FORMULATION OF TIME-TEMPERATURE DEPENDENT LONG-TERM FATIGUE STRENGTH OF CFRP LAMINATES

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Abstract

Statistical formulations of the master curves for the constant strain rate (CSR), creep and fatigue strengths of CFRP laminates which depend on time and temperature as well as the number of cycles to failure are proposed based on Christensen's theory which describes statistically the crack kinetics in viscoelastic body. The master curves of these strengths of the plain-woven fabric T300/vinylester CFRP, the flat fabric T700/vinylester CFRP and the multi-axial knitted fabric T700/vinylester CFRP laminates were measured by the accelerated testing methodology in our previous paper. The quantitative characteristics of CSR, creep and fatigue master curves for these CFRP laminates are discussed using the results of our statistical formulations. Finally, it is demonstrated that our formulation of the master curves is available and practical for long-term durability design for CFRP laminate.

1 Introduction

Recently carbon fiber reinforced plastics (CFRP) has been used for the primary structures of airplanes, ships, spacecrafts and others, in which the high reliability should be kept during the long-term operation. Therefore, it is strongly expected that the accelerated testing methodology (ATM) for the long-term life prediction of composite structures exposed under the actual environments of temperature and others is established.

The mechanical behavior of polymer resins exhibits time and temperature dependence, called viscoelastic behavior, not only above the glass transition temperature T_g but also below T_g. Thus, it can be presumed that the mechanical behavior of polymer composites significantly depends time and temperature. These examples have been shown by Aboudi and Cederbaum [1], Sullivan [2], Gates [3], Rotem and Nelson [4] and Kharrazi and Sarkani [5].

Our accelerated testing methodology [6-10] is based on the time-temperature superposition principle for polymer composites. This principle was originally developed for non-destructive material properties, but recent studies have shown that it can also be applied to failure properties of composite materials. Using this principle as the building block, we have developed a methodology to predict the long-term strength of composite materials, such as the static (constant strain rate: CSR), creep and fatigue strengths.

In our previous paper [11,12], the prediction of long-term flexural fatigue strength of CFRP laminates for advanced marine use was performed based on the accelerated testing methodology. CFRP laminates employed the plain-woven fabric T300/vinylester CFRP, the flat fabric T700/vinylester CFRP and the multi-axial knitted fabric T700/vinylester CFRP laminates. The three-point bending CSR and fatigue tests for these CFRP laminates were carried out at various temperatures and loading rates. As results, the flexural fatigue strength as well as CSR strength of these CFRP laminates depends clearly on time and temperature. The time-temperature superposition principle for the viscoelasticity of matrix vinylester resin holds for theses flexural strengths, and then the master curves can be obtained. The applicability of the accelerated testing methodology was confirmed experimentally.

Christensen and Miyano [13,14] developed a lifetime prediction methodology based upon kinetic crack growth in polymers and polymer composites showing the viscoelastic behavior. They formulated the master curves of CSR and creep strengths of polymer and polymer composites using their methodology as mentioned later. Their formulations were theoretically and statistically performed and the characteristic of polymer and polymer composites for the fracture were statistically cleared.

However, these formulations are not practical for the actual polymer composites because the fatigue strength can not be formulated by the method.

This paper is concerned with that the statistical formulations of the master curves for the constant strain rate (CSR), creep and fatigue strengths of CFRP laminates which depend on time and temperature as well as the number of cycles to failure are proposed based on Christensen's theory which describes statistically the crack kinetics in viscoelastic body. The master curves of these strengths of the plain-woven fabric T300/vinylester CFRP, the flat fabric T700/vinylester CFRP and the multi-axial knitted fabric T700/vinylester CFRP laminates measured by the accelerated testing methodology in our previous paper are formulated by our statistical formulation method. The quantitative characteristics of CSR, creep and fatigue master curves for these CFRP laminates are discussed using the results of formulations. Finally, it is demonstrated that our statistical formulation of the master curves is available and practical for long-term durability design for CFRP laminate.

2 Background of this study

2.1 Accelerated testing methodology for polymer and polymer composites

The time-temperature superposition principle has been widely employed to characterize the nondestructive properties, and recently, has shown remarkable success in characterizing the failure properties of polymer composites. In this case, elevated temperature states are used to accelerate the mechanical degradations, which occur under loads over long period of time at lower temperature. Figure 1 shows the outline of the accelerated testing methodology. Two scientific bases in this figure are the conditions that must be satisfied in order for the methodology to work properly. Here, a creep loading is considered as a fatigue loading with stress ratio of $R=\sigma_{min}/\sigma_{max}=1$, and a CSR loading is considered to be equivalent to a half cycle of fatigue loading with $R=0$.

