}\nu_{21}}{1-\nu_{12}\nu_{21}},$$

respectively. It is noted $E_{33}=E_{22}$ due to the transverse isotropy. In other words, the Toya's solution will be also applicable to a plane strain state problem with a transversely isotropic fiber, as long as Eqs. (16.8) are changed, respectively, to

$$\kappa_1=3-4\nu^m,\ \kappa_2=\frac{3-\nu_{23}^f-4\nu_{12}^f\nu_{21}^f}{1+\nu_{23}^f},\ \mu_1=\frac{E^m}{2(1+\nu^m)},\ \mu_2=\frac{E_{22}^f}{2(1+\nu_{23}^f)}. \quad (17)$$

Substituting Eqs. (16) together with Eqs. (11), (17), and $(\sigma_{22}^m)_{BM}$ into Eq. (10) results in

$$\hat{K}_{22}^t(\varphi)=\mathrm{Re}\{[e^{-i\psi}(\frac{a^2}{b}-b)(A_1-\frac{a^2k}{b^2}e^{-2i\psi}-((A_1-0.5)be^{i\psi}+B_1+\frac{C}{b}e^{-i\psi}+$$

$$\frac{D}{b^2}e^{-2i\psi})\chi(be^{i\psi}))-[\overline{N}_1\left(\frac{a^2}{\overline{a}'}\right)-\overline{N}_1\left(\frac{a^2}{\overline{b}'}\right)]-e^{-2i\psi}(N(a')-N(b'))]e^{-i\psi}$$

$$+2\operatorname{Re}[(N(b')-N(a'))e^{-i\psi}]\}\left[\frac{(V_f+0.3V_m)E_{22}^f+0.7V_mE^m}{2(b-a)(0.3E_{22}^f+0.7E^m)}\right], \tag{18.1}$$

$$N(z)=A_1z+\frac{a^2k}{z}-(z-ae^{i\psi})^{0.5+i\lambda}(z-ae^{-i\psi})^{0.5-i\lambda}[(A_1-0.5)-\frac{D}{a^2z}]\ , \tag{18.2}$$

$$N_1(z)=A_1z+\frac{a^2k}{z}+\frac{1}{v}(z-ae^{i\psi})^{0.5+i\lambda}(z-ae^{-i\psi})^{0.5-i\lambda}[(A_1-0.5)-\frac{D}{a^2z}], \tag{18.3}$$

where $a'=a(\cos\varphi+i\sin\varphi)$ and $b'=b(\cos\varphi+i\sin\varphi)$.

It remains a question how to assign the angle φ in Eq. (18.1) so that the transverse tensile SCF, denoted by $\hat{K}_{22}^t$, can be determined. As a transverse plane is isotropic, it is likely that a tensile load in this plane would result in a failure occurring along a direction where the maximum averaged stress is attained. In other words, $\hat{K}_{22}^t$ should correspond to the maximum of $\hat{K}_{22}^t(\varphi)$, which is recognized to be

$$\hat{K}_{22}^t=\hat{K}_{22}^t(\psi)=\max\{\hat{K}_{22}^t(\varphi),0^0\le\varphi\le 90^0\}. \tag{19}$$

Indeed, Hobbiebrunken et al.[8] have shown that the failure surface of the composite after an interface debonding was initiated from the crack end, as seen in **Fig. 2(b)**, which is consistent with the present choice for the integration line.

