A DIFFERENTIAL GEOMETRY APPROACH TO FORMING SIMULATION OF BIAXIAL PREFORMS

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Abstract

The forming simulation of biaxial woven fabrics is described in this paper. The simulation applies to a general surface defined numerically and for any choice of initial warp and weft paths.

The yarn path between consecutive warp and weft crossover points or nodes is assumed to be a geodesic. The algorithm computes the location of the nodes by solving for the intersection of two geodesic paths of given lengths on the surface.

1 Introduction

This paper is concerned with the problem of forming biaxial preforms on generally complex non-developable shapes from plane sheets. Simulation of forming processes of composite structures is necessary to verify the feasibility of the process for a given shape and also to locate the position of the reinforcements. The problem of mapping an initial plane, orthogonal network of inextensible fibers onto a given curved surface was first considered by Tchebychev [1]. He suggested a continuum model for cloth in which the yarns are continuously distributed and inextensible. More than half a century later, Mack & Taylor [2] proposed a more practical solution to the problem in the context of fitting woven cloth fabrics on a hemisphere. Subsequent authors [3, 4, 5, and 6] developed more general methodologies based on geometrical consideration and applicable for complex surfaces. The geometric models do not consider the mechanical properties of the fabrics. Fabric construction can have a significant impact on the deformed patterns. Several authors [7, 8, and 9] developed fabric mapping models based on fabric mechanics using FE techniques. Finite element based models simulate more realistically the forming processes as preform properties and specific boundary conditions are incorporated in the modeling. The main problem with FE based simulations is the CPU time which is of the order of hours while for geometric models typically CPU time is less than a minute.

The model presented here, applied to woven fabrics, can be considered as part of the family of fishnet type algorithms. However in this paper the yarns are made to follow geodesic paths, between nodes, on the complex surface to improve the accuracy of forming coarse fabrics.

Woven fabrics are composed of two set of interlaced yarns, the warp and the weft. The point of contact between a warp and weft is referred as node. In its un-deformed state, the warp and the weft yarns are perpendicular to each other. Woven fabrics undergo in-plane shear deformation and bending when forced to conform to a surface having double curvature.

2 Forming Problem

Forming biaxial preforms involves the mapping of an orthogonal network of yarns onto a 3D surface. This is illustrated in Figure 1 below.

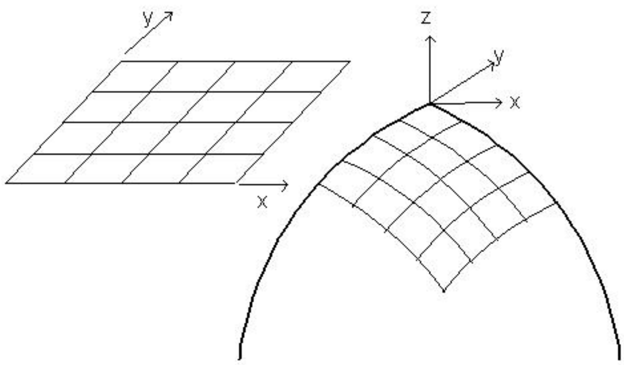

Fig. 1. Fitting of Biaxial Preform on a Surface

The forming problem as applied to woven fabrics is to locate the coordinates of the warp-weft intersection, i.e. nodes, on the 3D surface. In Figure

2 the coordinates of nodes A and B are known and it is required to determine the coordinates of point C on the surface with the condition that the lengths AC and BC are the same. Further the distances between the nodes are not changed by the mapping process which implies the assumption that there is no slippage between the warp and weft during the forming process.

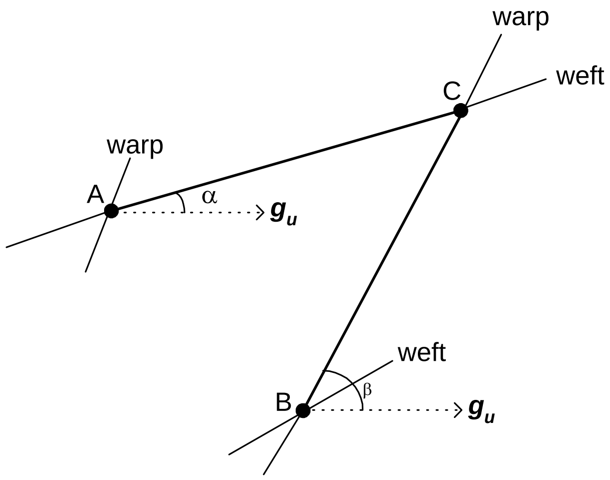

Fig. 2. Adjacent Fabric Nodes

Fishnet type algorithms assume that the paths AC and BC are straight lines. In this paper the paths AC are BC are assumed to be geodesics.

