SIZE EFFECTS ON THE STRENGTH OF NOTCHED COMPOSITES

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Abstract

This paper examines the use of a continuum damage model to predict strength and size effects in notched carbon-epoxy laminates. The effects of size and the development of a fracture process zone before final failure are identified in an experimental program. The continuum damage model is described and the resulting predictions of size effects are compared with alternative approaches. The results show that the continuum damage model is the most accurate technique to predict size effects in composites. Furthermore, the continuum damage model does not require any calibration and it is applicable to general geometries and boundary conditions.

1 Introduction

The introduction of advanced composite materials in new applications relies on the development of accurate analytical tools that are able to predict the mechanical response of composites under general loading conditions and geometries. In the absence of accurate analytical models, the design process has to rely on costly matrices of mechanical tests based on large numbers of test specimens.

The prediction of ultimate strength remains the main challenge in the simulation of the mechanical response of composite materials. The simulation of size effects on the strength of composites is of particular interest and relevance [1]: reliable analytical and numerical models must represent the variation of the ultimate strength when the structural dimensions increase [2].

Size effects in laminated composites occur at different material and structural levels. At the meso-mechanical level, it is observed that the transverse tensile and in-plane shear strengths of a ply constrained by sub-laminates depend on the ply thickness [3].

Size effects also occur at the macro-mechanical level. For example, it is shown in [4] that the strength of notched quasi-isotropic composite laminates decreases for increasing notch sizes when thin plies are used. This effect, usually known as the "hole size effect", is caused by the development and propagation of non-critical ply-level damage mechanisms that occur in the vicinity of the hole before the final collapse of the laminate. The ply-level damage mechanisms can be regarded as a fracture process zone that develops before final failure of the laminate. For very small specimens, the fracture process zone affects the entire width of the laminate. On the other hand, the size of the fracture process zone in large specimens is negligible when compared with the characteristic dimensions of the specimen. The relative dimension of the fracture process zone with respect to the specimen size justifies the different strengths observed in small and large specimens.

The calculation of macro-mechanical size effects is often based on semi-empirical methods that require calibration such as the point stress and average stress models proposed by Whitney and Nuismer [5]. The point stress model assumes that final failure occurs when the stress at a characteristic distance from the notch reaches the unnotched strength of the laminate.

Models based on continuum damage mechanics have the potential to provide the means for an effective methodology for the strength prediction of composite laminates. Continuum damage models are defined in the framework of the thermodynamics of irreversible processes. Generally speaking, the formulation of continuum damage models starts by the definition of a potential as a function of one or more damage variables that is the basis for establishing the relation between the stress and the strain tensors. It is also required to define the damage activation functions, i.e. the conditions that lead to the onset of inelastic response, and the damage evolution functions.

The objective of this paper is to investigate the use of a continuum damage model for the prediction of size effects in notched carbon-epoxy laminates loaded in tension. An experimental program is conducted to identify size effects occurring in laminates with different hole sizes. The recently proposed continuum damage model is described and analysis of open hole specimens subjected to tension loads are presented. The analyses results are compared with the experimental data and with predictions obtained using a strength of materials approach, Linear-Elastic Fracture Mechanics, and the point stress model.

2. Experimental program

2.1 Material selection and characterization

The material selected for is Hexcel's IM7-8552 carbon epoxy unidirectional tape with a nominal ply thickness of 0.131 mm. The elastic properties and strengths were measured using ASTM test standards. Five specimens were used for each test performed.

The mean measured values of the ply elastic properties are shown in Table 1. E_1 and E_2 are the longitudinal and transverse Young's modulus respectively, G_{12} is the shear modulus, and υ_{12} is the major Poisson's ratio.

Table 1. IM7-8552 elastic properties

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	υ_{12}
171.4	9.1	5.3	0.3

The measured ply strengths are shown in Table 2. X_T and Y_T^{ud} are the longitudinal and transverse tensile strengths, respectively. X_C and Y_C are the longitudinal and transverse compressive strengths, respectively. S_L^{ud} is the in-plane shear strength.

Table 2. IM7-8552 strengths (MPa)

X_T	X_C	Y_T^{ud}	Y_C	S_L^{ud}
2326	1200	62	200	92

The values of the transverse tensile strength (Y_T^{ud}) and of the in-plane shear strength (S_L^{ud}) measured in the test specimens correspond to the strengths of unconstrained unidirectional plies. The transverse tensile and shear strengths of constrained plies (in-situ strengths) are higher than the ones of an unidirectional ply and decrease when increasing the ply thickness. The in-situ strengths are calculated using models previously proposed by the authors, which are based on the mode I fracture toughness, G_{2+}, and on the mode II fracture toughness, G_6 [6]. The measured components of the fracture toughness are G_{2+}=0.28N/mm and G_6=0.80N/mm.