Finally, let us consider determination of the crack angle ψ. Due to Poisson's deformation of the matrix, there is a maximum compression applied on the fiber along the axis perpendicular to the tensile load. The crack angle ψ must be smaller than $\pi/2$. At the crack tip, the relative displacement between the fiber and the matrix along the radial direction must be zero. However, both England and Toya pointed out that the relative displacement at another point of the interface with a smaller central angle, $2\varphi=2(\psi-\gamma)$, was also zero[10,9]. According to Toya's solution, the last condition is expressed as

$$\operatorname{Re}\left[\left(G_0-\frac{1}{k}-\frac{2(1-k)}{k\exp(i\varphi)}\exp(2\lambda(\psi-\pi))\right)R(\exp(i\varphi))\right]_{\varphi=\psi-\gamma}=0, \tag{20}$$

$$G_0=\frac{1-(\cos\psi+2\lambda\sin\psi)\exp[2\lambda(\pi-\psi)]+(1-k)(1+4\lambda^2)\sin^2\psi}{2-k-k(\cos\psi+2\lambda\sin\psi)\exp[2\lambda(\pi-\psi)]}, \tag{21.1}$$

$$R(\exp(i\varphi))=[\exp(i(\varphi))-e^{i\psi}]^{0.5+i\lambda}[\exp(i(\varphi))-e^{-i\psi}]^{0.5-i\lambda}\exp(-i(\varphi)). \tag{21.2}$$

Substituting Eqs. (21.1) and (21.2) into Eq. (20), one gets

$$\operatorname{Re}\left\{\sqrt{\sin(\frac{\psi-\varphi}{2})\sin(\frac{\psi+\varphi}{2})}\left[G_0-\frac{1}{k}-\frac{2(1-k)}{k\exp(i\varphi)}\exp(2\lambda(\psi-\pi))\right]\right.$$

$$\left.\exp[\lambda(\pi-\psi)-i0.5\varphi+i\lambda\ln\big(\sin(0.5(\psi-\varphi))/\sin(0.5(\psi+\varphi))\big)]\right\}_{\varphi=\psi-\gamma}=0. \tag{22}$$

Following England[10] and Toya[9], the solution to Eq. (22) can be transformed into an equation for a phase angle to simplify the analysis. The solution to Eq. (22) is equivalent to that to the following condition (with $\varphi=\psi-\gamma$)

$$\tan^{-1}\left(\frac{2(1-k)\xi\exp(2\lambda\psi)\sin(\varphi)}{kG_0-1-2(1-k)\xi\exp(2\lambda\psi)\cos(\varphi)}\right)+\lambda\ln\left(\frac{\sin(0.5(\psi-\varphi))}{\sin(0.5(\psi+\varphi))}\right)-\frac{\varphi}{2}=\pm\frac{\pi}{2}. \tag{23}$$

Let us consider different values attained by ξ separately. When $\xi<1$, we have

$$k=\frac{\mu_1(1+\kappa_2)}{(1+\nu)(\mu_1+\kappa_1\mu_2)}=\frac{\mu_1+\kappa_2\mu_1}{\mu_1+\kappa_2\mu_1+\mu_2+\kappa_1\mu_2}<1,\ 2(1-k)\xi\exp(2\lambda\psi)>0\ ,$$

and $k^{-1}>1>0.5(1+4\lambda^2)\sin^2\psi$. It follows that

$$kG_0=\frac{1-(\cos\psi+2\lambda\sin\psi)\exp[2\lambda(\pi-\psi)]+(1-k)(1+4\lambda^2)\sin^2\psi}{2/k-1-(\cos\psi+2\lambda\sin\psi)\exp[2\lambda(\pi-\psi)]}<1.$$

This implies that Re(g)>0 and Im(g)<0, where g= $G_0 - \frac{1}{k} - \frac{2(1-k)}{k\exp(i\varphi)}\exp(2\lambda(\psi-\pi))$. In other words, we have

$$\tan^{-1}\left(\frac{2(1-k)\xi\exp(2\lambda\psi)\sin(\varphi)}{kG_0-1-2(1-k)\xi\exp(2\lambda\psi)\cos(\varphi)}\right)<0. \tag{24}$$