At first, the viscoelastic modulus (ex. creep compliance) of matrix resin should be measured at various temperatures and the viscoelastic modulus is shifted horizontally and vertically along the logarithmic scale to form a smooth curve called the master curve at an arbitrary chosen reference temperature. The amounts of horizontal and vertical shifts are called respectively as the time-temperature shift factor a_{T_0} (accelerating rate) and the temperature shift factor b_{T_0}. In the plot of the master curve, the vertical axis is the viscoelastic modulus, and the horizontal axis is the combination of temperature and time, which we call the reduced time. The significance of this master curve is that it can be used to predict the viscoelastic modulus of matrix resin under any combinations of temperature and time.

Scientific base A states that the same time-temperature superposition principle applies for all three types of strengths (CSR, creep and fatigue loadings). In other words, we can use the same shift factor of viscoelastic modulus of matrix resin for all three types of strengths for polymer composites. The CSR test data for polymer composites at a single strain rate at various temperatures are shifted along the logarithmic scale of time to form the master curve at an arbitrary chosen reference temperature. The fatigue test data to the case of $R=0$ for polymer composites at a single frequency at various temperatures are also shifted along the logarithmic scale of time to form the master curve. Scientific base B states that the extension from the CSR master curve to the creep master curve based on kinetic crack growth in viscoelastic body [13].

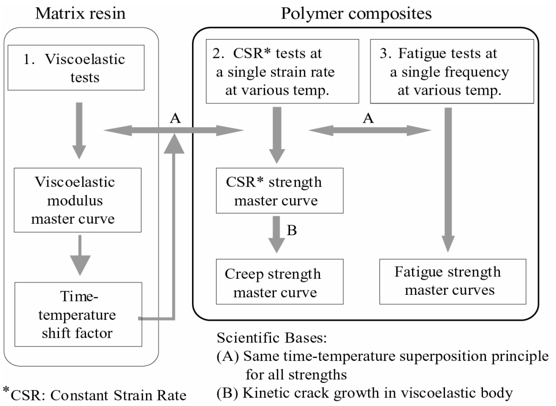


Fig. 1. Prediction procedure of fatigue strength for

2.2 Crack kinetic in polymer and polymer composites

In previous work, Christensen and Miyano [14] developed a lifetime prediction methodology based upon kinetic crack growth in polymers and polymer composites. The key condition in the kinetic aspect of the problem is the assumption of the power law form for the crack growth rate. When the general lifetime forms are specialized to the case of creep rupture, inboloving constant stress, the resulting creep rupture analytical solution for the lifetime is quite simple as below. The CSR case is formulated by

$$\log\frac{t'_s}{t'_1}=\log\left[\left(\frac{\sigma_i}{\sigma_s}\right)^{\frac{1}{n}+1}-\left(\frac{\sigma_i}{\sigma_s}\right)^2\right]+\log\left(\frac{1}{n}+2\right) \tag{1}$$

while the creep rupture case is formulated by

$$\log\frac{t'_c}{t'_1}=\log\left[\left(\frac{\sigma_i}{\sigma_c}\right)^{\frac{1}{n}+1}-\left(\frac{\sigma_i}{\sigma_c}\right)^2\right] \tag{2}$$

and where t'_s and t'_c are the failure time for CSR and creep cases, σ_s and σ_c are the applied stress for CSR and creep cases. Furthemore, t'_1 is the glassy reduced time, σ_i is the instantaneous static strength, and r is expressed by following equation.

$$r=\frac{1}{n}+1 \tag{3}$$

This formulation is very basic form, but this kinetic forms did not model the data in a satisfatory manner, except in the long time, power low range.

They revised this formulation to more general form[14]. The CSR case is formulated by

$$\frac{t'_s}{t'_1}=\frac{(r+1)\left[1-\left(\frac{\sigma_s}{\sigma_i}\right)^p\right]^q}{\left(\frac{\sigma_s}{\sigma_i}\right)^r} \tag{4}$$

while the creep rupture case is formulated by

$$\frac{t'_c}{t'_1}=\frac{\left[1-\left(\frac{\sigma_c}{\sigma_i}\right)^p\right]^q}{\left(\frac{\sigma_c}{\sigma_i}\right)^r} \tag{5}$$

and where p and q are the power law exponents. However, these formulations are not practical for the actual polymer composites because the fatigue strength can not be formulated by the method.