The in-situ strengths are calculated as functions of the fracture toughness and ply elastic properties using the models described in [6] with a shear response factor $\beta = 2.98\times10^{-8}$ MPa $^{-3}$. The calculated in-situ strengths are shown in Table 3.

Table 3. In-situ strengths (MPa)

Ply configuration	Y_T	S_L
Thin embedded ply	160.2	130.2
Thin outer ply	101.4	107.0

The continuum damage model also requires the fracture energies per unit surface for longitudinal failure, G_{1+} (tension) and G_{1-}

(compression). The measured fracture energies per unit surface are G_{1+} =81.5N/mm and G_{1-} =106.3N/mm.

2.2 Notched laminates

Tests of notched composite laminates were performed to quantify the size effect and to obtain empirical data to validate the numerical model. Quasi-isotropic laminates were manufactured in Hexcel IM7-8552 CFRP with a stacking sequence of $[90/0/\pm 45]_{3s}$.

Specimens with five different hole diameters, d =2mm, 4mm, 6mm, 8mm, 10mm and with a width-to-diameter ratio w/d equal to 6 were tested in a MTS servo-hydraulic machine following the ASTM D-5766 standard [7]. Five specimens were tested for each geometry.

Figure 1 shows the applied load and the cumulative number of AE signals as a function of time for one test specimen with a 8mm diameter hole.

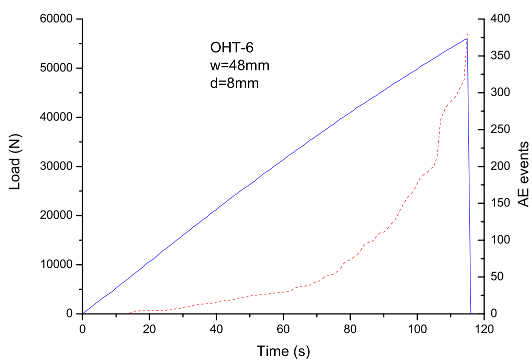

Fig. 1. Load-time-cummulative AE events relations.

From the AE signals shown in Figure 1, it can be concluded that non-critical damage mechanisms accumulate well before final failure of the specimen, creating a fracture process zone (FPZ).

The remote failure stress is defined using the failure load measured in the tests ($\overline{P}$) and the measured values of the specimen thickness (t_L) and width (w) as: $\overline{\sigma}^{\infty} = \dfrac{\overline{P}}{wt_L}$. The mean values of the remote failure stresses obtained for the different geometries are summarized in Table 4.

Table 4. Measured failure stresses (MPa)

Hole diameter (mm)	$\overline{\sigma}^{\infty}$
2	555.7
4	480.6
6	438.7
8	375.7
10	373.7

The failure mode observed in all specimens is net-section tension, as shown in Figure 2.

Fig. 2. Net-section tension failure mode.

The experimental results presented in Table 4 clearly identify a size effect: an increase in the hole diameter from 2mm to 10mm results in a 32.8% reduction in the strength.

3. Simulation of the effect of size on strength

Strength prediction methods uniquely based on stress or strain failure criteria are unable to predict the size effects observed in notched specimens. Consider for example a calculation of the final failure of a specimen with a central hole using the value of the longitudinal stress in the fiber direction (maximum stress criterion). The remote failure stress is given as [10]:

$$\overline{\sigma}^{\infty} = \frac{(1-d/w)X_T}{[2+(1-d/w)^3](\overline{Q}_{11}a^*_{11}+\overline{Q}_{12}a^*_{12})} \qquad (1)$$

Equation (1) demonstrates that the

application of the maximum stress criterion results in the same strength prediction for different hole diameters when the *d/w* ratio is held constant.

