A SMEARED CRACK MODEL FOR SIMULATING DAMAGE IN LAMINATED COMPOSITES

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Abstract

This paper presents a new three-dimensional damage propagation model for laminated composites. The initiation of failure is predicted using a state-of-the-art physically-based set of criteria, LaRC04. The novelty in this work lies in the modelling of failure propagation. So that the propagation of cracks with any orientation in a 3D space (as predicted by LaRC04) can be more accurately modelled, the strains are divided into their crack and elastic components. Mesh dependency is addressed using a smeared crack formulation with a characteristic length parameter. The model is implemented in the commercial software ABAQUS Explicit, so it can be readily used by designers and engineers. Validation and application examples will be presented at the conference.

1 Introduction

In many circumstances – crashworthiness is an extreme example – structures must continue to perform in the presence of damage. Under these situations, practitioners need reliable models which, besides predicting the initiation of failure, go beyond and predict its propagation. To solve complex problems, these models are typically numerical, often based on the finite element (FE) method. Some recent approaches to model propagation take into account that the response of the structure depends on the energy that each failure mode dissipates, and incorporate the fracture toughness associated with each failure mode in the numerical propagation model. When used together with failure criteria that predict the orientation of the macroscopic fractures in the composite, energy-based damage models account for the degradation of the traction components acting on the fracture plane [1]. The proposed framework provides a more accurate simulation of damage propagation, while ensuring that the solution is mesh-independent.

2 Objectives and approach

This study focuses on the initiation and propagation of damage in laminated composite materials reinforced with unidirectional plies. The study aims at predicting failure initiation and propagation in these composites more accurately than it is achieved currently. The approach relies on attempting developments concurrently in three interrelated areas, see Fig. 1: experimental, analytical and numerical.

Initiation is predicted using a previously published set of criteria, LaRC04 [2]. These criteria define the orientation of the fracture plane (e.g. matrix cracking) or failure band plane (e.g. fibre compressive kinking) for the failure mode predicted.

Failure propagation is predicted with a

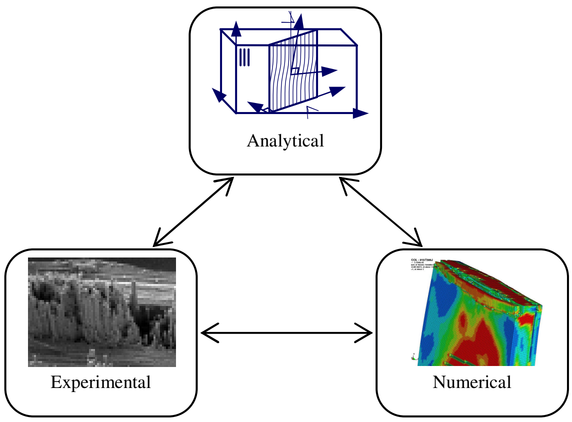

Fig. 1: Diagrammatic representation of the approach

Smeared Crack Model (SCM) recently coded into

FE, and is the main focus of this contribution. This SCM is a complete departure from earlier work [1], as it more correctly represents the presence of a macroscopic crack in the material by explicitly decomposing the total strain in its components related and not related to the macroscopic cracks being modelled [3].

3 Smeared crack model

In this SCM, the strain $\boldsymbol{\varepsilon}$ is explicitly divided into its elastic (i.e. not related to the macroscopic fracture process) component $\boldsymbol{\varepsilon}_e$ and the strain related to the fracture process $\boldsymbol{\varepsilon}_c$. Each fracture plane has an associated crack strain $\boldsymbol{\varepsilon}_{ci}$ and the total crack strain is therefore $\boldsymbol{\varepsilon}_c=\sum\boldsymbol{\varepsilon}_{ci}$. The elastic strain $\boldsymbol{\varepsilon}_e$ includes the linear components of strain as well as the strain resulting from material non-linearity but it does not include the strains related to the failure/fracture process. The analysis of a single failure feature will be summarised first.

