Damage Estimation in Nonlinear Laminates Subjected to a Transverse Concentrated Load

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ABSTRACT

A damage growth problem of nonlinear laminated plates subjected to a transverse concentrated load is analytically solved to estimate the significance of damage due to low velocity foreign object impact. The damaged portion is modelled as equally spaced multiply delaminated plates. A superposition technique is used to derive an approximated relation between the load and deflection. The first problem is a basic one, that is, a slightly nonlinear response of an intact global plate without damage pushed by a concentrated load at its center. The second problem is that of a nonlinear response of piled circular plates with the average radius and the thickness of the delaminated ligaments loaded by an equivalent force to the reduced bending stiffness due to the introduction of the damage. The periphery of the delaminated plate is assumed fixed, considering that the bending stiffness of the intact portion is much higher than the delaminated portion. Some nonlinear effect of the first problem is considered by introducing initial inplane stress caused by an equivalent relative displacement of the damaged portion of the first problem. The results are compared with finite element solutions and showed excellent agreement.

1 INTRODUCTION

Composite laminates having weak interfaces compared to their superior inplane performances are vulnerable to damage due to local bending when subjected to transverse impact and transverse concentrated loads [1, 2]. The damage causes significan**t** compressive strength reduction even when the damage is in a barely visible state (CAI) [3-6]. There are a number of numerical and/or experimental works to study impact damage problems of laminated composites due to its importance in the design of aeronautical structures [6-12], such as the relation between the damage and impact energy (or the impact force), scale effects, interlaminar toughness, stacking sequence, coupling between the delamination and matrix cracks, etc.

It is very helpful to estimate the various effects of material and structural parameters on impact damage problems of the composite laminates by some closed form expression. However, a limited number of such analytical works on the topic have been reported due to the geometrical complexity and its nonlinear nature. Suemasu and Majima obtained a closed form solution on the delamination propagation and quasi-static concentrated force for the linear problem [13] and Rayleigh-Ritz approximated solution for the nonlinear problem [14]. It is shown that large nonlinear effect must be considered to predict the impact damage. Olsson [15] obtained an analytical expression by separately considering bending and membrane components based on a similar idea to the present analysis. Suemasu et al. proposed a simple mathematical expression to estimate the significance of the impact damage in terms of impact load and impact energy [16]

A simple and more accurate form of the energy release rate is given for nonlinear plates in terms of applied force, damage size and various geometrical and material parameters in the present paper. Then, the solution is compared with finite element solutions to demonstrate the applicability of the present theory to the real problem.

2 ANALYSYS

Low velocity and large mass impact response may be replaced by a quasi-static concentrated load problem [16]. The load-displacement history of the impact damaged plates can be well expressed by that of the plate with an equivalent number of equally spaced multiple circular delaminations. In the present paper we consider circular quasi-isotropic laminates of radius R and thickness h fixed at their boundary. When the plate with the N-1 multiple circular delaminations of radius a is loaded at its center as shown in Figure 1(a), the delaminated portion deforms significantly and shows very large nonlinearity and the deflection of the intact portion is usually small and slight nonlinearity occurs. Introduction of N-1 multiple delaminations makes the damaged portion into N equal thickness ligaments and the bending stiffness reduce to $1/N^2$. But membrane stiffness reduction by introduction of the multiple delaminations is negligible. A concentrated load P is applied at its center as shown in Figure 1(a). The approximated response can be given by superposing three problems (b), (c) and (d). The sum of the applied load of three problems is same as that of problem (a).