3 Surface Definitions

The surface is divided into a set of curvilinear coordinates (u, v) and a specific method is needed to define the surface in the regions between the input dataset that characterize the surface.The position vector of an arbitrary point on the surface in terms of Cartesian coordinates is:

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \tag{1}$$

The corresponding covariant components are:

$$\mathbf{g_u} = \frac{\partial \mathbf{r}}{\partial u} = \frac{\partial x}{\partial u}\mathbf{i} + \frac{\partial y}{\partial u}\mathbf{j} + \frac{\partial z}{\partial u}\mathbf{k} \tag{2}$$

$$\mathbf{g_v} = \frac{\partial \mathbf{r}}{\partial v} = \frac{\partial x}{\partial v}\mathbf{i} + \frac{\partial y}{\partial v}\mathbf{j} + \frac{\partial z}{\partial v}\mathbf{k} \tag{3}$$

where u and v are the curvilinear coordinates, $\mathbf{g_u}$ and $\mathbf{g_v}$ are covariant base vectors. In practical applications the surface is characterized by a given set of Cartesian coordinates. In between the known coordinates the surface is defined by a cubic Hermite interpolation. For example the x-coordinate is defined in equation (4).

$$x = \begin{bmatrix} s^3 & s^2 & s & 1 \end{bmatrix} \mathbf{H}.\mathbf{X}.\mathbf{H}^{\mathrm{T}}.\begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix} \tag{4}$$

where

$$\mathbf{H} = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$\mathbf{H}^{\mathrm{T}}$ is the transpose of matrix $\mathbf{H}$.

$$\mathbf{X} = \begin{bmatrix} x_{00} & x_{01} & x_{00}^{t} & x_{01}^{t} \\ x_{10} & x_{11} & x_{10}^{t} & x_{11}^{t} \\ x_{00}^{s} & x_{01}^{s} & x_{00}^{st} & x_{01}^{st} \\ x_{10}^{s} & x_{11}^{s} & x_{10}^{st} & x_{11}^{st} \end{bmatrix}$$

The parameters s and t varies between 0 and 1 for the surface patch with vertices (x_{ij}, y_{ij}, z_{ij}) where indices i & j are either 0 or 1. The superscript in matrix $\mathbf{X}$ above refers to partial derivative of the Cartesian coordinate with respect to that parameter.

4 Generating Geodesic Paths

Given any two points on a surface there are an infinite number of paths that can link them. The path with the shortest length is a geodesic. On a plane surface the straight line and on a sphere the great circles are examples of geodesics. Geodesic paths on general complex surfaces can only be established by using numerical methods. Suppose a surface is described by curvilinear coordinates (u, v). Then the geodesic path from a given point in a given orientation is defined in equations (5) & (6) below.

$$\frac{d^2u}{ds^2} = -\Gamma^{u}_{uu}\left(\frac{du}{ds}\right)^2 - 2\Gamma^{u}_{uv}.\frac{du}{ds}.\frac{dv}{ds} - \Gamma^{u}_{vv}\left(\frac{dv}{ds}\right)^2 \tag{5}$$

$$\frac{d^2v}{ds^2} = -\Gamma^v_{uu}\left(\frac{du}{ds}\right)^2 - 2\Gamma^v_{uv}.\frac{du}{ds}.\frac{dv}{ds} - \Gamma^v_{vv}\left(\frac{dv}{ds}\right)^2 \tag{6}$$

To solve the nonlinear second order differential equations above, it is helpful to convert them to first order and then use numerical methods such as Runge-Kutta.

$$\frac{du}{ds} = p \tag{7}$$

$$\frac{dv}{ds} = q \tag{8}$$

$$\frac{dp}{ds} = -\Gamma^u_{uu}.p^2 - 2\Gamma^u_{uv}.p.q - \Gamma^u_{vv}q^2 \tag{9}$$

$$\frac{dq}{ds} = -\Gamma^v_{uu}.p^2 - 2\Gamma^v_{uv}.p.q - \Gamma^v_{vv}q^2 \tag{10}$$

In general to solve the above set of first order differential equations (7) to (10) we need initial values for u, v, p and q i.e. at $s = 0$. The coordinates u and v define the starting point of the geodesic path while p and q define the geodesic direction on the surface.

