STRENGTH DEGRADATION MODEL FOR FATIGUE LIFE PREDICTION

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Abstract

As application of composites in aerospace structures extends into primary structures, the needs for fatigue life prediction is critical for the reliability and safety of composite structures. Composite structures used in high cycle fatigue applications are often "over designed" and are somewhat heavier and more costly than necessary. Improved life prediction methodologies may results in more efficient use of these materials and may result in lower weight and lower cost structures.

Many models have been proposed to predict fatigue life of composite structures subjected to a variety of cyclic loading. Since composite laminates exhibit very complex failure processes, statistical life prediction methodologies have been prevailed.

In this paper we study the following statistical life prediction methodologies,

(1) Scatter factor in the recognition of structural configuration
(2) Consistency of estimated life to tested life (Introduction of Fudge factor)
(3) Simple fatigue life estimation method

The concerns in this study are (1) Weibull parameter conduction and its characteristics , (ii) Parameters necessary in the fatigue life estimations and (iii))Possibility of simple fatigue life estimation

1 Conduction of Weibull parameters and its characteristics

The Weibull parameters, i.e. the shape parameter and the scale parameter, are conducted from test data for static strength distribution and fatigue life distribution of carbon/epoxy laminates. The static strength data presented in this paper were obtained for various specimens of 0-degree tension(FL), 90-degree tension(FT), 0-degree compression(FL'), 90-degree compression(FT'), ±45-degree shear(FLT), laminate strengths of tension(Ftu), compression(Fcu), open hole tension(OHT) and crippling strength of flat plate(Fcc-Flat) or T-type(Fcc-T-type) shape.. The fatigue data were obtained for the specimens of open hole, single shear joint and double shear joint under constant amplitude loading of stress ratio R=－1.0. The configurations of fatigue test specimen are shown in fig.1.

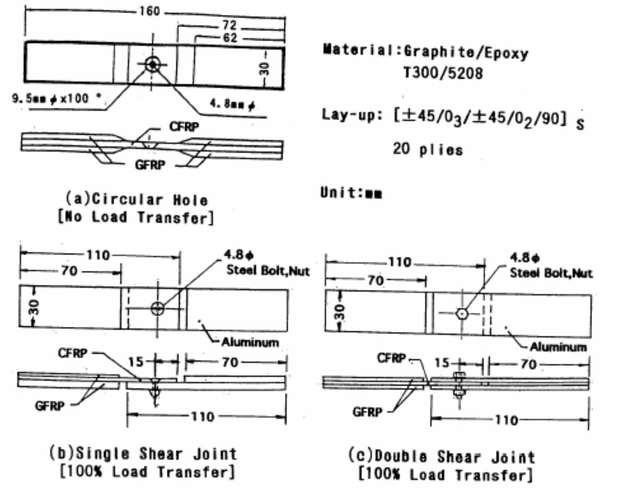
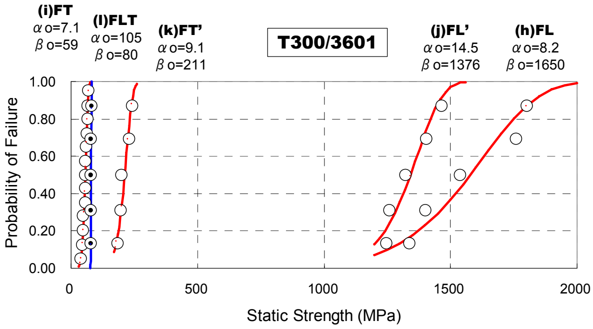
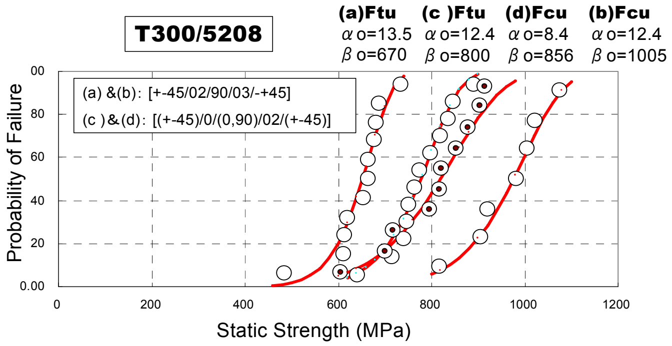