3 Formulations

3.1 Master curve for creep compliance of matrix resin

The horizontal and vertical shift factors $a_{T_0}(T)$ and $b_{T_0}(T)$ of master curve for creep compliance of matrix resin can be formulated by the following equations.

$$\log a_{T_0}(T)=a_1(T-T_0)+a_2(T-T_0)^2 \tag{6}$$

$$\log b_{T_0}(T)=b_1(T-T_0)+b_2(T-T_0)^2 \tag{7}$$

where a_1, a_2, b_1 and b_2 are all constants and T_0 is a reference temperature. The creep compliance D_c can be formulated by the following equation.

$$\log D_c=\log D_{c,0}(t_0,T_0)+\log\left[\left(\frac{t'}{t'_0}\right)^{m_g}+\left(\frac{t'}{t'_g}\right)^{m_r}\right] \tag{8}$$

where $D_{c,0}$ is a creep compliance at reference time t'_0 and temperature T_0, t'_g is a glassy reduced time on T_0 and m_g and m_r are the gradients in glassy and rubbery region of D_c master curve.

3.2 Master curves for CSR, creep and fatigue strengths

We propose the following equation as the practical formulation for the master curve of CSR strength σ_s.

$$\log\sigma_{s,th}=\log\sigma_{s,0}(t_0,T_0)-\log\left[1+\left(\frac{t'_s}{t'_1}\right)^{n_r}\right]+\frac{1}{\alpha_s}\log[-\ln(1-P_f)] \tag{9}$$

where, $\sigma_{s,0}$ is the CSR strength at reference time t_0 and temperature T_0, and n_r is the absolute value of the gradient in rubbery region of the CSR strength master curve. n_r means the time and temperature dependence on the reduction of CSR strength. P_f is the failure probability and α_s is the shape parameter in CSR strength, which is specified through a Weibull distribution.

The master curve of creep strength σ_c can be statistically and automatically formulated by the following equation using Christensen's theory [13] and Eq. 9,.

$$\log\sigma_{c,th}=\log\sigma_{s,0}(t_0,T_0)-\log\left[1+\left(\frac{1}{n_r}+2\right)^{n_r}\left(\frac{t'_c}{t'_1}\right)^{n_r}\right]+\frac{1}{\alpha_s}\log[-\ln(1-P_f)] \tag{10}$$

Additionally, we propose the following equation as the practical formulation based the master curve of CSR strength σ_f shown by Eq. 9.

$$\log\sigma_{f,th}=\log\sigma_{s,0}(t_0,T_0)-\log\left[1+\left(\frac{t'_s}{t'_1}\right)^{n_r}\left(\frac{t'_f}{t'_s}\right)^{n_c}\right]-\log\left(\frac{N_f}{N_0}\right)^{n_f}+\frac{1}{\alpha_f}\log[-\ln(1-P_f)] \tag{11}$$

where, n_c and n_f mean the dependences of time and number of cycles on the reduction of fatigue strength, respectively. α_f is the shape parameter in fatigue strength, which is specified through a Weibull distribution.

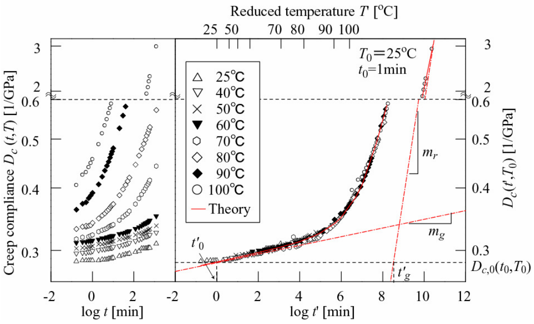
4 Demonstrations of formulation for CFRP laminates