The Christoffel symbol is defined as follows:

$$\Gamma^k_{ij} = \mathbf{g}^{\mathbf{k}}.\frac{\partial \mathbf{g_i}}{\partial x^j} \tag{11}$$

where indices i ,j & k are either of the curvilinear components u or v and $\mathbf{g}^{\mathbf{k}}$ is the contravariant base vector.

5 Computations of Initial Values of Geodesic Paths

Suppose the direction of the geodesic is angle α to the constant v-curves, at the starting point. Consider vector $\boldsymbol{\delta}\mathbf{r}$ of length δs along the constant local v-coordinate. Further consider vector $\mathbf{dr}$ of magnitude ds making an angle α with $\boldsymbol{\delta}\mathbf{r}$ as shown in Figure 3. Vectors $\mathbf{dr}$ and $\boldsymbol{\delta}\mathbf{r}$ can then be defined as in equations (12) and (13).

$$\mathbf{dr} = du.\mathbf{g_u} + dv.\mathbf{g_v} \tag{12}$$

$$\boldsymbol{\delta}\mathbf{u} = \delta u.\mathbf{g_u} \tag{13}$$

The length ds of vector $\mathbf{dr}$, the displacement δu and the angle α are given. The objective is to determine the values du and dv, hence the ratio $p = \frac{du}{ds}$ & $q = \frac{dv}{ds}$.

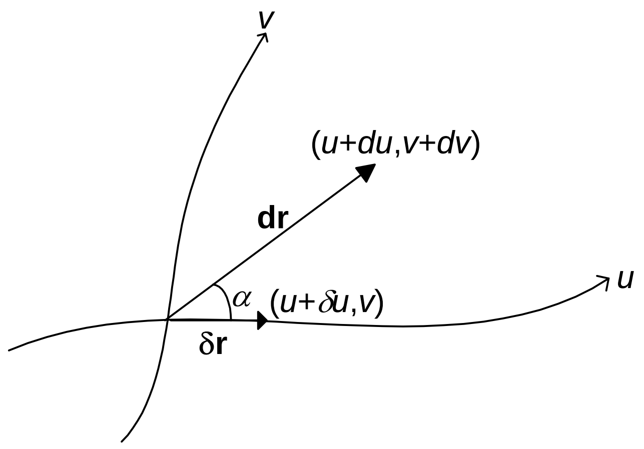

Fig. 3. Orientation of Geodesic at Starting Point

Taking the dot product of $\mathbf{dr}$ and $\boldsymbol{\delta}\mathbf{r}$ we get:

$$\delta s.ds.\cos\alpha = E.du.\delta u + Fdv.\delta u \tag{14}$$

Taking the cross product of d$\mathbf{r}$ and d$\mathbf{u}$ we get:

$$\delta s.ds.\sin\alpha = \sqrt{EG - F^2}\,|dv.\delta u| \tag{15}$$

where

$$E = \mathbf{g_u}.\mathbf{g_u} \tag{16}$$
$$F = \mathbf{g_u}.\mathbf{g_v} \tag{17}$$
$$G = \mathbf{g_v}.\mathbf{g_v} \tag{18}$$

Given a small value for δu:

$$\delta s = \delta u.\sqrt{E} \tag{19}$$

Given also the small value ds, the value dv can be computed from equation (15). By replacing the computed value of dv in equation (14), du can be worked out. Hence the initial values, p and q, can be calculated.

6 Initial Paths

The computation of the image of a fabric node on the surface requires the positions of two known nodes on the surface. To initiate the forming simulation, two intersecting yarn paths are chosen on the surface. Generally the paths make an angle of 90^o at the intersection, which is the starting point of the forming process. This is illustrated in the schematic diagram in Figure 4. One set of lines, say the verticals, represent the warp while the horizontals are the weft. The specification of the initial paths would affect the degree of deformation of the fabric as it made to conform to the surface and also the positions, if any, of wrinkles. The initial paths divide the surface into four quadrants and the forming in each quadrant is independent of each other.

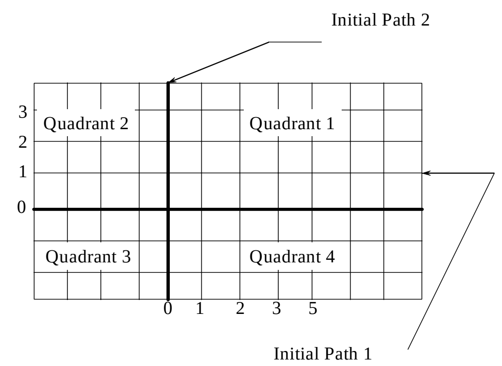

Fig.4. Initial Paths and Surface Quadrants

The nodes on the initial paths are computed as equidistant points on the geodesic paths which are obtained by integrating equations (7) to (10). The selection of the paths is effectively a decision on the forming start point and the orientation (α and β) of the two geodesics with respect to one of the curvilinear coordinates. The forming process within say quadrant 1 then follows by the computing nodes sequentially along either warp or weft lines.