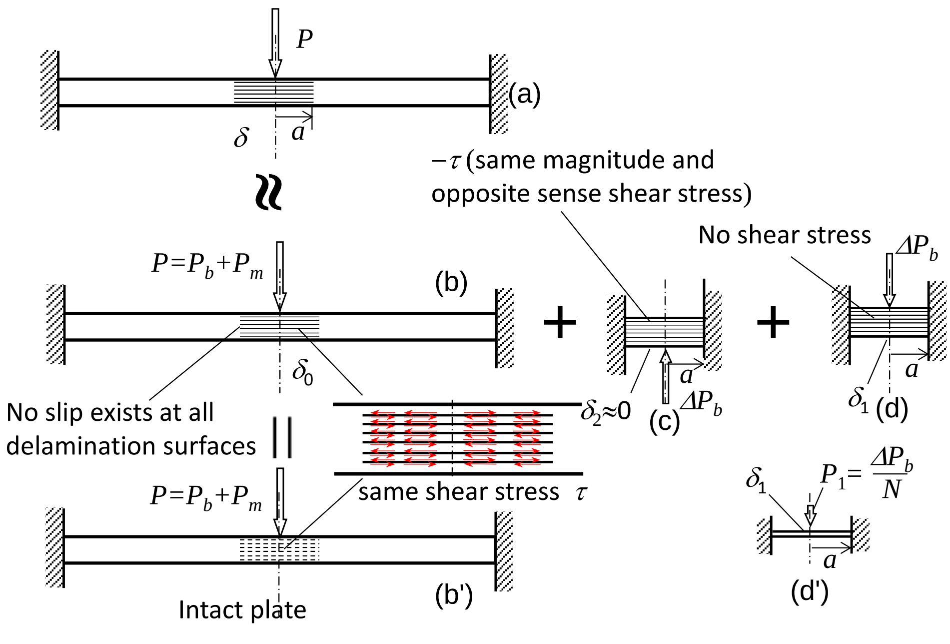

Figure 1: A circular plate with multiple circular delaminations subjected to a concentrated load at its center can be expressed as a Superposition of three problems

In the problem (b) a distributed shear stress is assumed at the delaminated surface which is equal to the shear stress existing at the corresponding interfaces in the intact plate. The solution of the problem (b) is same as that of the intact plate (b'). In the problem (b) the applied load can be expressed as a sum of bending linear term P_b and membrane nonlinear term P_m. The second and the third problems are piled N circular panels fixed at the delamination boundary. The opposite direction same magnitude loads are applied for the Problems (c) and (d). All the delaminated ligaments are assumed to deflect together. Since membrane stiffness is unchanged due to the introduction of the multiple delaminations, the necessary load to produce same deflection as the intact plate reduces same rate as the bending stiffness reduction. The load corresponding to bending stiffness reduction ΔP_b is written as

$$\Delta P_b = P_b \left(1 - \frac{ND_d}{D_0}\right) = P_b - P_b' \tag{1}$$

where P_b is the linear bending term of the load and D_0 and D_d are the bending rigidities of intact and delaminated ligaments, respectively. When the plate is homogeneous, $D_d = D_0/N^{3}$. In problem (c) an opposite sense shear stress to that of problem (b) is given at all the delamination surfaces. Linearized solution of this problem causes no deflection at the loading point. No constraint is assumed at the delaminated surface for problem (d). Then, we need only solutions of the load displacement relations of the two nonlinear plate problems, that is, a global base plate (b') and a thin delaminated plate (d').

The following relationship between a nondimensinal load p_0 and a normalized displacement q_0 can be obtained for an intact plate [16].

$$p_0 = q_0 + kq_0^{\gamma} \tag{2}$$

where

$$p_0 = \frac{PR^2}{16\pi D_0 h}, \qquad q_0 = \frac{\delta_0}{h} \tag{3}$$

The coefficients of the nonlinear term k and γ generally depend on the boundary conditions and shape of the plate and can be numerically determined. When the deformed shape does not change during the deflection, the parameter γ is equal to three.

As explained before, the boundary of the local additional deformation shown in Figure 1(d') can be assumed to be fixed. Then, same relation as Eq.2 between a nondimensional local load p and a normalized local displacement q can be derived when the effect of the global displacement is negligible.

$$p = q + kq^{\gamma} \tag{4}$$

where

$$p = \frac{\Delta P_b a^2}{16\pi D_1 t}, \qquad q = \frac{\delta_1}{t} \tag{5}$$

Since the additional displacement starts from the globally deformed condition as shown in Figure 2, the effect of this condition must be incorporated in the load displacement relation. The effect may be assumed that the additional displacement start from an equivalent state s and the additional normalized load p is given by the following equation.

$$p = g(q,s) = q + k\left\{(q+s)^{\gamma} - s^{\gamma}\right\} \tag{6}$$

The equivalent initial normalized displacement s must be a function of the global normal displacement q_0. It is not clear how to determine the relation between s and q_0. In the present paper we determine to use the normalized rise $\tilde{\delta}/t$ of the expected damaged portion, that is, the relative displacement from the damage boundary to the centre of the intact plate (Problem b') as illustrated in Figure 2. Then,

$$s = \frac{\tilde{\delta}}{t} \tag{7}$$

where

$$\tilde{\delta} = \delta_0 \mid_{r=0} - \delta_0 \mid_{r=a}$$

This assumption could express sufficiently well the nonlinear relationship between the load and displacement of the damaged plates [16]. If the deformed shape of the plate changes little during the loading (the nonlinearity is mild), the linear solution of the plate [12] can be used and the local rise of the circular plate fixed at its boundary is written as

$$s = \frac{\tilde{\delta}}{t} = N q_0 \alpha^2 (1 - 2\log\alpha) \tag{8}$$

where $\alpha = a/R$ is normalized radius of the delaminations.