Furthermore $\lambda\ln\left(\frac{\sin(0.5(\psi-\varphi))}{\sin(0.5(\psi+\varphi))}\right)<0,$ (25)

since $\lambda>0$. In light of Eqs (24) and (25), the right hand side of Eq. (23) should take -0.5π. Further simplification on Eq. (23) leads to

$$f(\gamma)=\exp\left\{\frac{1}{\lambda}\left[\frac{\psi-\gamma}{2}+\tan^{-1}\left(\frac{kG_0-1-2(1-k)\xi\exp(2\lambda\psi)\cos(\psi-\lambda)}{2(1-k)\xi\exp(2\lambda\psi)\sin(\psi-\lambda)}\right)\right]\right\}-\frac{\gamma}{2\sin(\psi)}=0. \tag{26}$$

In the last expression, the condition of $\sin(0.5\gamma)\approx0.5\gamma$ and $\sin(\psi-0.5\gamma)\approx\sin(\psi)$ has been used. As Eq. (26) or $\gamma f(\gamma)=0$ (since $\gamma>0$) represents the fact that the relative displacement of the fiber and matrix on the interface along a radial direction attains the minimum locally, it is required that

$$\frac{d(\gamma f(\gamma))}{d\gamma}=\left\{\gamma\exp\left[\frac{1}{\lambda}\left(\tan^{-1}(J_1/J_2)+\frac{\psi}{2}\right)\right]\frac{1}{\lambda}\left[\frac{J_3}{J_1^2+J_2^2}-\frac{1}{2}\right]\right.$$
$$\left.+\exp\left[\frac{1}{\lambda}\left(\tan^{-1}(J_1/J_2)+\frac{\psi}{2}\right)\right]-\frac{\gamma}{\sin(\psi)}\right\}=0, \tag{27}$$

where $J_1 = kG_0 - 1 - 2(1-k)\xi\exp(2\lambda\psi)\cos(\psi)$, (28.1)

$J_2 = 2(1-k)\xi\exp(2\lambda\psi)\sin(\psi)$, (28.2)

$J_3 = 2(1-k)\xi\exp(2\lambda\psi)[J_1\cos(\psi) - J_2\sin(\psi)]/J_2$. (28.3)

From Eq. (27), one obtains

$$\gamma=\frac{\lambda\exp\left[\frac{1}{\lambda}\left(\tan^{-1}(J_1/J_2)+\frac{\psi}{2}\right)\right]}{\exp\left[\frac{1}{\lambda}\left(\tan^{-1}(J_1/J_2)+\frac{\psi}{2}\right)\right]\left[\frac{1}{2}-\frac{J_3}{J_1^2+J_2^2}\right]+\frac{\lambda}{\sin(\psi)}}\approx\frac{2\lambda(J_1^2+J_2^2)}{J_1^2+J_2^2-2J_2J_3}, \tag{29}$$

because $\lambda << \exp\{\frac{1}{2\lambda}[2\tan^{-1}(J_1/J_2)+\psi]\}\sin(\psi)$.

If $\xi>1$, it follows that $\lambda=-ln(\xi)/(2\pi)<0$. Thus, $\gamma<0$. Physically, $\varphi=\psi-\gamma>\psi$, meaning that an oscillating point occurs outside the crack tip. In such a case, the condition that $\gamma f(\gamma)$ attains a minimum is no longer applicable. However, the function $f(\gamma)$ itself should assign a minimum. In other words, one has

$$\frac{f'(\gamma)}{\sin(\psi)}=\frac{1}{\gamma}\exp\left(\frac{1}{\lambda}\left[\frac{\psi-\gamma}{2}+\tan^{-1}(J_1/J_2)\right]\right)\frac{1}{\lambda}\left(-\frac{1}{2}+\frac{J_3J_2}{J_1^2+J_2^2}\right)$$
$$-\frac{1}{\gamma^2}\exp\left(\frac{1}{\lambda}\left[\frac{\psi-\gamma}{2}+\tan^{-1}(J_1/J_2)\right]\right)=0, \tag{30}$$

which leads to

$$\gamma=-\frac{2\lambda(J_1^2+J_2^2)}{J_1^2+J_2^2-2J_2J_3}. \tag{31}$$

Finally, $\xi=1$ implies $\lambda=0$. Both Eq. (29) and Eq. (31) give $\gamma=0$. Substituting it into Eq. (20) results in none unique solution for the crack angle ψ. Hence, $\xi=1$ corresponds to a singular crack. However, any practical measurement for the fiber and matrix properties involves in a deviation. Adjusting a fiber or matrix property so that $\xi\neq1$ can leads to a unique solution for the angle ψ.

