

# DESIGN ANALYSIS OF THE OUTWATER-MURPHY SINGLE-FIBER SPECIMEN

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## Abstract

*The Outwater-Murphy single fiber specimen is examined as candidate test to determine the fracture energy required for debonding of a fiber from the interface. The analysis is based on a simple mechanics of materials model where the compliance change of the specimen is determined as a function of debonded area. A successful debond test requires that the debond fracture occurs prior to any other failure mechanisms such as global buckling, fiber microbuckling and net-section yielding. Calculations based on model indicate that the levels of the energy release rate attainable are quite small, except for specimens with a small cross section area. Specimens with high fiber/matrix adhesion and low yield strength will fail in compression before the fiber debonds from the matrix. Published estimates of the debonds toughness of carbon and glass fibers in thermoset resin matrices indicate that the specimen could be viable over narrow ranges of cross section dimensions and open hole diameters.*

## 1. Introduction

Long term durability is a major concern for polymeric composite materials used in Naval applications. Polymer composites do not corrode like metals but they absorb moisture which degrades the strength of matrix and the adhesion at the fiber/matrix interface. As the structure will be immersed continuously for 10-20 years, moisture absorption saturation level will be reached. Therefore, the effect of the absorbed moisture on the long term durability needs to be determined. Several investigations have shown that fiber/matrix interface may be especially susceptible to degradation by moisture, and absorption of moisture by capillary action (wicking). Many test methods have been proposed to characterize the interface [1].

The first technique proposed was the fiber pull-out test where a single fiber is pulled out from a block of matrix [2]. This test was devised in early stages of composite research when fiber diameters were larger making handling easier than for many of the commercial fibers of today. In the load and deformation during the pull-out process are monitored continuously, until debonding of the fiber is completed. Data recorded are converted into fiber/matrix interfacial shear strength.

Another popular method is the Single Fiber Fragmentation test [3,4]. A single fiber is totally embedded in the polymeric matrix formed into a small tensile specimen. In this test, tensile load from the loaded ends of the specimen is transferred into the fiber through shear stresses at the fiber/matrix interface. If the fiber tensile strength of fiber is exceeded, the fiber fractures inside the matrix. Upon continued loading the failure process is repeated, producing shorter and shorter fragments until the remaining fragment lengths are no longer sufficient in size to enable stress build-up and fracture of the fragment. A simple shear-lag analysis is used to analyze the experimental data based on the saturation length of the fiber fragments, the fiber diameter, and the fiber tensile strength, to determine the interfacial shear strength.

Another method devised in 1960 by Outwater and Murphy [5], which looks promising, but has not been much explored, consists of embedding one single fiber aligned axially in a rectangular prism of resin, see Fig. 1. A small hole is drilled in the center of the specimen through the fiber. As the specimen is loaded in compression, interfacial shear stresses are generated near the fiber ends as a result of the discontinuity at the hole. The initiation and propagation of an interfacial crack is supposed to occur once the specimen is loaded sufficiently. To the authors' knowledge, experimental data generated from the OM test have not been published. In this study a simple design analysis based on linear elastic

fracture mechanics is conducted to assess the viability of the method as a fiber/matrix interface test.

## 2. Fracture Mechanics Analysis

Consider the symmetry section of the OM test shown on Fig. 2. The total length of the section is  $L/2$ , and the hole diameter,  $D$ . To analyze debond propagation we will assume the presence of a debond length,  $a$ , at the edge of the hole along the fiber/matrix interface, see the front view of the half OM specimen in Fig. 3. Application of compressive load to the specimen results in a total displacement,  $\delta$ , of the specimen,  $\delta$  may be partitioned in three parts corresponding to the three regions of length,  $L_1$ ,  $a$ , and  $L_3$  in Fig. 3,

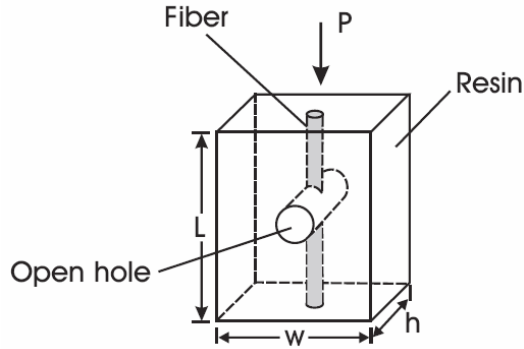


Fig. 1 Outwater-Murphy (OM) specimen[5].