Fig.1 Configuration of fatigue test specimens

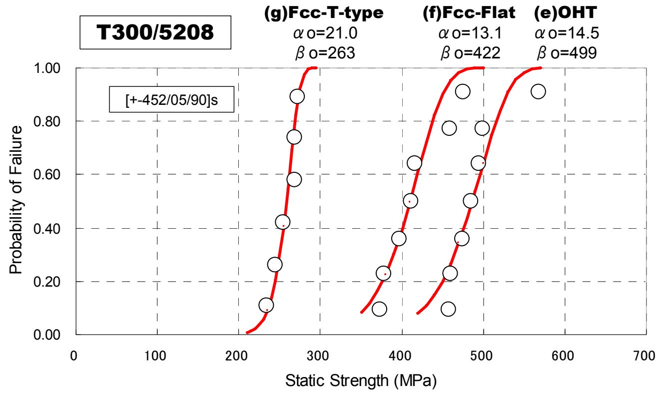
The Weibull parameters of static strength are shown in fig.2.



(a) Unidirectional properties



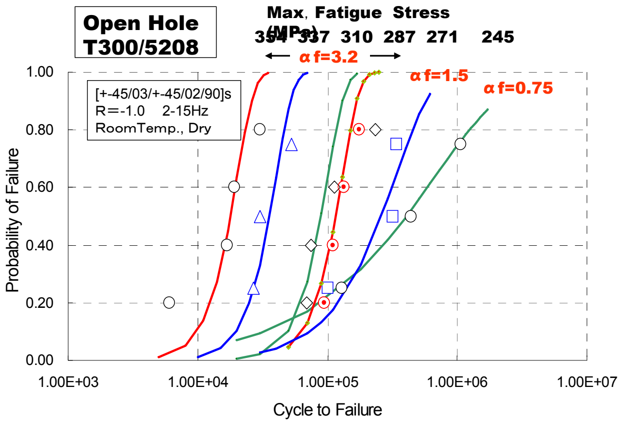
(b) Laminate properties



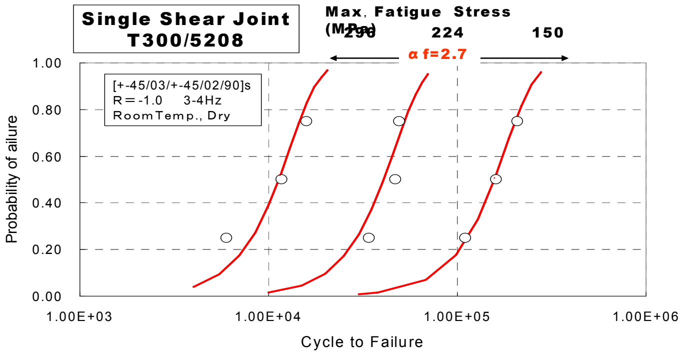
(c) Element properties

Fig.2 Weibull parameters of static strength

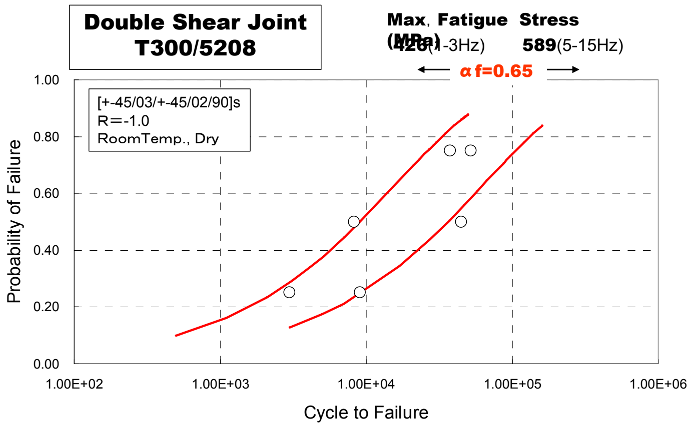
The Weibull parameters of fatigue life are shown in fig.3.



(a) Open hole



(b) Single Shear Joint



(c) Double Shear Joint

Fig.3 Weibull parameters of fatigue life

Fig.3(a) indicates shape parameter of fatigue life does not always have the same value for stress change.