Substituting Eq. (29) or (31) into Eq. (20), a crack angle is obtained, and the transverse tensile SCF of the matrix with the debonded interface is determined from Eq. (19).

5 INTERFACE DEBONDING DETERMINATION

Let a UD composite be subjected to a transverse tensile load, σ_{22}^{0}, up to an ultimate failure. The measured transverse tensile strength of the composite is Y. Suppose that the fiber/matrix interface of the composite is initially bonded perfectly. When the load is increased to a critical level, e.g. $\hat{\sigma}_{22}^{0}$, a stable crack with a central angle of 2ψ occurs on the interface. Many reports have pointed out that an unstable propagation from an initial interface crack to the last stable angle is short[8], with no significant change in the applied load. Thus, we can safely assume that at a transverse load level smaller than $\hat{\sigma}_{22}^{0}$ the interface is in perfect bonding.

By Bridging Model, the transverse stress in the matrix when the crack occurs reads

$$\hat{\sigma}_{22}^{m} = \frac{0.3E_{22}^{f} + 0.7E^{m}}{(V_f + 0.3V_m)E_{22}^{f} + 0.7V_m E^{m}} \hat{\sigma}_{22}^{0} . \tag{32}$$

Further, the longitudinal stress of the matrix at the critical load level is obtained as

$$\hat{\sigma}_{11}^{m} = \frac{V_f A_{12}}{(V_f + V_m A_{11})(V_f + V_m A_{22})} \hat{\sigma}_{22}^{0} . \tag{33}$$

No other stress in the matrix exists. Supposing that the transverse matrix stress corresponding to the composite failure is denoted by $\tilde{\sigma}_{22}^{m}$, one has

$$\hat{K}_{22}^{t}(\tilde{\sigma}_{22}^{m} - \hat{\sigma}_{22}^{m}) + K_{22}^{t}\hat{\sigma}_{22}^{m} = \sigma_{u,t}^{m} , \tag{34.1}$$

where $$\tilde{\sigma}_{22}^{m} - \hat{\sigma}_{22}^{m} = \frac{0.3E_{22}^{f} + 0.7E^{m}}{(V_f + 0.3V_m)E_{22}^{f} + 0.7V_m E^{m}} (Y - \hat{\sigma}_{22}^{0}) . \tag{34.2}$$

From Eqs. (32)-(34), the critical transverse tensile load is found to be

$$\hat{\sigma}_{22}^{0} = \frac{\hat{K}_{22}^{t} Y}{\hat{K}_{22}^{t} - K_{22}^{t}} - \frac{(V_f + 0.3V_m)E_{22}^{f} + 0.7V_m E^{m}}{(0.3E_{22}^{f} + 0.7E^{m})(\hat{K}_{22}^{t} - K_{22}^{t})} \sigma_{u,t}^{m} . \tag{35}$$

If it is near to, equal to, or greater than the transverse tensile strength, Y, the fiber and matrix system is said to have a perfect interface bonding up to failure. No interface modification is necessary. Otherwise, the system will undergo an earlier interface crack and further such modification is preferred.