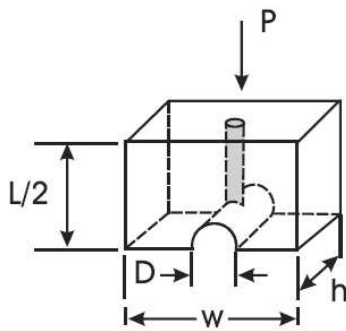


Fig. 2 Symmetry section of OM specimen.

$$\delta = \delta_1 + \delta_2 + \delta_3 \quad (1)$$

According to Hooke's law

$$\delta = \frac{PL_1}{A_1E_1} + \frac{Pa}{A_2E_2} + \frac{PL_3}{A_3E_3} \quad (2)$$

where  $E$  refers to Young's modulus and  $A$  is the cross section area, and subscripts 1,2 and 3 refer to the regions indicated in Fig. 3. The moduli,  $E_i$ , are given by

$$E_1 = E_fV_f + E_mV_m \quad (3a)$$

$$E_2 = E_mV_m \quad (3b)$$

$$E_3 = E_m \quad (3c)$$

where  $V_f$  and  $V_m$  are the volume fraction of the fiber and matrix in regions 1 and 2, and subscripts  $f$  and  $m$  refer to fiber and matrix respectively.

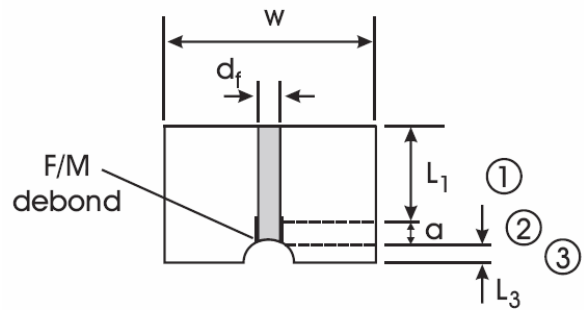


Fig. 3 Element considered for fracture analysis.

$$V_f = \frac{\pi d_f^2}{4w^2}, \quad V_m = 1 - V_f \quad (\text{Region 1}) \quad (4)$$

$$V_f = 0, \quad V_m = 1 - \frac{\pi d_f^2}{4w^2} \quad (\text{Region 2}) \quad (5)$$

where  $d_f$  is the fiber diameter, and from now on we specifically consider a square specimen, i.e.,  $h = w$  in Fig. 1.

For the purpose of calculation of the strain energy release rate,  $G$ , available for debond propagation, it is necessary to calculate the compliance of symmetry section of the OM specimen shown in Fig. 3.

$$C(a) = \frac{L_1}{E_1A_1} + \frac{a}{E_2A_2} + \frac{L_3}{E_mA_3} \quad (6)$$

If the debond extends an increment  $da$ , the length of the region 1 will decrease by  $da$ , while the length of the region 2 increases by the same amount. Hence,

$$C(a + da) = \frac{L_1 - da}{E_1 A_1} + \frac{a + da}{E_2 A_2} + \frac{L_3}{E_m A_3} \quad (7)$$

Further, the increase in debond area,  $dA$ , is given by

$$dA = \pi d_f da \quad (8)$$

The energy release rate,  $G$  is given in terms of compliance [6],

$$G = \frac{P^2}{2} \frac{dC}{dA} \quad (9)$$

Combination of Eqs.(6)-(9) yields

$$G = \frac{P^2}{2 \pi d_f} \left( \frac{1}{E_2 A_2} - \frac{1}{E_1 A_1} \right) \quad (10)$$

For a square cross section of the OM specimen in Fig. 1;  $h = w$  and  $A_1 = A_2 = w^2$ . The expression for  $G$  becomes:

$$G = \frac{P^2}{2 \pi d_f w^2} \left( \frac{1}{E_2} - \frac{1}{E_1} \right) \quad (11)$$

With  $E_1 = E_f V_f + E_m V_m$  and  $E_2 = E_m V_m$ , with  $V_m$  being close to unity we can write  $E_1 = E_2(1 + \varepsilon)$  where  $\varepsilon \ll 1$  is given by

$$\varepsilon = \frac{E_f V_f}{E_m V_m} \quad (12)$$

Hence,  $G$  is given by,

$$\begin{aligned} G &= \frac{P^2}{2 \pi d_f w^2 E_2} \left( 1 - \frac{1}{1 + \varepsilon} \right) \approx \frac{P^2 \varepsilon}{2 \pi d_f w^2 E_2} \\ &= \frac{P^2 E_f V_f}{2 \pi d_f w^2 E_2 E_m V_m} \end{aligned} \quad (13)$$

This expression reveals that  $G$  does not depend on the debond length,  $a$ . It is noted that  $G$  increases quadratically with applied load and inversely in proportion to the square of the side length,  $w$ , of the square OM specimen. Consequently a load high enough and small cross section should cause debonding. The expression for  $G$  will be utilized in a design analysis of the OM specimen in next section.