The strain in the material coordinate system can be expressed as

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_c \,. \tag{1}$$

The stress $\boldsymbol{\sigma}$ relates to the elastic strain $\boldsymbol{\varepsilon}_e$ by

$$\boldsymbol{\sigma} = \mathbf{D}_e \boldsymbol{\varepsilon}_e = \mathbf{D}_e \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_c\right). \tag{2}$$

When expressed in a coordinate system aligned with the crack, Fig. 2, the only non-zero crack strains are $\mathbf{e}_c = \left(\varepsilon_n^c, \gamma_{mn}^c, \gamma_{nl}^c\right)^T$. The crack strains in the local crack coordinate system are related to the crack strains in the material coordinate system

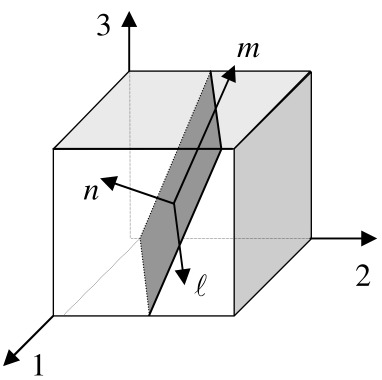

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Fig. 2: Local coordinate system aligned with the crack

$$\boldsymbol{\varepsilon}_c = \mathbf{T}_c \mathbf{e}_c \tag{3}$$

where $\mathbf{T}_c$ is the 6x3 transformation matrix

$$\mathbf{T}_c = \begin{bmatrix} n_1^2 & n_1 m_1 & n_1 \ell_1 \\ n_2^2 & n_2 m_2 & n_2 \ell_2 \\ n_3^2 & n_3 m_3 & n_3 \ell_3 \\ 2n_1 n_2 & n_1 m_2 + m_2 m_1 & n_1 \ell_2 + n_2 \ell_1 \\ 2n_2 n_3 & n_2 m_3 + m_3 m_2 & n_2 \ell_3 + n_3 \ell_3 \\ 2n_3 n_1 & n_1 m_3 + m_3 m_1 & n_1 \ell_3 + n_3 \ell_1 \end{bmatrix} \tag{4}$$

The constitutive law in Eq. (2) can then be written as

$$\boldsymbol{\sigma} = \mathbf{D}_e \left(\boldsymbol{\varepsilon} - \mathbf{T}_c \mathbf{e}_c\right). \tag{5}$$

The traction acting on the fracture plane $\mathbf{s} = \left(\sigma_n, \tau_{mn}, \tau_{nl}\right)^T$ can be obtained from $\boldsymbol{\sigma}$ and $\mathbf{T}_c$ as

$$\mathbf{s} = \mathbf{T}_c^T \boldsymbol{\sigma} \,. \tag{6}$$

A propagation law relates $\mathbf{s}$ to $\mathbf{e}_c$ such that the fracture energy is correctly modelled. A general law is of the form

$$\mathbf{s} = \mathbf{s}_o + \hat{\mathbf{D}}_c^* \mathbf{e}_c \tag{7}$$

or, in a rate form

$$\dot{\mathbf{s}} = \hat{\mathbf{D}}_c \dot{\mathbf{e}}_c \tag{8}$$

In Eqs. (7) and (8) , if $\hat{\mathbf{D}}_c^*$ does not depend on $\mathbf{e}_c$, then $\hat{\mathbf{D}}_c^* = \hat{\mathbf{D}}_c$. For the total formulation, an expression for the crack strain $\mathbf{e}_c$ can be obtained by equating Eqs. (6) and (7) , resulting in [3]

$$\mathbf{e}_c = \left(\mathbf{T}_c^T \mathbf{D}_e \mathbf{T}_c + \hat{\mathbf{D}}_c^*\right)^{-1} \left(\mathbf{T}_c^T \mathbf{D}_e \boldsymbol{\varepsilon} - \mathbf{s}_o\right) \tag{9}$$

or

$$\mathbf{e}_c = \left(\hat{\mathbf{D}}_e + \hat{\mathbf{D}}_c^*\right)^{-1}\left(\mathbf{T}_c^T \mathbf{D}_e \boldsymbol{\varepsilon} - \mathbf{s}_o\right) \tag{10}$$

with

$$\hat{\mathbf{D}}_e = \mathbf{T}_c^T \mathbf{D}_e \mathbf{T} \tag{11}$$

For the incremental formulation, the corresponding expression for $\dot{\mathbf{e}}_c$ is

$$\dot{\mathbf{e}}_c = \left(\hat{\mathbf{D}}_e + \hat{\mathbf{D}}_c^*\right)^{-1}\left(\mathbf{T}_c^T \mathbf{D}_e \boldsymbol{\varepsilon}\right) \tag{12}$$