Which quantity should be chosen to assess an interface crack of the composite at any other load condition? It is likely that a critical Mises stress of the matrix would be most suitable, which is evaluated through

$$\hat{\sigma}_{e}^{m} = \sqrt{(\hat{\sigma}_{11}^{m})^2 + (K_{22}^{t}\hat{\sigma}_{22}^{m})^2 - K_{22}^{t}\hat{\sigma}_{11}^{m}\hat{\sigma}_{22}^{m}} . \tag{36}$$

When any other composite made of the same fiber/matrix system is subjected to an arbitrary but planar load, a Mises true stress of the matrix at a load step is determined through

$$(\bar{\sigma}_{e}^{m})_l = \sqrt{(\bar{\sigma}_{11}^{m})_l^2 + (\bar{\sigma}_{22}^{m})_l^2 - (\bar{\sigma}_{11}^{m})_l(\bar{\sigma}_{22}^{m})_l + 3(\bar{\sigma}_{12}^{m})_l^2} , \tag{37}$$

$$(\bar{\sigma}_{11}^{m})_l = (\bar{\sigma}_{11}^{m})_{l-1} + d\sigma_{11}^{m} , \tag{38.1}$$

$$(\bar{\sigma}_{22}^{m})_l = (\bar{\sigma}_{22}^{m})_{l-1} + K_{22}^{eq} d\sigma_{22}^{m} , \tag{38.2}$$

$$(\bar{\sigma}_{12}^{m})_l = (\bar{\sigma}_{12}^{m})_{l-1} + K_{12} d\sigma_{12}^{m} , \tag{38.3}$$

$$K_{22}^{eq} = \begin{cases} K_{22}^{t}, if\ d\sigma_{22}^{m} > 0\ and\ (\sigma_{e}^{m})_{l-1} < \hat{\sigma}_{e}^{m} \\ \hat{K}_{22}^{t}, if\ d\sigma_{22}^{m} > 0\ and\ (\sigma_{e}^{m})_{l-1} \geq \hat{\sigma}_{e}^{m} \\ K_{22}^{c}, if\ d\sigma_{22}^{m} < 0 \end{cases} . \tag{39}$$

l is a load step, $\{ d\sigma_{11}^{m}, d\sigma_{22}^{m}, d\sigma_{12}^{m} \}$ are the stress increments in the matrix due to the l^{-th} load increment applied on the composite calculated by Bridging Model. An interface crack occurs if and only if

$$(\bar{\sigma}_{m}^{1})_l > 0 \text{ and } (\bar{\sigma}_{e}^{m})_l \geq \hat{\sigma}_{e}^{m} , \tag{40}$$

where $\bar{\sigma}_{m}^{1}$ is the first principal true stress of the matrix. Furthermore, by virtue of Tsai-Wu's criterion, a matrix failure is govern by

$$F_1[(\sigma_{11}^m)_l^2+(\sigma_{22}^m)_l^2-(\sigma_{11}^m)_l(\sigma_{22}^m)_l]+F_2(\sigma_{12}^m)_l^2+F_3[(\sigma_{11}^m)_l+(\sigma_{22}^m)_l]\geq 1, \quad (41.1)$$

$$F_1=1/(\sigma_{u,t}^m\sigma_{u,c}^m),\ F_2=1/(\sigma_{u,s}^m)^2,\ F_3=1/\sigma_{u,t}^m-1/\sigma_{u,c}^m. \quad (41.2)$$

On the other hand, a fiber failure is detected using a generalized maximum normal stress failure criterion[3]. Namely, a fiber tensile failure is attained if

$$(\sigma_{eq,t}^f)_l\geq\sigma_{u,t}^f, \quad (42.1)$$

$$(\sigma_{eq,t}^f)_l=\begin{cases}(\sigma_f^1)_l, & if\quad (\sigma_f^3)_l<0,\\ [(\sigma_f^1)_l^3+(\sigma_f^2)_l^3]^{\frac{1}{3}}, & if\quad (\sigma_f^3)_l=0\end{cases} \quad (42.2)$$

A fiber compressive failure is assumed when

$$(\sigma_{eq,c}^f)_l\geq-\sigma_{u,c}^f, \quad (43.1)$$

$$(\sigma_{eq,c}^f)_l=\begin{cases}(\sigma_f^3)_l, & if\quad (\sigma_f^1)_l>0,\\ (\sigma_f^3)_l-(\sigma_f^1)_l & if\quad (\sigma_f^1)_l\leq 0\end{cases} \quad (43.2)$$