4.1 Materials

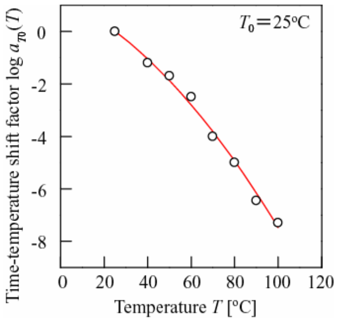
The formulations of the master curves for CSR, creep and fatigue strengths are demonstrated for three kinds of CFRP laminates in this section. First laminate is T300/VE-P textile laminate, which consists of plain-woven fabric T300 carbon cloth (Toray Industries, Inc.) and vinylester resin Derakane Momentum 411-350 (The Dow Chemical Company). This combination of fibers and matrix is the most familiar as CFRP laminates for marine use. Second laminate is T700/VE-F textile laminate, which consists of flat fabric T700 carbon cloth and vinylester resin. This carbon cloth is more innovative than the plain-woven fabric cloth for marine use because the crimp of flat fabric fibers is less than that of plain-woven fabric fibers. Third laminate is T700/VE-K textile laminate, which consists of multi-axial knitted fabric T700 carbon cloth and vinylester resin. This carbon cloth has more thick than the conventional fabric, and good mechanical property is expected by its no-crimp configuration and smooth warping performance.

4.2 Master curve for creep compliance of matrix resin

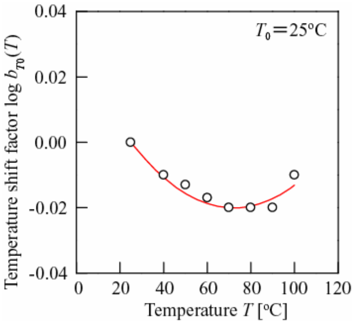
The creep test as one of viscoelastic tests were performed at various temperatures for the neat resin [11]. Figure 2-(a) shows the formulation of the master curve of the creep compliances D_c of matrix resin using Eq. 8 obtained by shifting horizontally and vertically. The time-temperature shift factor a_{To} and the temperature shift factor b_{To}, which formulated by Eqs. 6 and 7, are shown in Figures 2-(b) and (c). The parameters in these equations are shown on Table 1.



(a) Master curve of creep compliance



(b) Time-temperature shift factor



(c) Temperature shift factor

Fig. 2. Master curve of creep compliance of matrix resin VE

Table 1. Parameters of creep compliance master curve for VE resin in Eqs. 6-8

T_0 [°C]	25
t'_0 [min]	1
t'_g [min]	3.1x10^8
$D_{c,0}$ [GPa^{-1}]	0.28
m_g	1.3x10^{-2}
m_r	0.51
a_1	-6.2x10^{-2}
a_2	-5.0x10^{-4}
b_1	-8.5x10^{-4}
b_2	9.0x10^{-6}

4.3 Master curves for CSR, creep and fatigue strengths

The three-point bending tests for three kinds of CFRP laminates were performed for CSR, creep and fatigue loadings at various temperatures. These strengths for three kinds of CFRP laminates are shown in Figure 3. The left, center and right of these figures are the master curves of CSR, creep and fatigue strengths, respectively.