Both (10) and (12) allow the determination of the crack strain $\mathbf{e}_c$ for a given strain state provided $\hat{\mathbf{D}}_c^*$ or $\hat{\mathbf{D}}_c$ do not depend on $\mathbf{e}_c$. For a general formulation of the constitutive law expressed in Eqs. (7) and (8) in the spirit of damage mechanics, it will be seen in the following that $\hat{\mathbf{D}}_c^*$ and $\hat{\mathbf{D}}_c$ do depend on $\mathbf{e}_c$. Specific models with $\hat{\mathbf{D}}_c^*$ and $\hat{\mathbf{D}}_c$ independent of $\mathbf{e}_c$ during damage propagation are possible, but not all reproduce automatically the physics of the decohesion process, ie. complete decohesion attained simultaneously for all traction components. For a general situation, all three components of $\mathbf{e}_c$ can be non-zero.

The decomposition of the total strain in elastic and crack components is represented in Fig. 3(a) for a one-dimensional case. The figure shows a tensile strain, applied at constant rate. In the elastic regime, the crack strain is zero and the total and elastic strains therefore coincide. During the propagation of failure, the crack strain increases while the elastic strain decreases. As the crack strain increases, the traction on the fracture surface decreases and eventually becomes zero at total failure. Since the only non-zero crack strains are the components of $\mathbf{e}_c$, then $\mathbf{s}$ and $\mathbf{e}_c$ are work-conjugate.

A simple linear cohesive law is presented in Fig. 3(b), and can be expressed as

$$\sigma_c = \sigma^o - \frac{\sigma^o}{\varepsilon^f}\varepsilon_c \tag{13}$$

with

$$\varepsilon_f = 2\frac{E_c}{\sigma^o} \tag{14}$$

For a 3D situation, this law could be expressed as

$$\begin{Bmatrix} \sigma_n \\ \tau_{mn} \\ \tau_{n\ell} \end{Bmatrix} = \begin{Bmatrix} \sigma_n^o \\ \tau_{mn}^o \\ \tau_{n\ell}^o \end{Bmatrix} + \underbrace{\begin{bmatrix} -\dfrac{\sigma_n^o}{\varepsilon_n^f} & 0 & 0 \\ 0 & -\dfrac{\sigma_{mn}^o}{\varepsilon_{mn}^f} & 0 \\ 0 & 0 & -\dfrac{\sigma_{n\ell}^o}{\varepsilon_{n\ell}^f} \end{bmatrix}}_{\hat{\mathbf{D}}_c^*} \begin{Bmatrix} \varepsilon_n^c \\ \varepsilon_{mn}^c \\ \varepsilon_{n\ell}^c \end{Bmatrix} \tag{15}$$

being

$$\varepsilon_i^f = 2\frac{E_c}{\left\|\mathbf{s}_o\right\|^2}\sigma_i^o \quad \text{with } i = n, mn, n\ell \tag{16}$$

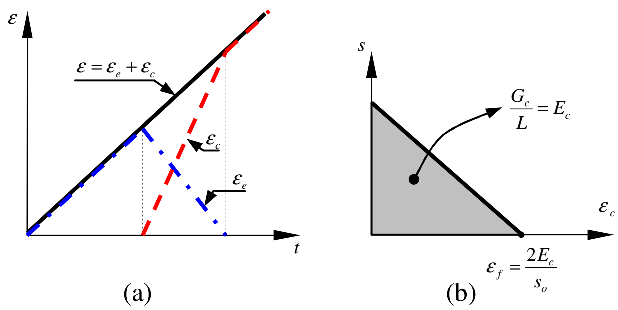

Fig. 3: (a) Elastic and crack components of the strain; (b) cohesive law and fracture toughness

In Fig. 3(b), the area under the curve is E_c, and it is an energy per unit volume. To simulate the fracture process accurately, this energy is calculated from the ratio between the critical energy release rate for the failure mode being simulated and a characteristic length. This characteristic length is the ratio between the volume associated with the integration point and the area of fractured surface in the same volume. The critical energy release rate for each failure mode is measured experimentally [4].

