

GENERALIZED ESHELBY SOLUTION FOR GRADIENT MODEL OF INTERPHASE LAYER AND ITS APPLICATION TO PREDICTING THE PROPERTIES OF MICRO- AND NANOCOMPOSITES

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SUMMARY

The generalized Eshelby problem is extended to the higher-order continuum theory, which takes into account the specific properties of interphase layer. This solution has the properties of the natural cylindrical “transverse isotropy” and can be very effective for predicting the properties of micro- and nanocomposites with whiskerized fibres.

Keywords: Interphase layer, multiscale, modelling, nanocomposites, Eshelby solution

INTRODUCTION

The problem of the physical modeling for the composites reinforced by the micro- and nano-inclusions is associated with the description of the specific properties of the interphase layer which define the structure and properties of the media. To describe the spectrum of the local cohesion and superficial phenomena in the neighborhood of inclusions we will use formal theoretical strain gradient model of the interphase layer [1-3]. This model enables us to analyze various length-scale effects, e.g., cohesive interactions; surface effects in both volume and surface of the media. The Eshelby theory (Eshelby solution and Eshelby tensor) has fundamental role in the analysis of the wide class of composites with deterministic and random structures, containing inclusions of any shape and orientation and subjected to external fields of various physical nature. In this work, the generalized Eshelby problem for the isolated spheroid inclusions with arbitrary number of intermediate layers is considered. It is assumed that the stress field at infinity can be expressed as a polynomial of degree N with respect to coordinated. Therefore, the consideration is not limited by the case of uniform stress field at infinity.

METHODOLOGY

Solution of the generalized Eshelby problem was based on the special analytical representation for the solution of the gradient model (generalised Papkovitch-Neuber

representation). This representation gives the strong solution of the gradient model in the form of differential operator under auxiliary potentials, satisfying Helmholtz and Laplace equations. The main difficulty lay in approximation of these potentials and in determining their analytical form subject to all conditions of contact. In this work, the potentials were expressed in the terms of Laurent and Taylor series separately for the field of inclusions and for the field outside the inclusion. Then these expansions were “stitched” on the interfaces by means of the four conjunction conditions. It was proved that the stitching problem at the boundary of the inclusions has an exact solution. Therefore, the generalized Eshelby problem has the analytical solution in the closed form. For the practical solution of the Eshelby problem, two approaches can be used. The first approach is a semi-analytical method of stitching the solutions through a modified functional energy. It can be used for inclusions which are different from the spherical shape. The second method is the fully analytical method, where the conjunction conditions are written in the form of matrix equation for the unknown factors taking into account the analytic properties of approximating special functions. The first approach is easier for implementation, but the second approach gives the most accurate result in a closed form.

APPLICATIONS

When modelling reinforced composites, the generalized Eshelby solution was combined with the known technique for predicting the effective properties. The asymptotic average solution for gradient model was obtained by asymptotic homogenization procedure for composites with periodic structure. The effective constants were calculated for various properties of the matrix and the inclusions. The approach was also applied to a 3-D problem using the generalized Eshelby solution. Analytical solution to the generalized Eshelby problem for cylindrical inclusions, which is a part of the multiscale model, was defined as a function of the elastic constants and one additional parameter. This parameter determines the length of the zone of the local scale effect along the normal direction with respect to interphase, where the solution varies exponentially. This solution has the property of natural cylindrical “transverse isotropy” and can be very effective for predicting the properties of micro- and nanocomposites with whiskerized or bristled fibres [4]. The four-component analytical model developed in [4] assumes that a whiskerized micro- or nanofibre consists of three different parts: a solid core, a solid coating (homogeneous shell), and a “bristled coating” (composite shell). The fourth component in the model is the matrix. Three out of four components are homogeneous materials, e.g., EPON 828 epoxy matrix, SiO₂ solid coating and CdTe solid fibre core, with known properties. However, the bristled coating is itself a composite consisting of, e.g., the EPON 828 matrix reinforced by SiO₂ nanowires. The prediction for effective properties of this component can be improved using the model described in this paper.

References

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