There are two approaches that can be used with Linear Elastic Fracture Mechanics (LEFM) to calculate the effect of size on the strength of notched composite laminates. In the first approach, it is assumed that the length a of a pre-existing crack in the laminate is scaled in the same proportion of the hole diameter and specimen width and that the critical value of the laminate's stress intensity factor, K_{Ic}, is independent of the crack length. Under these circumstances, the failure stress of a specimen with a hole diameter d_2 can be calculated from the failure stress of the specimen with a hole diameter d_1 as:

$$\bar{\sigma}_2^{\infty} = \bar{\sigma}_1^{\infty}\sqrt{\frac{d_1}{d_2}} \tag{2}$$

The second approach to predict size effects using LEFM is the inherent flaw model (IFM) proposed by Waddoups et al. [11]. It is considered that the non-critical damage mechanisms occurring before ultimate failure of a composite laminate can be lumped into a "inherent flaw" of length a. The strength of the laminate containing an open-hole is predicted using two parameters: the length of the inherent flaw, a, that needs to be calculated from a baseline specimen, and the unnotched tensile strength of the laminate, X_T^L, as:

$$\bar{\sigma}^{\infty} = X_T^L / f(a,R) \tag{3}$$

The point-stress model (PSM) proposed by Whitney and Nuismer [5], considers that ultimate failure occurs when the stress at a given distance from the hole boundary, r_{ot}, reaches the unnotched strength of the laminate, X_T^L. The strength predicted using the PSM is [10]:

$$\bar{\sigma}^{\infty} = X_T\left\{\frac{2+(1-\frac{d}{w})^3}{6(1-\frac{d}{w})}A(\bar{Q}_{11}a_{11}^* + \bar{Q}_{12}a_{12}^*)\right\}^{-1} \tag{4}$$

with:

$$A = 2 + \left(\frac{d}{d+2r_{ot}}\right)^2 + 3\left(\frac{d}{d+2r_{ot}}\right)^4 \tag{5}$$

Failure is predicted using two parameters: the characteristic distance in tension r_{ot}, and the longitudinal tensile strength of the ply, X_T.

4. Continuum damage model

The continuum damage model used here is based on previous work by the authors. The full details of the derivation of the model can be found in [12]-[13].

4.2 Constitutive model

The proposed definition for the complementary free energy density of a ply is:

$$\begin{aligned} G \;=\; & \frac{\sigma_{11}^2}{2(1-\mathrm{d}_1)E_1} + \frac{\sigma_{22}^2}{2(1-\mathrm{d}_2)E_2} - \\ & \frac{\nu_{12}}{E_1}\sigma_{11}\sigma_{22} + \frac{\sigma_{12}^2}{2(1-\mathrm{d}_6)G_{12}} + \\ & (\alpha_{11}\sigma_{11} + \alpha_{22}\sigma_{22})\Delta T + (\beta_{11}\sigma_{11} + \beta_{22}\sigma_{22})\Delta M \end{aligned} \tag{6}$$

where the damage variable d_1 is associated with longitudinal failure, d_2 is the damage variable associated with transverse matrix cracking, and d_6 is the damage variable associated with longitudinal and transverse cracks. β_{11} and β_{22} are the coefficients of hygroscopic expansion in the longitudinal and transverse directions, respectively. ΔT and ΔM are the differences of temperature and moisture content with respect to the corresponding reference values. The strain tensor is calculated as:

$$\varepsilon = \frac{\partial G}{\partial \sigma} = H : \sigma + \alpha\Delta T + \beta\Delta M \tag{7}$$

The lamina compliance tensor can be

represented as:

$$H = \frac{\partial^2 G}{\partial \sigma^2} = \begin{bmatrix} \frac{1}{(1-d_1)E_1} & -\frac{\upsilon_{12}}{E_1} & 0 \\ -\frac{\upsilon_{12}}{E_1} & \frac{1}{(1-d_2)E_2} & 0 \\ 0 & 0 & \frac{1}{(1-d_6)G_{12}} \end{bmatrix} \quad (8)$$

4.3 Damage activation functions

It is assumed that the elastic domain is enclosed by four surfaces, each of them accounting for one damage mechanism: longitudinal and transverse fracture under tension and compression. Those surfaces are formulated by the damage activation functions based on the LaRC failure criteria [14].

4.4 Damage evolution laws and numerical implementation

Local strain-softening constitutive models that do not take into account the finite element discretization produce results that are mesh-dependent, i.e. the solution is non-objective with respect to the mesh refinement and the computed energy dissipated decreases with a reduction of the element size. An effective solution to assure objective solutions consists of using a characteristic length of the finite elements (l^*) in the definition of the constitutive model [15].