Since the applied load P consists of the linear bending component P_b and the nonlinear inplane component P_m, the bending load reduction ΔP_b can be given as a linear function of global normal deflection q_0 as

$$\Delta P_b = \frac{16\pi D_0 h}{R^2}\left(1 - \frac{1}{N^2}\right) q_0 \tag{9}$$

The normalized local load p due to the bending stiffness reduction is derived as a linear function of q_0 as

$$p = \frac{N^3 a^2}{16\pi D_1 t}\Delta P_b = N\left(N^2 - 1\right)\alpha^2 q_0 \,. \tag{10}$$

where the same normalizing rule is used as the global case
The relation between the local and global displacement is obtained by substituting Eq.10 into Eq.6.

$$q + k\left\{(q+s)^{\gamma} - s^{\gamma}\right\} = N\left(N^2 - 1\right)\alpha^2 q_0 \tag{11}$$

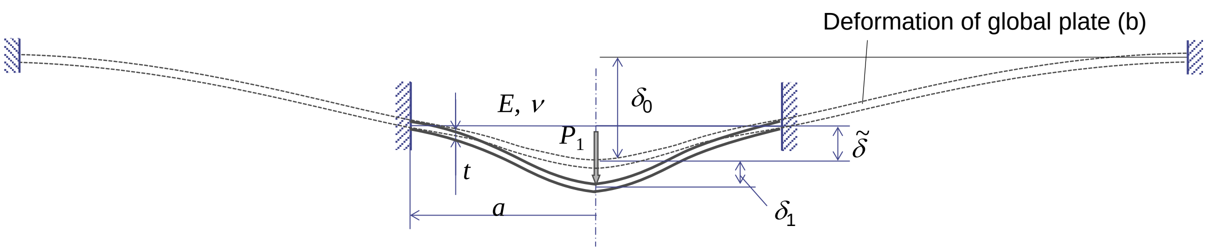

Figure 2: Local model of damaged portion.

When the size of the damage is constant, a stored complimentary energy is obtained by integrating the displacement δ of the loading point by the applied load P.

$$\Pi_C(P|\alpha) = \int_0^P \delta(P)\,dP \tag{12}$$

The total displacement of the loading point δ is a sum of the global displacement δ_0 and local displacement δ_1 and

$$\Pi_C(P|\alpha) = \int_0^P (\delta_0 + \delta_1)\,dP = \Pi_{C0} + \Pi_{C1} \tag{13}$$

$$\Pi_{C0} = \int_0^P \delta_0(P)dP$$
$$\Pi_{C1} = \int_0^P \delta_1(P_b)dP$$

Considering Eq.2, the complimentary energies Π_{C0} and Π_{C1} are written as

$$\begin{aligned}\Pi_{C0} &= \frac{16\pi\kappa D_0 h^2}{R^2}\int_0^{q_0} q_0 \frac{dp_0}{dq_0} dq_0 \\ &= \frac{16\pi D_0 h^2}{R^2}\left(\frac{1}{2}{q_0}^2 + \frac{\gamma}{\gamma+1} k_0 {q_0}^{\gamma+1}\right)\end{aligned}$$

$$\begin{aligned}\Pi_{C1} &= \frac{16\pi\kappa D_0 h^2}{NR^2}\int_0^{q_0} q \frac{dp_0}{dq_0} dq_0 \\ &= \frac{16\pi D_0 h^2}{NR^2}\int_0^{q_0} q\left(1+\gamma k {q_0}^{\gamma-1}\right)dq_0\end{aligned} \quad (13)$$

where the local displacement q is a function of q_0 and the damage size α. As Π_{C0} is independent of the damage size α, the energy release rate of uniform simultaneous growth of all delaminations can be given by differentiating the energy Π_{C1} with respect to delamination area.