7 Forming Algorithm

The output of the forming algorithm is the angles α and β, i.e. the angles the geodesic paths of given length from A and B make with the curvilinear component u. The flowchart in Figure 5 illustrates the methodology used.

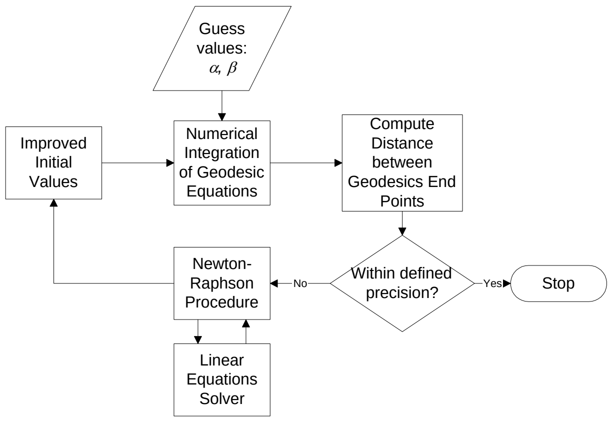

Fig. 5. Forming Algorithm

The coordinates of the point intersection of the geodesic paths is the mapping of a fabric node on the surface. This known location is then used to locate subsequent nodes on the fabric and the process is repeated until the surface boundary is reached or some other termination condition is specified such as onset of wrinkling which occurs when angle between warp & weft is less than some value, which is a characteristic of the fabric.

7.1 Estimates of Initial Values

The computation time as well as whether the Newton-Raphson's iteration will converge depend on the choice of guess estimates for the two initial values α and β.

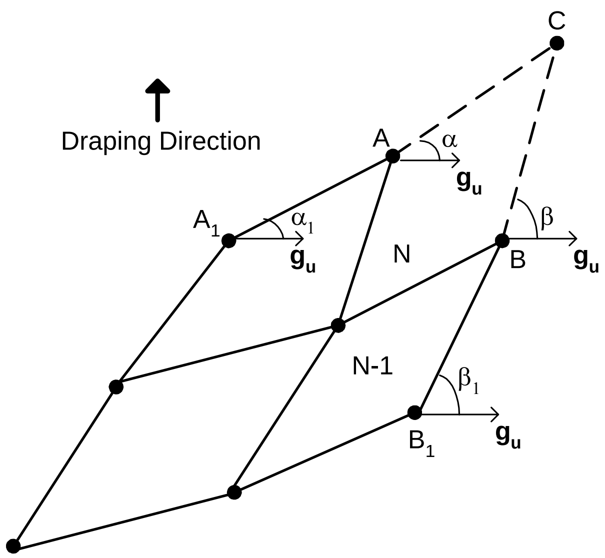

Fig. 6. Estimates of Orientation of AC & BC

With reference to Figure 6, the initial estimates of α and β are the same as the orientation of geodesics A_1A and B_1B to the covariant vector $\mathbf{g_u}$, i.e. α_1 and β_1.

An improved estimate is provided by application of local Gauss-Bonnet Theorem [10] which relates the excess angle of a polygon bounded by geodesics on a surface to the Gaussian curvature and the area of the polygon. For example a better guess value for α is obtained as follows

$$\alpha = \alpha_1 + \sum_{i=1}^{N-1} K_i A_i \qquad (20)$$

where

K_i is the Gaussian Curvature computed at the vertex of fabric unit cell.
A_i is the area of the unit cell.

The cells are numbered in the draping direction with the cell adjacent to the initial path being numbered one. A similar expression holds for angle β.

8 Forming Examples

Figures 7 and 8 show the yarns' paths over a spherical surface and a complex part respectively. The colorbar indicates the values of the shear deformation in a fabric unit cell. Shear deformation is the angular change from the warp-weft angle in the plane cloth.