The energetic regularization of the model proposed requires the fracture energies associated with the four fracture planes used in the model. The exponential damage evolution laws proposed by the authors [12]-[13] are expressed in the following general form:

$$d_M = 1 - \frac{1}{f_N(r_N)} \exp\{A_M[1 - f_N(r_N)]\} f(r_K) \quad (9)$$

where the function $f_N(r_N)$ is selected to force the softening of the constitutive relation and it is taken as being independent of the material. The term $f(r_K)$ represents the coupling factor between damage laws and elastic threshold domains.

The regularization of the energy dissipated is performed by integrating the rate of energy dissipation for each failure mode. The energy dissipated in each failure mode must be independent of the element size, and must be equal to the fracture energy measured in the experiments:

$$\int_1^\infty \frac{\partial G}{\partial d_M} \frac{\partial d_M}{\partial r_M} dr_M = \frac{G_M}{l^*}, M = 1+,1-,2+,2-,6 \quad (10)$$

The parameters A_M that assure a mesh-independent solution are calculated solving the previous equation.

The constitutive model was implemented in the ABAQUS Finite Element (FE) code [16] as a user-written UMAT subroutine.

4.5 Simulation of quasi-isotropic open hole tension specimens

Finite element models of all specimens tested were created using ABAQUS [16] four-node S4 shell elements. The difference between the working and reference temperatures used to calculate the residual thermal stresses was -155° C. An implicit dynamic analysis was subsequently performed with a 2mm/min loading rate.

The models simulate the fracture process from the onset of damage up to structural collapse. Figure 3 shows the plane of localized deformation (fracture plane).

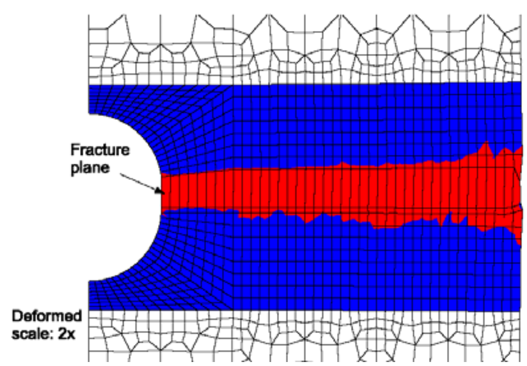

Fig. 3. Predicted fracture plane.

The remote failure stresses measured in the experimental program and predicted by the numerical model are shown in Table .

5. Comparison of approaches

The four methods previously described, i.e. strength of materials, LEFM-scaled, LEFM-inherent flaw model, point stress model, and continuum damage model were applied to predict the size effect for the specimens tested. The predictions of the normalized strength as a function of the hole diameter obtained using the different models are shown in Figure 4.

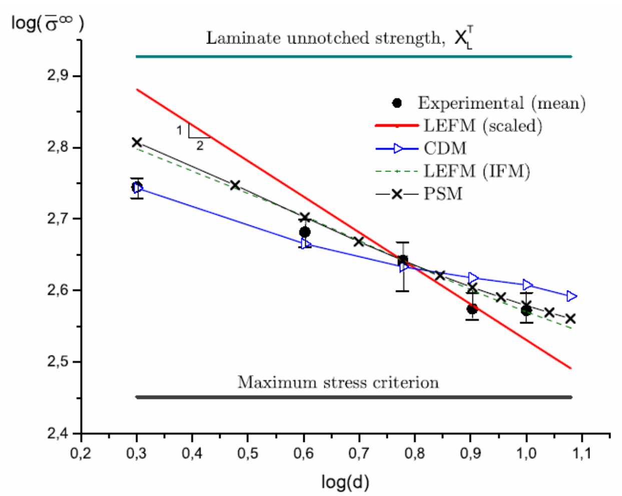

Fig. 4. Prediction of size effects in CFRP.

It can be observed that both the point stress and LEFM-IFM models can predict with reasonable accuracy the size effect law of notched composite laminates. The point stress and inherent flaw models accurate for specimens with hole diameters close to the diameter used to calculate the characteristic distance (PSM) and the length of the inherent flaw (6mm in both cases). For specimens with small hole diameters, the predictions lose accuracy. Therefore, to accurately predict the notched strength of laminates these models require the calculation of the characteristic distance and length of inherent flaw for different geometries [17].

The continuum damage model can predict the size effect law observed in the experiments, especially for specimens with hole diameters smaller than 6mm. Unlike the point stress and inherent flaw models, the continuum damage model does not require any adjustment parameter and only uses material properties that are measured at the ply level as well as the fracture energies.

The use of the LEFM-scaled model results in accurate predictions for hole sizes between 6mm and 10mm. However, the strength is overpredicted for small hole diameters. For small specimens, the damaged region in the vicinity of the hole cannot be considered to be negligible when compared with the characteristic dimensions of the specimen, and LEFM is not applicable.