$$\begin{aligned}G &= \left[\frac{\partial \Pi_{C1}}{\partial A}\right]_{P=const} = \left[\frac{\partial \Pi_{C1}}{2\pi(N-1)a\partial a}\right]_{P=const} = \left[\frac{\partial \Pi_{C1}}{2\pi(N-1)R^2\alpha\partial\alpha}\right]_{p_0=const} \\ &= \frac{1}{N(N-1)}\frac{8Dh^2}{R^4}\int_0^{q_0}\frac{1}{\alpha}\left[\frac{\partial q}{\partial\alpha}\right]_{P=const}\left(1+\gamma k {q_0}^{\gamma-1}\right)dq_0\end{aligned} \quad (13)$$

Differentiating both sides of Eq. 6 by α under the condition P=constant, the following relation is derived after some manipulations.

$$\frac{1}{\alpha}\left[\frac{\partial q}{\partial\alpha}\right]_{P=const} = \frac{2N\left(N^2-1\right) - \left(\frac{\partial g}{\partial s}\right)\left(\frac{1}{\alpha}\frac{\partial g}{\partial\alpha}\right)}{\partial g/\partial q} q_0 \quad (14)$$

where

$$g(q,s) = q + k\left\{(q+s)^\gamma - s^\gamma\right\}$$
$$\frac{\partial g}{\partial q} = 1 + \gamma k (q+s)^{\gamma-1}$$
$$\frac{\partial g}{\partial s} = \gamma k\left\{(q+s)^{\gamma-1} - s^{\gamma-1}\right\}$$
$$\frac{\partial g}{\partial\alpha} = -4Nq_0\alpha\log\alpha$$

Substituting Eq.14 into Eq.13 yields a normalized energy release rate Γ as

$$\Gamma = \frac{R^4}{8Dh^2}G = 2(N+1)\int_0^{q_0}\frac{1-\frac{2\log\alpha}{N^2-1}\gamma k\left\{(q+s)^{\gamma-1} - s^{\gamma-1}\right\}}{1+\gamma k(q+s)^{\gamma-1}}\left(1+3k_0{q_0}^2\right)q_0 dq_0 \quad (15)$$

The value Γ can be derived by integrating Eq.15 numerically. The normalized energy release rate Γ is a function of q_0, that is, the applied load p_0 since q and s are functions of q_0. The equilibrium path of P and δ when $\Gamma = \Gamma_{cr}$ can be obtained numerically with increasing the parameter α.

When the nonlinear terms is neglected, the present solution coincides with the theoretical solution given in the reference [13].

3. RESULTS AND DISCUSSIONS

Nonlinear relation between the applied load and the center deflection is shown in Figure 3 for a circular thin isotropic fixed plate. The thick line is the FEM result. The FEM result is well fitted by a power form equation (k=0.425 and γ=2.8), while the cubic polynomial cannot agree well the curve at highly nonlinear portion. The following analyses are conducted by using the values k=0.425 and γ=2.8.

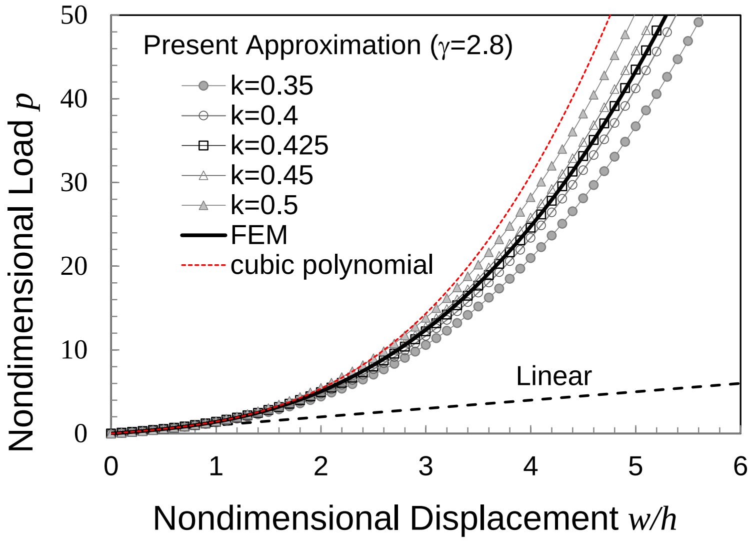

Figure 3: Nonlinear Relation between the load and deflection for the fixed circular plate