It must be pointed out that this simple model does not consider the possible consideration from friction at the interface. Friction combined with residual radial stresses due to cure shrinkage or cool-down after cure of the resin would lead to shearing tractions opposing the debond process. The contributions from friction to the pull-out force in single fiber pull-out tests have been discussed by Di Francia et al [7] and Gao et al [8]. Obviously, this effect is potentially important also in OM test and needs to be further examined.

For successful testing, it is required that a debond initiates before the specimen yields in compression. Yielding is expected to occur at the minimum cross section of the OM specimen, which, for the square shape considered, is given by,  $A_{net} = w(w - D)$ , see Fig. 2. The stress average acting on this cross section is

$$\sigma = \frac{P}{w^2 - wD} \quad (14)$$

where  $w$  is side length of the OM specimen and  $D$  is diameter of the hole. It is noted that a larger hole diameter will increase the stress. This expression will be utilized to examine the potential failure by yielding of the OM specimen.

### 3. Results and Discussion

Consider two fibers, viz. E-glass and carbon, and a vinylester matrix with diameters and mechanical properties listed in Table 1.

Table1. Diameters and mechanical properties of typical E-glass and carbon fibers and vinylester.  $d_f$  = fiber diameter,  $E$  = Young's modulus,  $\sigma_{ys}$  = yield strength, [9]

Material	$d_f$ , $\mu\text{m}$	$E$ , GPa	$\sigma_{ys}$ , MPa
E-glass	10	70	N/A
Carbon	7	230	N/A
Vinyl ester	N/A	4.4	75.8

Calculations of the energy release rate,  $G$ , were conducted for OM specimens with total length,  $L = 12.7$  mm. To examine the influence of the side length,  $w$ , on  $G$ , calculations were conducted for  $w = 4, 6, 8$  and  $12.7$  mm for loads,  $P$ , up to 5kN. The results are presented in semi-logarithmic form due to the large range of  $G$  values. Figure 4 shows  $G$  vs.  $P$

for the carbon/vinyl ester and E-glass/vinyl ester specimens. The curves show that the side length has a very strong influence on the energy release rate available for debonding. For a side lengths,  $w = 8$  and  $12.7$  mm the levels of  $G$  remain small even for loads in the  $5$  kN range. Critical value of  $G$  to initiate a debond in carbon/epoxy have been presented by Di Francia et al [7]. They reported  $G_c = 50$  J/m<sup>2</sup> for an unspecified combination of carbon/epoxy and glass/epoxy. Such a debond toughness, Fig. 4a provides critical loads of  $392$  and  $883$  N for carbon/vinyl ester OM specimens with  $w = 4$  and  $6$  mm respectively.

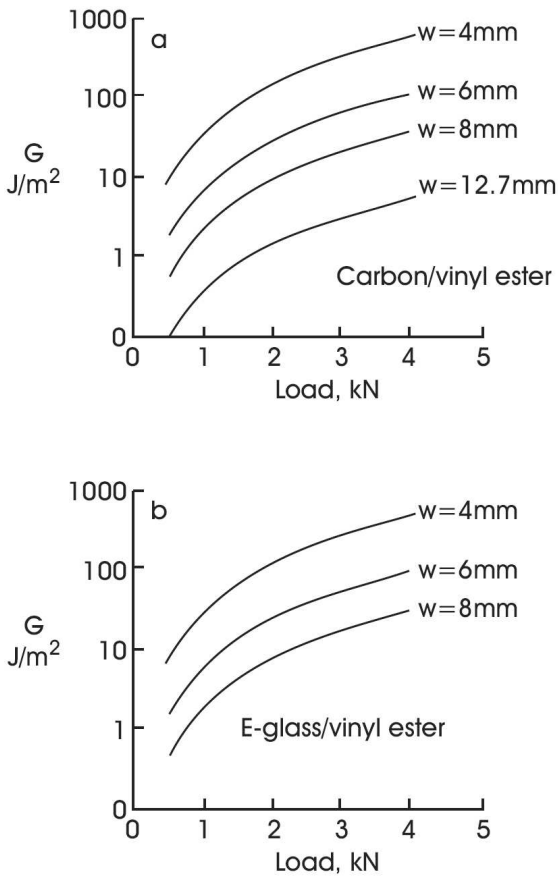


Fig. 4 Energy release rate available for debonding a) carbon/vinyl ester and (b) E-glass/vinylester.

