GENERATION OF TRANSVERSAL MATERIAL RANDOMNESS IN FIBRE REINFORCED COMPOSITES

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1 Introduction

In micromechanics, the modelling of the transverse section of fibre reinforced composites has been a problem with multiple answers. While some authors (eg. [1]) consider periodicity of the fibre distribution, others consider that the fibres are randomly distributed in the matrix (eg. [2]). Matsuda et al. [3] studied the influence of such decision on the elastic-viscoplastic behaviour of long fibre reinforced laminates and proved that random distribution has a significant effect on the microscopic distribution of stress.

Until recently, in order to model accurately the stochastic distribution of fibres, analysts used either mirroring techniques of small 'hand-distributed' samples or digital image analysis techniques through which it is possible to acquire the exact fibre spatial distribution of a real material. Algorithms involving random point generators do not allow the analyst to achieve high fibre volume fractions (> 50%), making them useless for composites such as carbon fibre reinforced polymers (CFRPs), for example.

The present contribution provides analysts with a new tool capable of generating random fibre distributions with a high fibre volume fraction more rapidly and efficiently than the processes mentioned above. Statistical spatial descriptors and functions are used to demonstrate that the generated fibre distribution is stochastically valid. Finite element analyses (FEA) are also performed to demonstrate the transverse isotropy typical of long fibre reinforced materials.

2 Algorithm's Description

The algorithm here presented was written in Matlab® and is divided in three steps which are run sequentially. The sequence is then repeated in as many iterations as necessary to achieve the desired fibre volume fraction.

2.1 Step 1 – Hard-core Model

The first step is the hard-core algorithm. Points are generated stochastically inside a quadrangular area. This area is defined in micromechanics as a representative volume element (RVE). Each of the generated points corresponds to the centre of a fibre. Fibre radius, dimension of the RVE and minimum distance between fibres are all input variables. The hard-core model safeguards that there will be no overlapping of fibres during generation of coordinates, but it does not allow going beyond ~ 50% of fibre volume fraction.

2.2 Step 2 – Stirring the fibres

The second step is in fact a heuristic. It will move the fibres in the following manner: depending on the iteration number, each fibre shall look for the closest, second closest or third closest fibre. It will then shift its position towards the chosen fibre by a random distance between 0 and the maximum distance allowed to avoid violation of the minimum distance between fibres specified. In the fourth, fifth, etc iterations, each fibre will look for the second and third closest fibre respectively, and so on. This process is applied to every fibre once per iteration and it opens empty spaces between fibres.

2.3 Step 3 – Fibres in the Outskirts

The last step has the objective of increasing the fibre volume fraction on the RVE without adding fibres to it. This is achieved by pushing the sliced fibres positioned along the edges of the RVE inwards, always safeguarding the minimum distance between fibres and no overlapping.

The algorithm stops in any of these steps if the desired fibre volume fraction is achieved. Figure 1 shows two generated RVEs with a fibre volume fraction of 65%. The algorithm requires only one minute to achieve a fibre volume fraction of 59% using a standard desktop computer. A fibre volume fraction of 65% is achieved in less then 3 minutes.

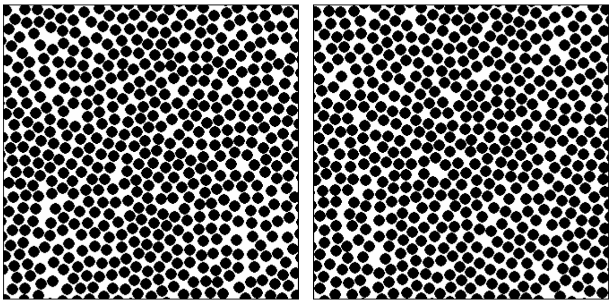

Fig. 1. Two examples of random fibre spatial distribution with volume fraction of 65%

3 Statistical Characterization

The generated fibre spatial distributions were compared to those generated by other algorithms [4], using several spatial descriptors such as average of Voronoi polygon areas and average of neighbouring distances between Voronoi cells, Ripley's *K* function, pair distribution function and nearest neighbour functions. For all these spatial descriptors, the proposed algorithm achieved a random spatial distribution of fibres faster and with a better statistical representation.

4 Material Characterization

FEAs were performed in some of the generated RVEs. Both fibre and matrix were considered linear elastic isotropic materials. Their respective properties are shown in Table 1.

Table 1. Elastic properties of the constituents, from [5]

	Fibre	Matrix
Young's Modulus [GPa]	74	3.35
Poisson's ratio	0.2	0.35

The objective of the analyses was to determine the effective properties on the transverse plane and to demonstrate that the generated fibre spatial distributions were able to capture the transverse isotropy of the composite material.

Stress and strain tensors are considered to remain constant throughout the longitudinal direction; hence, generalized plane strain elements were chosen from ABAQUS® element library. Boundary conditions were applied such that all four sides would remain straight. These are very restrictive boundary conditions and will cause high stress concentrations along the edges. Hence the effective properties were calculated using elements distanced from the edges by $0.15\times a$ where a is the size length of the RVE.

Following Trias et al. [2], the dimension of the RVE was chosen to be $50\times R$, where R represents the fibre radius. The same value was used to generate the fibre spatial distribution in Figure 1.

The transverse effective properties calculated for four different fibre distributions and measured values [5] are summarised in Table 2.

Table 2. Calculated and measured effective properties for a fibre volume fraction of 60%

	E_{22} [MPa]	ν_{23}	E_{33} [MPa]	ν_{32}	G_{23} [MPa]
RUN_01	15135	0.373	15185	0.367	5483
RUN_02	14517	0.373	14768	0.390	5496
RUN_03	15029	0.370	14883	0.368	5476
RUN_04	14371	0.356	14685	0.371	5247
Exper. [5]	16200	0.400	16200	0.400	5786

5 Conclusions and Future Work

A new algorithm to generate random spatial distribution of inclusions in fibre reinforced composites is presented. The algorithm was characterized both statistically and materially, and has proven to have better performance and provide better and faster results than existing methodologies.

The next step within this work is to determine all effective mechanical and thermal properties of the material and to perform new FEAs using a visco-elastic model for the matrix. Including a failure criterion for the matrix and considering interfacial decohesion is also envisaged.

References

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