The relationships between the nondimensional applied load p_0 and the normalized deflection δ/h are plotted in Figures 4 and 5. In the Figure 4, the solutions obtained by neglecting the equivalent initial normalized displacement s are also shown to indicate that its effect on the solution is significant. The present solution considering the equivalent initial normalized displacement s showed good agreement with the finite element solution for both cases N=4 and 8 even for large α (=0.4). Though the present definition of s is not rigorous, the relationship between the load and the additional displacement can be sufficient to obtain the rough estimate of the damaged plate response. To obtain more physically reliable solution, the effect s must be obtained based on a rigorous mechanical development and Eq.6 shall be refined. In Figure 5 the solutions of the present expression are compared with a solution based on the cubed power assumption [16] when N=8. The results show strong nonlinearity even when the normalized load p_0 is small. The present solution agrees very well with the finite element results for this nonlinear level, while the former results based on a cubed power equation depart from the finite element solutions at highly nonlinear region as expected. The most serious point of the solution is that the deflection of α=0.4 becomes smaller than that of α=0.2 when p_0 exceeds 3.5. Owing to the effect, the energy release rates derived at this region becomes negative.

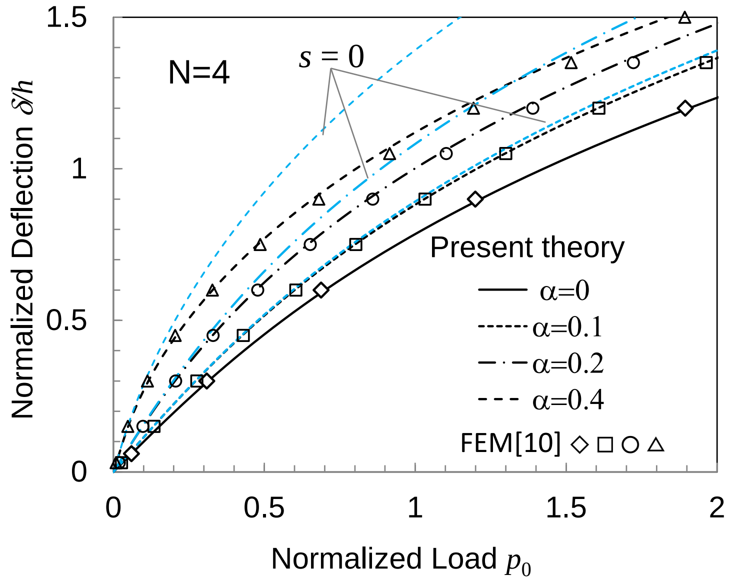

Figure 4: Load-deflection relationship of a plate of N=4. The solutions neglecting the effect of initial inplane effect s are also plotted by blue lines.

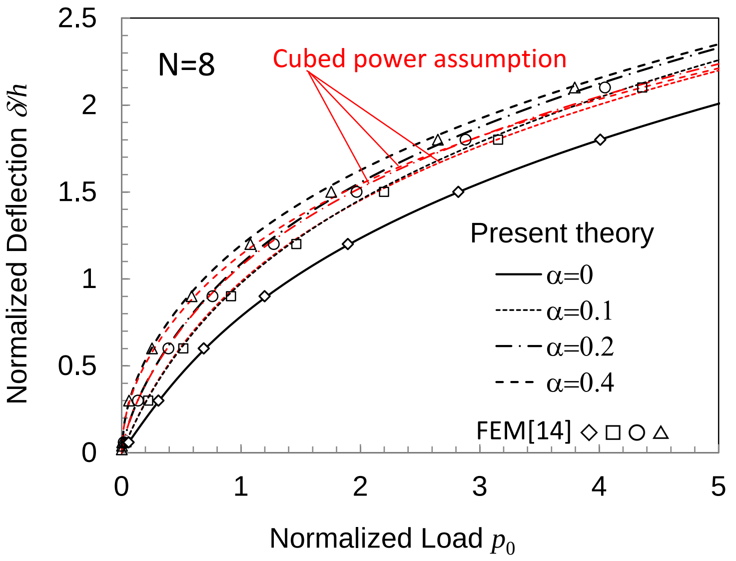

Figure 5 Load-deflection relationship of a plate of N=8. Solutions based on a cubic power approximation of the nonlinear term are also shown by red lines to indicate the effect of the value of the power γ