Eqs. (15) and (16) define $\hat{\mathbf{D}}_c^*$. Since $\hat{\mathbf{D}}_c^*$ does not depend on $\mathbf{e}_c$, then $\hat{\mathbf{D}}_c^* = \hat{\mathbf{D}}_c$ for this model. However, a problem with this constitutive cohesive model is that, for a non-proportional deformation, complete decohesion is not attained at the same time for all components of the traction vector. Anyway, Eqs. (10) and (12) can be used to determine the crack strain (increment), and Eq. (5) or its incremental form can be used to determine the stress (increment).

An alternative damage model to the one in Eq. (15) that guarantees full decohesion simultaneously for all crack strain components takes the simple form

$$\begin{Bmatrix} \sigma_n \\ \tau_{mn} \\ \tau_{n\ell} \end{Bmatrix} = (1-\omega)\begin{Bmatrix} \sigma_n^o \\ \tau_{mn}^o \\ \tau_{n\ell}^o \end{Bmatrix} \tag{17}$$

Neglecting irreversibility for the moment, the damage variable ω would be a function of $\mathbf{e}_c$, eventually through an equivalent strain $\tilde{\varepsilon}$. The damage variable ω would be given as

$$\omega = \frac{\tilde{\varepsilon}}{\tilde{\varepsilon}_f} \tag{18}$$

where $\omega = \tilde{\varepsilon}_f$ is the equivalent strain at complete decohesion. It is not trivial how this model could be used to explicitly (ie. without iterating) determine $\mathbf{e}_c$ using an equation similar to Eq. (10). Alternative damage formulations of the form

$$\mathbf{s} = (1-\omega)\mathbf{K}_c\mathbf{e}_c \tag{19}$$

or

$$\mathbf{s} = \mathbf{s}_o + \mathbf{h}\tilde{\varepsilon} \tag{20}$$

all have in common not resulting in a trivial determination of $\mathbf{e}_c$. In practical terms, this means that a series of iterations would be needed for determining $\mathbf{e}_c$ within each time step.

These limitations suggest that a model equal or similar to the one in Eq. (15) might still be preferable. Additionally, a simple modification guarantees full decohesion at the same time for all traction components. The modification consists in updating the orientation and magnitude of $\mathbf{e}_c^f$ so that all traction components vanish simultaneously at full decohesion. In Eq. (15), decohesion will happen at the same time for all traction components only if the crack strain at complete decohesion has the same orientation as

$$\mathbf{e}_c^f = \begin{Bmatrix} \varepsilon_n^f \\ \gamma_{mn}^f \\ \gamma_{n\ell}^f \end{Bmatrix} \tag{21}$$

given by Eq. (16). It is clear that $\mathbf{e}_c^f$ as determined from from Eq. (16) is parallel to $\mathbf{e}_c^o$. However, at time step (*i*), the crack strain corresponding to an earlier time step (*j*) is known (for now, suppose for instance *j* = *i* - 1), and its direction is a best estimate