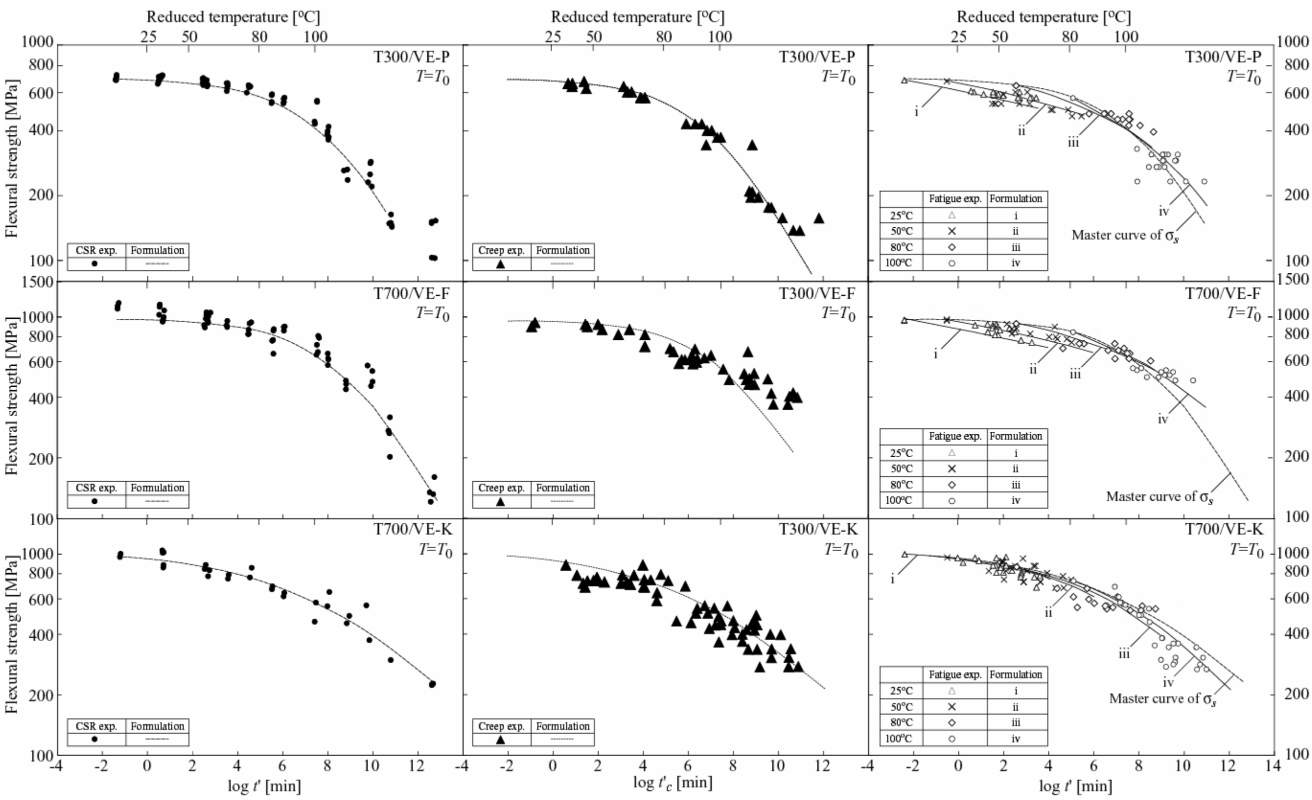
The results of formulation for these master curves, which are defined using Eqs. 9, 10 and 11, are also shown in these figures. The Weibull distributions obtained through the formulation of these master curves for CSR and fatigue strengths are shown in Figures 4 and 5. The parameters using Eqs. 9, 10 and 11 are shown on Table 2.

From these figures and table, the following results are demonstrated. At first, it is quantitatively characterized that CSR strengths $\sigma_{s,0}$ for T700/VE-F and T700/VE-K CFRP laminates are higher than that for T300/VE-P CFRP laminates. It is attributable to that fiber strength of T700 carbon is stronger than that of T300 and the crimps of fiber for T700/VE-F and T700/VE-K CFRP laminates are less than that for T300/VE-P CFRP laminates.

Table 2 Parameters of CSR, creep and fatigue master curves for CFRP laminates

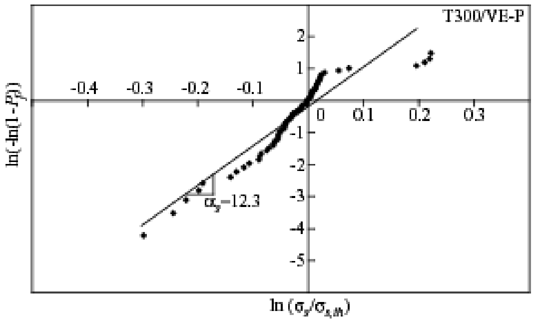
	T300/VE-P	T700/VE-F	T700/VE-K
t'_1 [min]	2.9x10^8	1.3x10^9	1.9x10^8
$\sigma_{s,0}$ [MPa]	712	1030	1030
N_0	0.5	0.5	0.5
n_r	0.22	0.22	0.12
n_c	0.17	0.16	0.14
n_f	0.019	0.017	-0.001
α_s	12.3	9.2	10.3
α_f	11.4	10.5	10.9

It is qualitatively cleared from Figure 3 that the flexural fatigue strength of the fabric CFRP laminates decreases strongly with the increasing number of cycles to failure while that of the non-crimp CFRP laminates decreases scarcely with the increasing number of cycles to failure. It is also quantitatively confirmed from Table 2 that the value of n_f for the non-crimp CFRP laminate is less than these values of n_f for the fabric CFRP laminates while values of n_c for these CFRP laminates are almost same. Therefore, it is summarized that the long-term durability of flexural fatigue strength for