The conducted Weibull parameters of static strength are listed in table 1 and fig.4 with probability density function and also those of fatigue life are listed in table 2 and fig.5 with cumulative distribution function.

Table 1 Weibull parameters of static strength

Shape Parameter: Static Strength

Strength	Shape Parameter, αo
Bending Specimen	28.0
Ftu	13.5 12.4
Fcu	12.4 8.4
OHT	14.5
Fcc (Flat Plate)	13.1
Fcc (T-type)	21.0
FL (0 tension)	8.2
FT (90 tension)	7.1
FL' (0 compression)	14.5
FT' (90 compression)	9.1
FLT (±45 shear)	105.0

Table 2 Weibull parameters of fatigue life

Shape Parameter: Fatigue Life

Configuration	Shape Parameter, αf
Bending Specimn	0.63 1.18
Open Hole	3.2 1.5 0.75
Single Shear Joint	2.7
Double Shear Joi	0.65

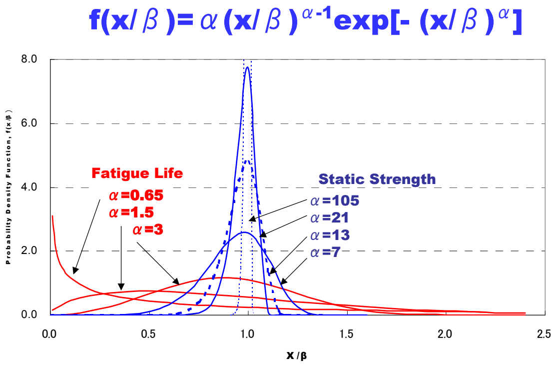


Fig.4 Probability density Function

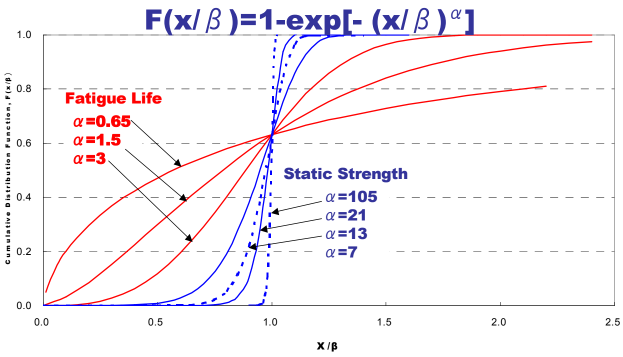


Fig.5 Cumulative distribution function

2 Scatter factor in the recognition of structural configuration

2.1 Conduction scatter factor

As shown in table 2, the shape parameters of fatigue life vary with every configuration of structural element. The scatter band is very wide, Therefore scatter factor should be established for the each structural configuration. Scatter factor depends on he shape parameter as follows,

Fatigue life distribution ,

$F_N(n) = 1 - \exp[-(n/N)^{\alpha f}]$

$(n/N)^{\alpha f} = -\ln[1 - F_N(n)]$

Scatter factor SF,

$$SF = (n/N) = [-\ln[1 - F_N(n)]]^{1/\alpha f} \quad (1)$$

MIL-HDBK-17 provides the two kinds of probability of failure $F_N(n)$ in confidence level 95%.

A value = Probability of failure $F_N(n)$ = 1%

B value = Probability of failure $F_N(n)$ = 10%

Mean value = Probability of failure $F_N(n)$ = 63.2% for Weibull distribution

The scatter factors were calculated by (1) and are shown in fig.6 and table 3.

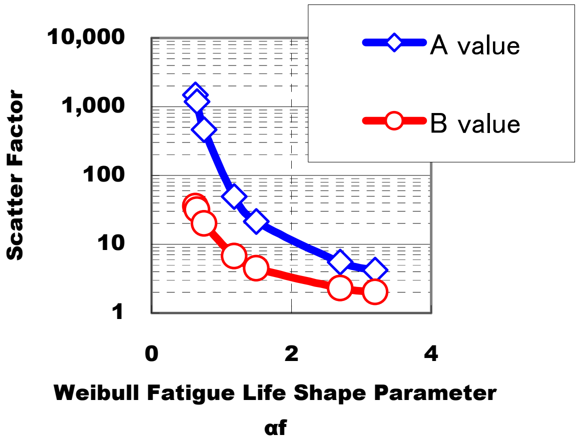


Fig.6 Scatter factor vs shape parameter

Table 3 Scatter factor for A value and B value

Configuration	αf	Scatter Factor	
		A value	B value
Bending Specimen	0.63	1,483	36
	1.18	49	6.7
Open Hole	3.2	4.2	2.0
	1.5	21	4.5
	0.75	461	20
Single Shear Joint	2.7	5.5	2.3
Double Shear Joint	0.65	1,184	32