$$\begin{Bmatrix} \sigma_n \\ \tau_{mn} \\ \tau_{n\ell} \end{Bmatrix}^{(i)} = \begin{Bmatrix} (\sigma_n)^{(j)} \dfrac{\varepsilon_n^f}{\varepsilon_n^f - (\varepsilon_n)^{(j)}} \\ (\sigma_n)^{(j)} \dfrac{\gamma_{mn}^f}{\gamma_{mn}^f - (\gamma_{mn})^{(j)}} \\ (\sigma_n)^{(j)} \dfrac{\gamma_{n\ell}^f}{\gamma_{n\ell}^f - (\gamma_{n\ell})^{(j)}} \end{Bmatrix} + \begin{bmatrix} \dfrac{-(\sigma_n)^{(j)}}{\varepsilon_n^f - (\varepsilon_n)^{(j)}} & 0 & 0 \\ 0 & \dfrac{(\tau_{mn})^{(j)}}{\gamma_{mn}^f - (\gamma_{mn})^{(j)}} & 0 \\ 0 & 0 & \dfrac{(\tau_{n\ell})^{(j)}}{\gamma_{n\ell}^f - (\gamma_{n\ell})^{(j)}} \end{bmatrix} \begin{Bmatrix} \varepsilon_n^c \\ \varepsilon_{mn}^c \\ \varepsilon_{n\ell}^c \end{Bmatrix}^{(i)} \tag{22}$$

to the direction of $\mathbf{e}_c^f$ than $\mathbf{e}_c^o$. Using the crack strain corresponding to time step (*j*), see also Fig. 4, the cohesive law in Eq. (22) follows.

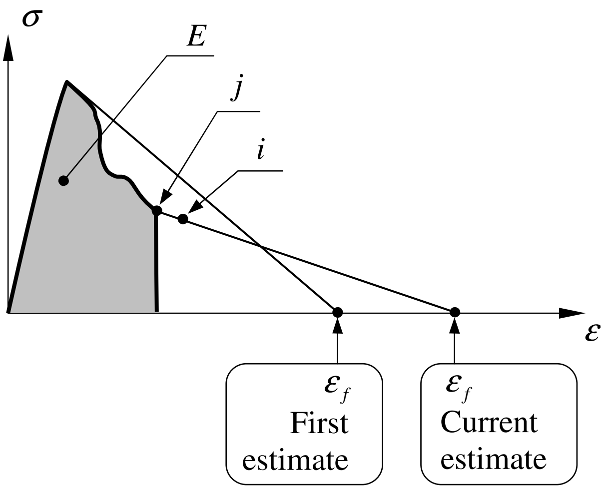

Fig. 4: Updating the estimate of $\mathbf{e}_c^f$

In order to guarantee correct energy absorption, the magnitude of $\mathbf{e}_c^f$ at time step (*i*) is obtained from

$$\left\|\mathbf{e}_c^f\right\|^{(i)} = \left\|\mathbf{e}_c\right\|^{(j)} + \frac{2\left(E_c - E^{(j)}\right)}{\left\|\mathbf{s}\right\|^{(j)}} \tag{23}$$

and the three components are recovered as

$$\left(\mathbf{e}_c^f\right)^{(i)} = \frac{\mathbf{s}(j)}{\left\|\mathbf{s}_o\right\|^2}\left\|\mathbf{e}_c^f\right\|^{(i)} \tag{24}$$

In Eq. (23), the variable $E^{(j)}$ is the energy E at time step (*j*), and is given by

$$E = \int \mathbf{s}^T d\mathbf{e}_c \tag{25}$$

4 Irreversibility and interpenetration

The model as formulated so far does not account for irreversibility and contact of the crack faces in compression. Interpenetration can easily be accounted for by replacing Eq. (5) by

$$\boldsymbol{\sigma} = \mathbf{D}_e\left(\boldsymbol{\varepsilon} - \mathbf{T}_c\left\langle \mathbf{e}_c \right\rangle_1\right). \tag{26}$$

where the symbol $\langle\cdot\rangle_1$ represents the McCauley bracket applied to the first component of $\mathbf{e}_c$.