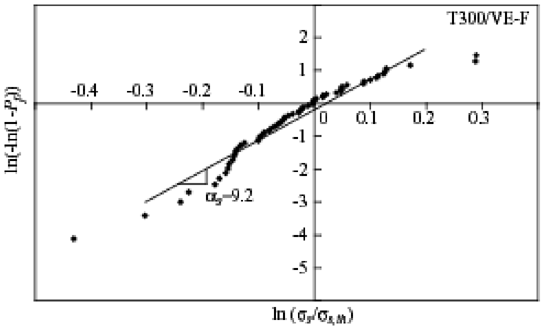


(a) CSR strength (b) Creep strength (c) Fatigue strength

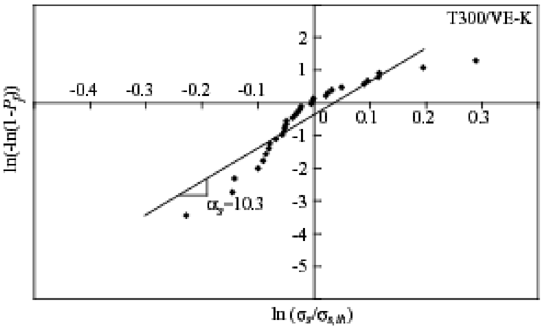
Fig. 3. Master curves of flexural CSR, creep and fatigue strengths 6f three kinds of CFRP laminates



(a) T300/VE-P laminates



(b) T700/VE-F laminates



(c) T700/VE-K laminates

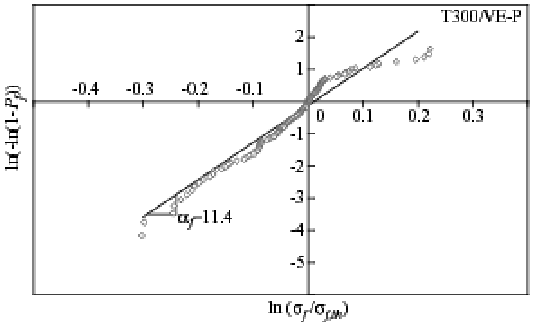
Fig. 4. Weibull distribution of CSR strength

the non-crimp CFRP laminate is superior to that for the fabric CFRP laminates because n_f means the dependence of number of cycles to failure on the long-term durability.

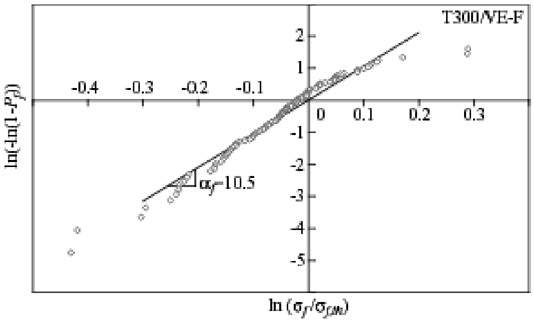
It is confirmed from Figures 4 and 5 and Table 2 that the shape parameters α_s and α_f of the CSR and fatigue strengths for three kinds of CFRP laminates are found to be almost same value and do not change with the loading type and the kind of materials.

4 Conclusions

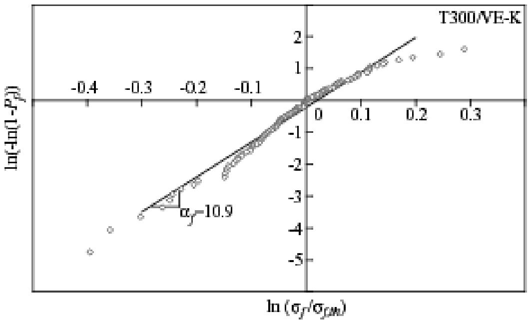
Statistical formulations of the master curves for CSR, creep and fatigue strengths of CFRP laminates which depend on time and temperature as well as the number of cycles to failure are proposed based on Christensen's theory which describes statistically the crack kinetics in viscoelastic body. We apply the formulation to the master curves of these strengths for two kinds of fabric CFRP laminates and the non-crimp CFRP laminates which were measured by the accelerated testing methodology in our previous paper. The quantitative characteristics of CSR, creep and fatigue master curves for these CFRP laminates are discussed using the proposed formulations. Consequently, it is quantitatively summarized that the long-term durability of flexural CSR and fatigue strengths for the non-crimp CFRP laminate is superior to that for the fabric CFRP laminates from the parameters using our formulations. Finally, it is demonstrated that our formulation of the master curves inclusive statistics is available and practical for long-term durability design for CFRP laminate.



(a) T300/VE-P laminates



(b) T700/VE-F laminates



(c) T700/VE-K laminates

Fig. 5. Weibull distribution of fatigue strength

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