The corresponding loads for  $w = 4$  and  $6$  mm for E-glass/vinyl ester OM specimens, obtained from Fig. 4b, are  $595$  and  $1340$  N. To examine the possibility for yielding, graphs of compressive stress vs. applied load were constructed for OM specimens with  $1$  and  $2$  mm open holes, respectively. The calculations

were done for side lengths using Eq.(14)  $w = 4, 6$  and  $8$  mm. Figs. 5 displays the results for the two hole diameters. It is noted that stress decreases with increasing side length and decreasing diameter.

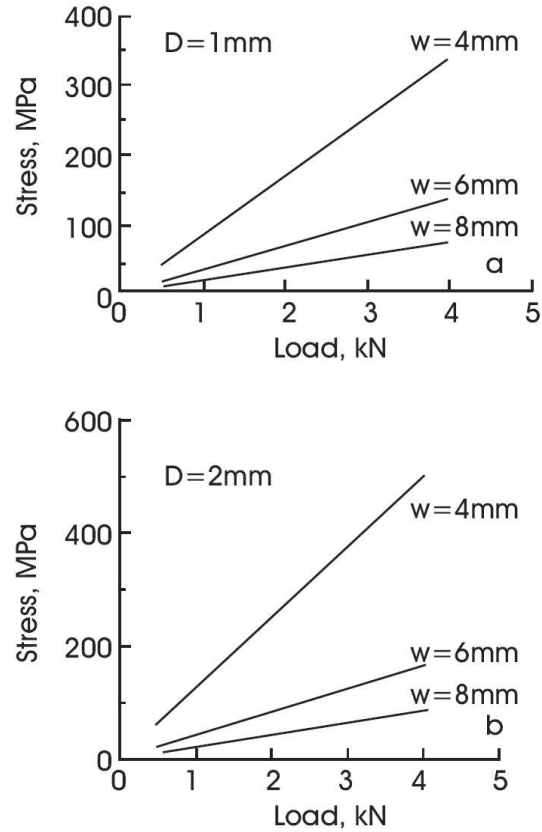


Fig. 5 Stress vs. load a) hole diameter = 1, b) hole diameter = 2mm.

Based on the critical loads calculated for the carbon/vinylester and E-glass/vinylester specimens with  $w = 4$  and  $6$  mm , compressive stress results are listed in Table 2.

Table 2. Compressive stresses (MPa) at onset of fiber/matrix debonding

D , mm	w, mm	Carbon/vinyl ester	E-glass / vinyl ester
1	4	32.8	49.6
	6	29.2	44.6
2	4	49.0	74.4
	6	36.8	55.8

These values may be compared to the yield stress of vinylester in Table 1. For all the specimens with a 1mm hole the compressive stress is below yield. The compressive stresses for all specimens are below the yield strength of the vinylester matrix, although the  $w = 4\text{mm}$  E-glass/vinylester specimen is very close to yielding at the onset of debond initiation. Also the  $w = 6\text{mm}$  E-glass/vinylester specimen is close to yielding.

Hence, for initial design it is recommended to use specimens with a side length of  $w = 6\text{mm}$  and maximum a 1mm diameter hole in the center. The larger cross section (larger  $w$ ) and smaller hole diameter are also beneficial for resisting buckling. A simple Euler estimate assuming both ends simply supported neglecting the hole and fiber yields buckling stresses of 360 and 808 MPa, for 12.7 mm long specimens with  $w = 4$  and 6 mm, respectively. Consequently, the stability of a  $w = 6\text{mm}$  specimen should not be an issue.

#### 4. Conclusions

The Outwater-Murphy single fiber specimen has been examined as candidate test to determine the fracture energy required for debonding of a fiber from the interface. The analysis is based on a simple mechanics of materials model where the compliance change of the specimen is determined as a function of debonded area. The calculations indicate that specimens with high fiber/matrix adhesion and low strength will fail in compression before the fiber debonds from the matrix. Published estimates of the debonds toughness of carbon and glass fibers in thermoset resin matrices indicate that specimen could be tailored to achieve the desired failure mechanism.

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