In the above, $(\sigma_f^1)_l$, $(\sigma_f^2)_l$ and $(\sigma_f^3)_l$ ($\sigma_f^1\geq\sigma_f^2\geq\sigma_f^3$) are the current three principal stresses of the fiber evaluated from

$$(\sigma_{11}^f)_l=(\sigma_{11}^f)_{l-1}+d\sigma_{11}^f,\ (\sigma_{22}^f)_l=(\sigma_{22}^f)_{l-1}+d\sigma_{22}^f,\ (\sigma_{12}^f)_l=(\sigma_{12}^f)_{l-1}+d\sigma_{12}^f. \quad (44)$$

6 IllUSTRATION

The interface crack angle, ψ, the transverse tensile SCF of the matrix after the crack, $\hat{K}_{22}^t$, and the critical transverse tensile load, $\hat{\sigma}_{22}^0$, of each of the 9 UD composites from **Table 1** are calculated, and are shown in **Table 4**. The interface crack angles of all the 9 composites were close to each other, nearly equal to 72^0. For every composite, $\hat{K}_{22}^t$ is much higher than K_{22}^t, as shown in **Tables 3** and **4**, explaining why the load carrying ability of a composite after an interface crack is decreased significantly.

	E-Glass LY556	E-Glass MY750	AS4 3501-6	T300 BSL914C	IM7 8511-7	T300 PR319	AS Epoxy	S2-Glass Epoxy	G400-800 5260
ψ (^{0})	70.4	70.5	73.5	73.5	72.8	70.8	72.8	70.4	72.0
$\hat{K}_{22}^t$	8.47	7.94	5.46	5.57	5.99	7.68	6.00	8.08	6.24
$\hat{\sigma}_{22}^0$ (MPa)	22.50	30.08	37.65	4.10	60.96	27.59	15.81	61.43	78.89

Table 4: Transverse tensile SCFs of the matrices in the 9 UD composites after interface crack

Two UD composites, Kevlar-49 fiber/epoxy and E-glass fiber/8804 epoxy systems, were subjected to off-axial tensile loads up to failures. Constituent properties and transverse tensile strengths of the two composites together with fiber volume fractions were provided in [11-13], respectively, and are listed in **Table 5**. Using them, the calculated SCFs of the matrices and the critical transverse tensile loads of the composites are shown in **Table 6**. It is seen that at initial perfect bonding, the SCFs of the Kevlar system are close to 1. This is because the transverse modulus of the Kevlar fiber is comparable to that of the matrix. Nevertheless, the transverse tensile SCF of the matrix in the Kevlar system after the interface debonding is still significantly higher than that with a perfect interface bonding. As both the critical loads are smaller than the corresponding transverse tensile strengths, the two composites will undergo an interface debonding. However, the Kevlar system will be debonded much earlier.

		E_{11}(GPa)	E_{22}(Gpa)	ν_{12}	G_{12}(Gpa)	ν_{23}	$\sigma_{u,t}$(MPa)	$\sigma_{u,c}$(MPa)	Y(MPa)
Kevlar-49/epoxy V_f=0.55	Fiber	124.1	4.1	0.35	2.9	0.35	2060	-	27.7
	Matrix	3.45	3.45	0.35	1.28	0.35	69	120	

E-glass/8804 V_f=0.51	Fiber	71	71	0.26	28.2	0.26	1500	-	45.3
	Matrix	3.1	3.1	0.29	1.12	0.29	70	86	

Table 5: Original properties of two material systems[11-13]

	K_{22}^{t}	K_{12}	K_{22}^{c}	$\hat{K}_{22}^{t}$	$\hat{\sigma}_{22}^{0}$ (MPa)
E-glass/8804	2.98	1.38	2.02	6.01	41.6
Kevlar-49/epoxy	1.13	1.17	1.06	2.79	1.7