2.2 Sample of safe life estimation

It is so surprised that the scatter factor of double shear joint is very large in comparison with one of single shear joint. However we have a good information that the tested lives of double shear joint is superior to single shear joint as shown in fig.7.

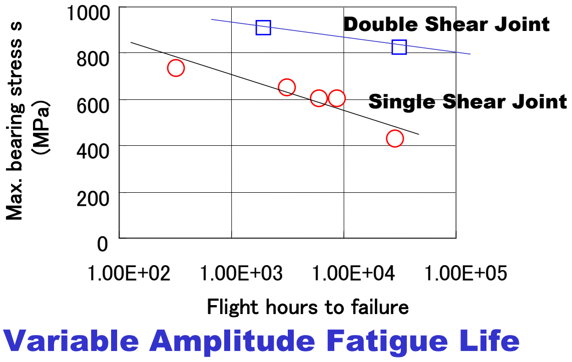


Fig.7 Tested life of double shear joint and single shear joint

Safe lives were calculated for the maximum bearing stress 800MPa and shown in table 4.

Safe Life = Tested Life / Scatter Factor

Table 4 Safe life comparison between single shear joint and double shear joint

Sample of Safe Life Calculation at max.Fbr 800MPa

Joint Configuration	Tested Life (Flight hours)	Scatter Factor (B value)	Safe Life (FLT hours)
Single Shear Joint	2.00E+02	2.3	87
Double Shear Joint	1.00E+05	32	3,125

The safe life of double shear joint is much longer than that of single shear joint in spite of the large scatter factor. Safe life depends deeply both on mean fatigue life (tested life in above) and scatter factor (shape parameter in table 3).

3 Comparison between Prediction and Test

3.1 Fatigue life estimation method

In this paper we study the following four estimation models,

(1) Miner's Rule
(2) Linear Residual Strength Reduction Fatigue Model [1]
(3) Residual Strength Degradation Parameter Model [2]
(4) Residual Strength Degradation Rate Model [3][4]

3.2 Parameters necessary to estimate fatigue life

The necessary parameters of each model are listed in a table 5. Residual strength degradation parameter " ν ", cycle mix constant "Cm" and load sequence parameter " γ " are introduced supplementarily into the life estimation to improve the accuracy.

Table 5 Parameters necessary in fatigue life estimation

		Prediction Items		
study model	Methodology	Residual Strength	Fatigue Life	Probability of Failure
	Miner's Rule	NA	ni/Ni	NA
model 1	Linear Residual Strength Reduction Fatigue Model	ni/Ni, βo	Residual Strength >Spectrum Stress	NA
model 2	Residual Strength Degradation Parameter ν	ni, Ni, βo, ν , Cm	Residual Strength >Spectrum Stress	ni, Ni, αo, αf, ν
model 3	Residual Strength Degradation Rate	ni, Ni, βo, αo, αf, γ	Residual Strength >Spectrum Stress	ni, Ni, βo, αo, αf, γ

ni = number of cycle in segment i
Ni = fatigue life scale parameter for segment i
βo = static strength scale parameter
αo = static strength shape parameter
αf = fatigue life shape parameter
ν = residual strength degradation parameter
Cm = cycle mix constant
γ = load sequence parameter

The procedure needs new disinclined test data to fit in to the degradation rate or the load sequence effects. Therefore the estimation becomes complicated and troublesome.