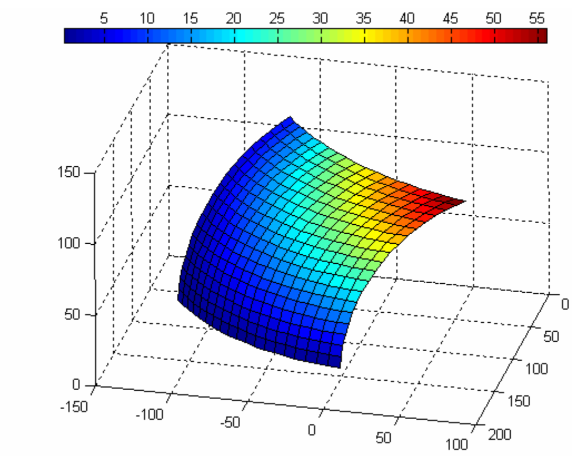

Fig. 7. Forming over a Spherical Surface

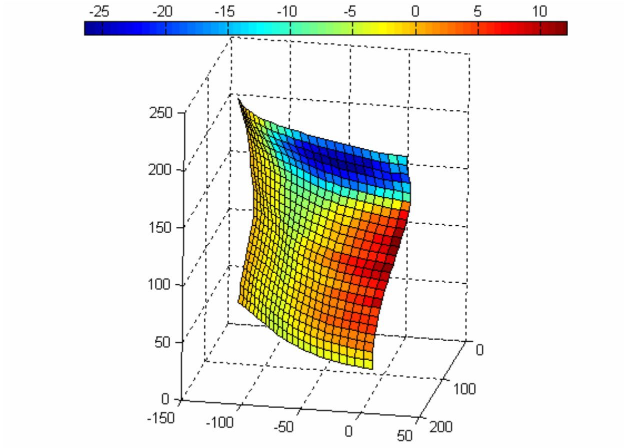

Fig. 8. Forming over a Complex Part

Shear deformation is measured at a specific vertex for all the unit cells that make up the fabric. Positive shear deformation implies decrease of the angle between warp and weft while negative shear deformation indicates angular increase.

In Figure 7 the surface is part of a sphere which has a constant positive curvature. The shear deformation is always positive and is highest at the furthest point from the simulation start position.

The complex part of Figure 8 contains areas of positive and negative curvature. The blue areas in Figure 8 indicate negative curvature and negative shear deformation and the red areas illustrates a decrease in warp-weft angle at regions of positive curvature.

9 Conclusions

Fishnet type algorithms specify straight line distance between nodes such as B and C in Figure 1, effectively putting the yarn below a surface of positive Gaussian curvature. By reducing the fishnet interval to minimize this error one would actually be using a different material. The algorithm described here measures lengths on the surface and so this error which would be significant for sharp areas or for coarse material mesh, is completely avoided. Further to reduce computation time and the risk of non-convergence of forming over sharp areas, a method of estimating initial values for geodesic orientation, based on Gaussian curvature in that area has been implemented.

References

[1] Tchebychev PL. “Sur la coupe des vêtements”. *Oeuvres II*, Acd. Sci.,St. Petersbourg , pp 708,1907.

[2] Mack C., Taylor H.M. “The fitting of woven cloth to surfaces”. *Journal of the Textile Institute*, Vol. 47, pp 477-488, 1956.

[3] Heisey F.L, Haller K.D. “Fitting woven fabric to surfaces in three dimensions”. *Journal of the Textile Institute*, pp 250-253, 1988.

[4] Van West B.P., Pipes R.B., Keefe M. “A simulation of the draping of bi-directional fabrics over arbitrary surfaces”. *Journal of the Textile Institute*, Vol. 81, pp 448-460, 1990.

[5] Aono M, Breen B.E, Wozny M.J. Modeling methods for the design of 3D broadcloth over arbitrary surfaces. *Computer Aided Design*, Vol. 33, pp 989-1007, 2001.

[6] Bourouchaki H., Cherouat A. “Drapage géométrique des composites”. *Comptes Rendus Méchanique*, Vol. 331, pp437-442, 2003.

[7] Boisse P., Borr M., Cherouat A. “Finite element simulations of textile composite forming including the biaxial fabric behaviour”. *Composites Part B*, Vol. 28, pp 453-464, 1997.

[8] De Luca P., Lefébure P., Pickett A.K. “Numerical and experimental investigation of some press forming parameters of two fibre reinforced thermoplastics: APC2-AS4 and PEI-CETEX”. *Composites Part A*, Vol.29, pp101-110, 1998.

[9] Boubaker B.B., Haussy B., Ganghoffer J. “Modèles discrets de tissues tissées: Analyse de stabilité et de drapé”. *Comptes Rendus Méchanique*, Vol. 330, pp 871-877, 2002.

[10] Dineen S. “*Multivriate calculus and geometry”*. Springer 2001, pp 221-223.