The square root of the nondimensional energy release rate $\sqrt{\Gamma}$ is plotted against the applied load for several cases of α when N=4 and 8 and compared with a finite element results in Figures 6 and 7. The present results excellently agree with the finite element solutions for a wide range of the load. The present analysis is appropriate to evaluate the stability of delaminations during the indentation loading. The larger the delamination radius α is, the less the energy release rate increase with the load becomes. This tendency is more obvious when the delamination number N is large. It is because the membrane term becomes dominant with the increase of delamination size and number and the effect of the delaminations growth on the stored energy release rate decreases. It means that the load must be increased to keep the delaminations to grow. The results showed that the initiation of the delamination occurs at lower load when number of delamination is more but the load must be increased rapidly to keep the delaminations to grow when N is large.

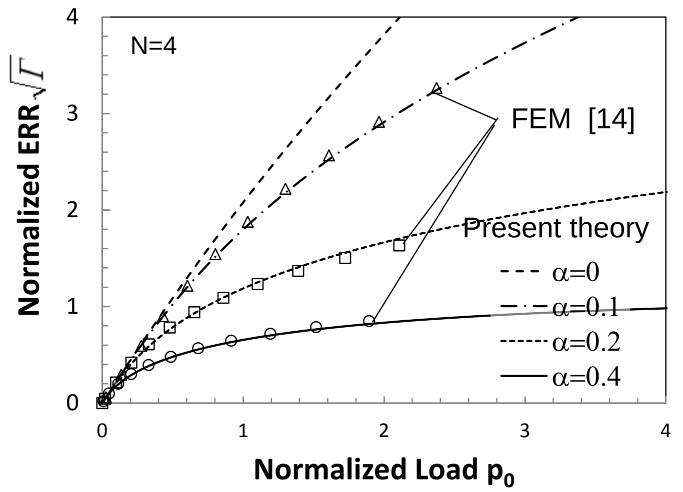

Figure 6 Relationships between the load and square root of the energy release rate for N=8.

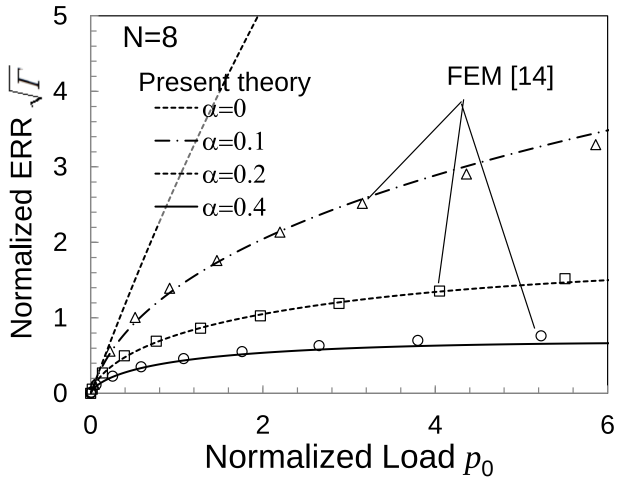

Figure 7 Relationships between the load and square root of the energy release rate for N=8.

The load – deflection histories obtained under the condition that the delamination propagate when $\Gamma=\Gamma_c$ are plotted for N = 4 and N = 8 in Figures 8 and 9, respectively. The dotted thin lines in the figures are the equilibrium paths of intact panel and fully delaminated panel. The equilibrium points of damaged panels locate between the two dotted lines. The present results may not be very accurate when the damage size α exceeds 0.5 considering its assumption. The load increases with the deflection and also with the damage size. The damage starting load is nearly proportional to $\sqrt{\Gamma_{cr}/(N+1)}$, that is, the critical load for N = 4 is almost $3/\sqrt{5} \approx 1.34$ times larger than that of N = 8. However, the load must be rapidly increased for N=8 to keep the delamination to grow. So, the necessary load for N=8 becomes higher than that for N = 4 when normalised displacement increases without plate breakage.

As the applied energy equals the area below the load-deflection curve, the total damage area can be given as a function of the applied energy, that is, the low velocity impact energy. The total delaminated areas are plotted against a consumed energy in Figure 10. Total delamination area is roughly proportional to the critical interfacial toughness. With the increase of the applied energy, the total area of the delaminations increases linearly and then decelerated due to the decrease of the bending effect in the deformation.