For irreversibility, loading is defined as a situation for which the failure function ϕ (corresponding to the failure mode taking place) increases, thus resulting in the elastic domain being defined by the history variable κ as

$$\kappa(t) = \max_{\tau \le t} \phi(\tau) = r(\tau) \tag{27}$$

where $r(t)$ is the elastic domain threshold. Time step (*j*) in Eq. (22) should be the time step corresponding to the last time step when κ was modified. Assuming loading in a first instance, Eq. (26) can be used with the cohesive law given in Eq. (22). Then the loading assumption can be verified. If the assumption was wrong and the situation was of unloading, then the following cohesive law can be used to compute $\mathbf{e}_c$ in Eq. (10):

$$\begin{Bmatrix} \sigma_n \\ \tau_{mn} \\ \tau_{n\ell} \end{Bmatrix}^{(i)} = \begin{bmatrix} \dfrac{(\sigma_n)^{(j)}}{(\varepsilon_n)^{(j)}} & 0 & 0 \\ 0 & \dfrac{(\tau_{mn})^{(j)}}{(\gamma_{mn})^{(j)}} & 0 \\ 0 & 0 & \dfrac{(\tau_{n\ell})^{(j)}}{(\gamma_{n\ell})^{(j)}} \end{bmatrix} \begin{Bmatrix} \varepsilon_n^c \\ \varepsilon_{mn}^c \\ \varepsilon_{n\ell}^c \end{Bmatrix}^{(i)} \tag{28}$$

where time step (*j*) corresponds to the last time step when κ was modified.

5 More than one failure event

If more than one failure event takes place in the material (eg. matrix cracking and fibre breaking), then the formulation has to account for it. The stresses are in this case

$$\boldsymbol{\sigma} = \mathbf{D}_e \left(\boldsymbol{\varepsilon} - \sum_i \mathbf{T}_{ci} \langle \mathbf{e}_{ci} \rangle_1 \right). \tag{29}$$

The traction acting on fracture surface p is

$$\mathbf{s}_p = \mathbf{T}_{cp}^T \boldsymbol{\sigma}. \tag{30}$$

Thus one gets

$$\boldsymbol{\sigma} = \left(\mathbf{C}_e + \sum_i \mathbf{T}_{ci} \left(\hat{\mathbf{D}}_{ci}^* \right)^{-1} T_{ci}^T \right)^{-1} \left(\boldsymbol{\varepsilon} - \sum_i \mathbf{T}_{ci} \left(\hat{\mathbf{D}}_{ci}^* \right)^{-1} \mathbf{s}_i^o \right) = \left(\mathbf{C}_e + \overline{\mathbf{T}}_c \left(\overline{\hat{\mathbf{D}}}_c^* \right)^{-1} \overline{\mathbf{T}}_c^T \right)^{-1} \left(\boldsymbol{\varepsilon} - \overline{\mathbf{T}}_c \left(\overline{\hat{\mathbf{D}}}_c^* \right)^{-1} \overline{\mathbf{s}}^o \right) \tag{31}$$

where

$$\overline{\mathbf{T}}_c = \left[\mathbf{T}_{c1} \quad \mathbf{T}_{c2} \quad \dots \quad \mathbf{T}_{cn} \right], \tag{32}$$

$$\overline{\hat{\mathbf{D}}}_c^* = \begin{bmatrix} \hat{\mathbf{D}}_{c1}^* & & & \\ & \hat{\mathbf{D}}_{c2}^* & & \\ & & \dots & \\ & & & \hat{\mathbf{D}}_{cn}^* \end{bmatrix}. \tag{33}$$

and

$$\overline{\mathbf{s}}^o = \left[\mathbf{s}_1^o \quad \mathbf{s}_2^o \quad \dots \quad \mathbf{s}_n^o \right]. \tag{34}$$

Solving the previous equations implies inverting a matrix of order $3n$ where n is the number of fracture planes. Since this matrix is diagonal, this does not constitute a difficulty. However, a 6×6 matrix has to be inverted afterwards.

Loading can be assumed first, and once the stresses have been found, it is still necessary to verify that no interpenetration or unloading occurs. The crack strain can be retrieved for each crack i using

$$\mathbf{e}_{ci} = \left(\hat{\mathbf{D}}_{ci}^* \right)^{-1} \left(\mathbf{s}_i - \mathbf{s}_i^o \right). \tag{35}$$

6 Application examples

Application examples will be shown during the conference.

7 Conclusions

It is possible to formulate a model for damage propagation in composites which, while being suitable for implementation in standard FE codes, it can represent the propagation of cracks in the material.

8. Acknowledgements

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