The maximum stress criterion is unable to predict size effects and always underpredicts the strength of notched laminates.

6. Conclusions

By comparing the experimental data with the different models that are commonly used for the strength prediction of composites, it can be concluded that fiber-based failure criteria cannot predict size effects. In addition, the strength of materials approach always underpredicts the strength of notched composites, with errors as high as -49.1% for a specimen with a 2mm hole diameter.

The Linear Elastic Fracture Mechanics approach using a hole diameter of 6mm for calibration predicts the size effect accurately for specimens with hole diameters between 6mm and 10mm. However, Linear Elastic Fracture Mechanics should not be used for the strength prediction of larger specimens whose failure stresses tend to a constant value.

The point stress and inherent flaw models are simple approaches that provide reasonable predictions for the range of hole diameters tested. However, the accuracy of these models relies upon the measurement of the characteristic distance and length of the inherent flaw for each lay-up.

The continuum damage model proposed predicts with good accuracy hole size effects in composite laminates subjected to tension. The continuum damage models provides not only the final failure load, but also information concerning the integrity of the material during the load history.

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References

[1] M.R. Wisnom, “Size effects in the testing of fibre-reinforced composites”. *Composites Science and Technology*. Vol. 59, pp 1937-1957, 1999.

[2] Z.P. Bažant, “Size effect”. *International Journal of Solids and Structures*, Vol. 37, pp 69-80, 2000.

[3] A. Parvizi, K. Garrett, J. Bailey. “Constrained cracking in glass fibre-reinforced epoxy cross-ply laminates”, *Journal of Material Science*, Vol. 13, pp. 195-20, 1978.

[4] B.G. Green, M.R. Wisnom, S.R. Hallet, “An experimental investigation into the tensile strength scaling of notched composites”, *Composites-Part A*, Vol. 38, pp. 867-878, 2007.

[5] J. M. Whitney, R. J. Nuismer, R.J., “Stress fracture criteria for laminated composites containing stress concentrations”, *Journal of Composite Materials*, Vol. 8, pp. 253-265, 1974.

[6] P.P Camanho, C.G. Dávila, S.T. Pinho, L. Iannucci, P. Robinson, “Prediction of in-situ strengths and matrix cracking in composites under transverse tension and in-plane shear”, *Composites-Part A*, Vol . 37, pp. 165-176, 2006.

[7] Open hole tensile strength of polymer composite laminates, ASTM D 5766/D 5766M-02a, American Society for Testing and Materials (ASTM), West Conshohocken, PA, U.S.A.

[8] S.C. Tan, “Stress concentrations in laminated composites”, Technomic Publishing Company, Inc., 2004.

[9] C.T. Herakovich, “Mechanics of fibrous composites”, John Wiley and Sons, Inc., 1998.

[10] P.P. Camanho, P. Maimí, C.G. Dávila, “Prediction of size effects in notched laminates using continuum damage mechanics”, *Composites Science and Technology*, in press, 2007.

[11] M.E. Waddoups, J.R. Eisenmann, B.E. Kaminski, “Macroscopic fracture mechanics of advanced composite materials”. *Journal of Composite Materials*, Vol. 5, pp. 446-454, 1971.

[12] P. Maimí, P.P. Camanho, J.A. Mayugo, C.G. Dávila, “A continuum damage model for composite laminates: part I - constitutive model”. *Mechanics of Materials*, in press, 2007.

[13] P. Maimí, P.P. Camanho, J.A. Mayugo, C.G. Dávila, “A continuum damage model for composite laminates: part II - computational implementation and validation”. *Mechanics of Materials*, in press, 2007.

[14] C.G. Dávila, P.P Camanho, P.P., C.A. Rose, C.A., “Failure criteria for FRP laminates”. *Journal of Composite Materials*, Vol. 39, pp 323-345, 2005.

[15] Z.P. Bažant, M. Jirásek, “Nonlocal integral formulations of plasticity and damage: survey of progress”. *Journal of Engineering Mechanics*. Vol. 128, pp. 1119-1149, 2002.

[16] ABAQUS 6.5 User's Manual, ABAQUS Inc., Pawtucket, RI, U.S.A. (2005).

[17] P.P. Camanho, M. Lambert, “A design methodology for mechanically fastened joints in laminated composite materials”. *Composites Science and Technology*, Vol. 66, pp. 3004-3020, 2006.