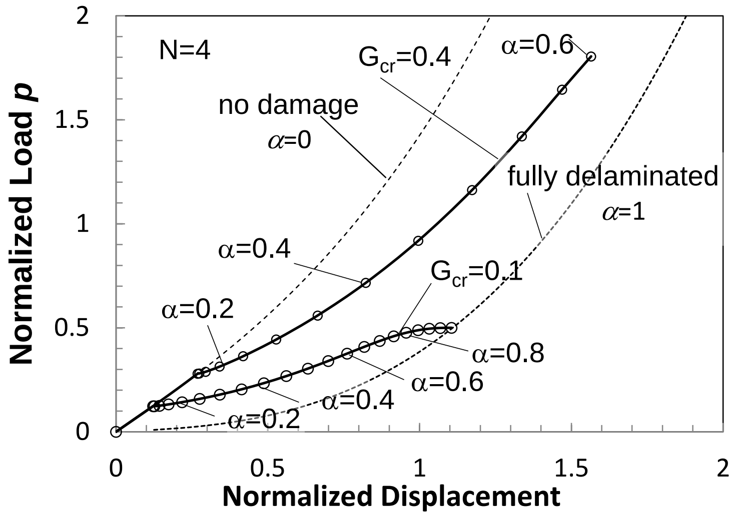

Figure 8: Relationships between the deflection and the applied force when the delamination growth occurs under constant fracture resistance for N =4.

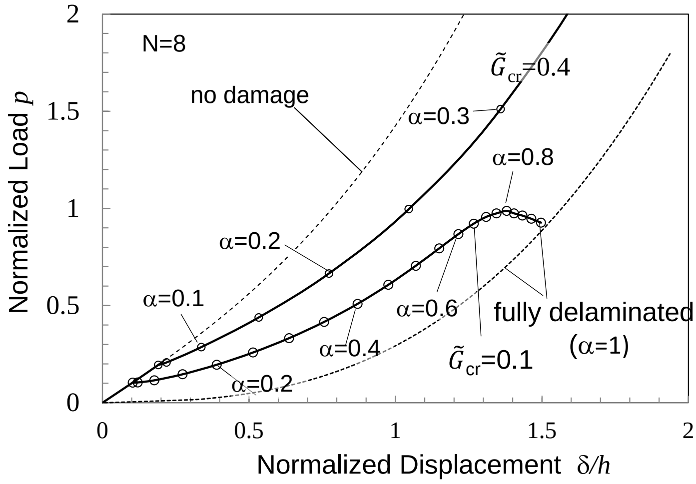

Figure 9: Relationships between the deflection and the applied force when the delamination growth occurs under constant fracture resistance for N = 8.

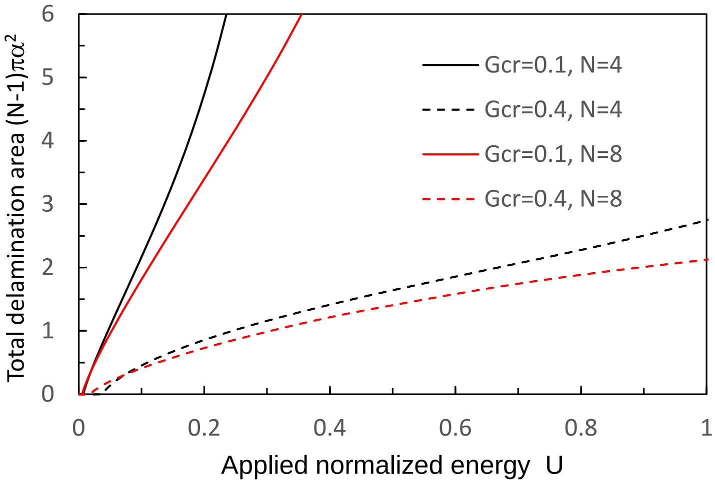

Figure 10 Relationships between the applied energy and the total delamination area.

4. CONCLUSIONS

An analytical solution is proposed for the impact damage problem, where the energy release rate for the simultaneous growth of multiple circular delaminations is given as an integral form of applied displacement. The solutions agree well with finite element results. The expression can be used for the rough estimate of the significance of impact damage in terms of applied energy, interlaminar toughness and dimensions of the laminates. The effect of stacking sequence may be taken into account from the number of the delaminated ligaments *N* in the thickness direction.

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