Table 6: SCFs and critical transverse tensile loads

The predicted and measured ff-axial tensile strengths of the two composites are plotted in **Figs. 3** and **4**, respectively. In order to display the predicted results at most off-axial angles more clearly, the predictions at angles smaller than 10^0 are not shown. Three kinds of predictions have been made. One is done with a perfect interface bonding assumption, another is without any SCFs of the matrix taken into account, and the third is incorporated with the interface debonding. As expected, the predictions without any SCFs are far away from the experiments, whereas those with the interface debonding incorporated agree the best with the measured data. The perfect bonding assumption for both of the composites results in the predictions lied in between the other two kinds of predictions.

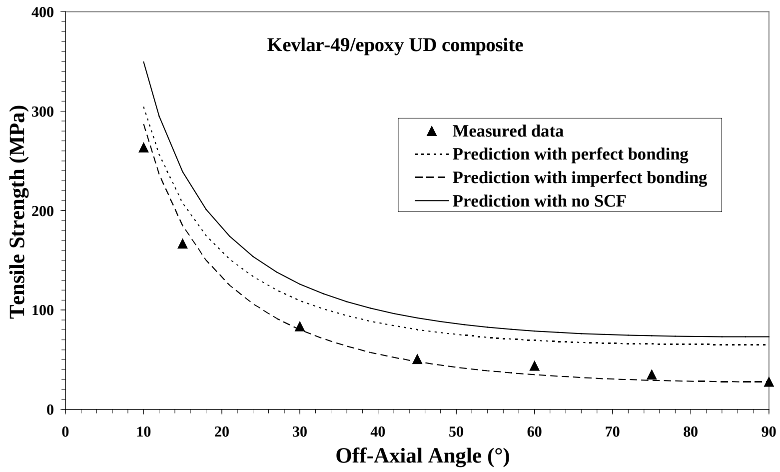


Figure 3: Comparison of different schemes' predictions with experiments[14] (Pindera et al., 1986) for off-axial tensile strengths of a Kevlar-49/epoxy UD composite

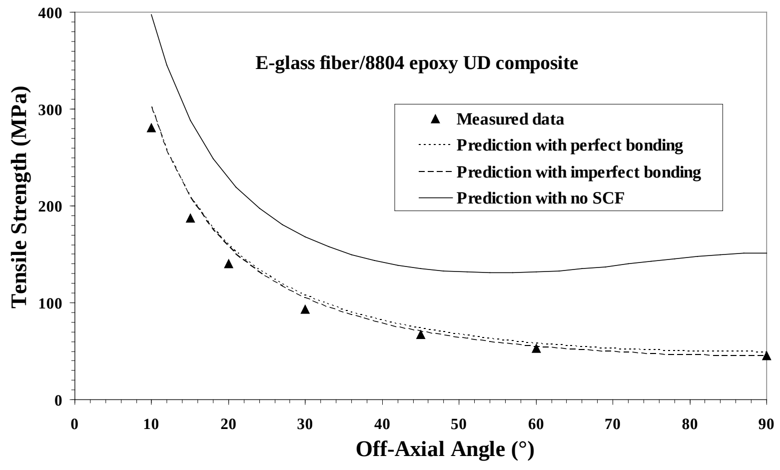


Fig. 4. Comparison of different schemes' predictions with experiments[12] for off-axial tensile strengths of a E-glass/8804 UD composite

7 CONCLUSION

The SCF of the matrix in a composite with a debonded fiber/matrix interface has been derived in the paper. A straightforward result of this work is in correlating an interface debonding with an arbitrary load applied on the composite. Only a transverse tensile test on a UD composite made from the same constituents is necessary, in addition to the original mechanical properties of the fiber and matrix. More other benefits can be gained from this work in failure analysis of a composite. For instance, a prediction accuracy for a composite strength with a less perfectly bonded interface can be much more increased in accuracy only using the original constituent information.

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