3.3 Consistency of estimated life to tested life

For the joint test articles as illustrated in fig.8, the ratios of calculated lives to the tested lives are 2.16 for Linear Residual Strength Reduction Fatigue Model (LRSRFM) and 2.12 for Residual Strength Degradation Parameter Model (ν=1.0). However, the ratio is 4.91 for Miner's Rule. The applicability of strength degradation models is better than Miner's Rule. In the case of ν=1.0, Residual Strength Degradation Parameter Model gives almost the same life as Linear Residual Strength Reduction Fatigue Model (LRSRFM).

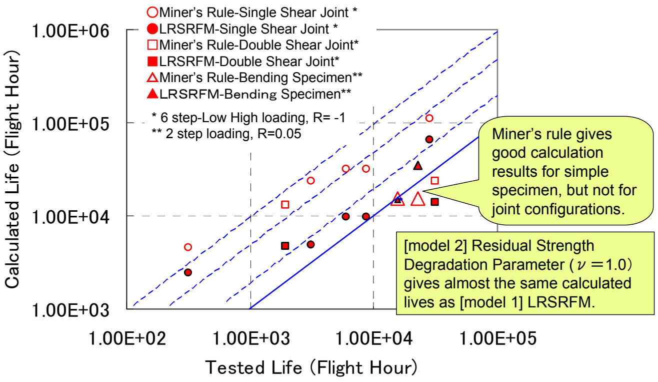


Fig.8 Consistency with test results

3.4 Introduction of fudge factor

The consistency of estimation model to test result is defined as the fudge factor,

Fudge Factor=Calculated Life / Tested Life

For the joints,

Fudge Factor = 5 to 10 for Miner's rule
Fudge Factor = 2 for LRSRM
Fudge Factor = 2 for Residual Strength Degradation Parameter Model (ν =1.0)

When we can get the Weibull parameters for static strength distribution and fatigue life distribution of constant amplitude loading, the residual strength distribution under variable amplitude loading can be predicted using the strength degradation models.

The Linear Residual Strength Reduction Fatigue Model (LRSRFM) is easier to calculate the fatigue life and can give the about twice longer life, but can not give the probability of failure.

The Residual Strength Degradation Parameter Model and the Residual Strength Degradation Rate Model can give both the fatigue life and the probability of failure. By introducing the parameters ν , Cm, γ , it is expected to improve the accuracy of life estimation. However, its calculation is rather complicated and troublesome.

An extensive program needs to evaluate more the applicability of the residual strength degradation models under the varying amplitude fatigue loadings. And also it is desired for simple calculations not using the fitting parameters ν , Cm, γ , because it needs hard testing and difficult interpretation of test results.

4 Proposal of simple fatigue life estimation method

We can estimate safe lives simply using the scatter factor (paragraph 2.1) and the fudge factor (paragraph 3.4). The schematic illustration is shown in fig.9.
That is,
Safe Life = Calculated Life/(Scatter Factor x Fudge Factor)

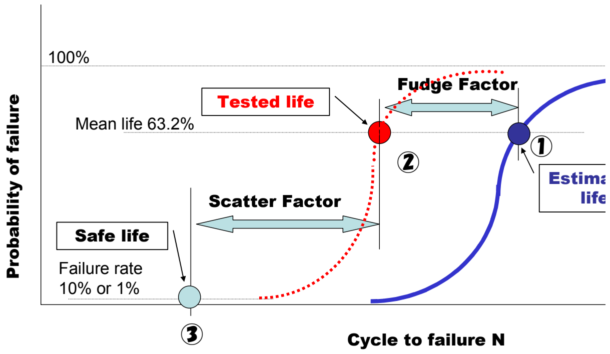


Fig.9 Schematic illustration of simple fatigue life estimation method

5 Conclusion

(1) Shape factors of fatigue life vary widely with every configuration of structural element. Therefore scatter should be established for each structural configuration.
(2) Safe life depends deeply both on mean life (tested life) and scatter factor (shape parameter). As shown in the sample estimation, the safe life of double shear joint is much longer than that of single shear joint in spite of the very large scatter factor (32 vs. single shear joint 2.3).
(3) Fudge factor depends on the statistical life prediction methodologies. The more tested parameters are used, the better consistencies to test life (fudge factor ~1.0) can be got. However it needs hard testing and difficult interpretation of test results.
(4) Safe life can be estimated simply by using the scatter factor and the fudge factor. Safe life=Calculated Life/(Scatter Factor x Fudge Factor)

References

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