STRUCTURAL DYNAMICS OF CIRCULAR COMPOSITE PLATES WITH DISCRETE STIFFENERS

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Keywords: *structural dynamics, vibrations, acoustics, composite materials*

Abstract

In an effort to better understand the consequences of using anisotropic materials in modern structures, a study of the structural dynamics of CFRP laminated plates is necessary. Analytical solutions for the resonant frequencies and mode shapes of isotropic plates are presented as a means of validating finite element modeling (FEM) efforts. The FEM work is then expanded to generate predictions of vibration characteristics in circular CFRP laminated plates. In addition, the use of intelligently designed discrete stiffeners is investigated through the use of FEM. The use of stiffeners to influence acoustic emissions is also discussed. Preliminary experimental work has been completed and results agree with the FEM and analytical predictions.

1 Introduction

The dynamic response of a structure to a time-varying load is intimately tied to its structural integrity and acoustic properties. Fiber reinforced composite materials are becoming more popular all the time. The use of carbon fiber reinforced polymers (CFRP), in particular, has expanded beyond specialized aerospace and military applications. The automotive and advanced sporting goods industries, among others, are finding an increasing number of ways to create useful structures with laminated composite materials. The low density and relatively high strength and stiffness of CFRPs make them a popular choice for many structures, but the anisotropic nature of these materials demands that more care be taken with analysis and design. Even when laminates are designed to have in-plane quasi-isotropic material properties, the out-of-plane properties are generally not isotropic. CFRPs constructed with the commonly used $[0 \pm 45\ 90]_s$ stacking sequence are an example of such a laminate.

2 Analytical Solutions

2.1 Isotropic Plates

The equation of motion for transverse deflections in a thin, isotropic vibrating plate is well known and has been presented by [1]. Equation 1 shows the dependence on spatial coordinates and time. D is the flexural rigidity of the plate, u_3 is the displacement coordinate perpendicular to the plate surface, ρ is the plate density, h is the plate thickness, t is the time variable, and P is the applied load.

$$D\nabla^4 u_3 + \rho h \frac{\partial^2 u_3}{\partial t^2} = P \qquad (1)$$

For isotropic plates the flexural rigidity, D, is defined by

$$D = \frac{Eh^3}{12(1-\upsilon^2)} \qquad (2)$$

where h is the (constant) plate thickness, υ is Poisson's ratio, ρ is the density of the plate material, and u_3 is the displacement coordinate perpendicular to the surface of the plate.

In special cases where the boundary conditions and material properties of the plate can be expressed in simple mathematical terms, the equation of motion can often be solved as an eigenvalue problem. The eigenvalues and eigenvectors represent resonant frequencies and mode shapes, respectively.

For a circular plate clamped at its boundary, the mode shapes and natural frequencies can be expressed in terms of Bessel functions (see Equation 4) [1]. According to [1] and [2], the resonant frequencies of an isotropic circular plate with a clamped boundary condition are given by

$$\omega_{mn} = \frac{(\lambda a)^2_{mn}}{a^2}\left(\frac{D}{\rho h}\right)^{1/2} \quad m = 0,1,2,3... \quad n = 0,1,2,3... \tag{3}$$

where n is the number of nodal diameters, m is the number of nodal circles (excluding the boundary), and a is the radius of the plate. The mode shapes are given by

$$u_3 = A\left[J_n(\lambda r) - \frac{J_n(\lambda a)}{I_n(\lambda a)} I_n(\lambda r)\right]\cos(\theta - \phi) \tag{4}$$

or, alternatively, the theta dependence can be written as a linear combination of two sine functions, separated by a phase angle of $\pi/2$ radians. In this case, the transverse displacement is given by

$$u_3 = B\left[J_n(\lambda r) - \frac{J_n(\lambda a)}{I_n(\lambda a)} I_n(\lambda r)\right]\sin(n\theta - \frac{\pi}{2}) + C\left[J_n(\lambda r) - \frac{J_n(\lambda a)}{I_n(\lambda a)} I_n(\lambda r)\right]\sin(n\theta) \tag{5}$$

where u_3 is the transverse displacement, r and θ are polar coordinates with the origin at the center of the plate, J_i is Bessel function of the first kind, and I_i is a modified Bessel function of the first kind.

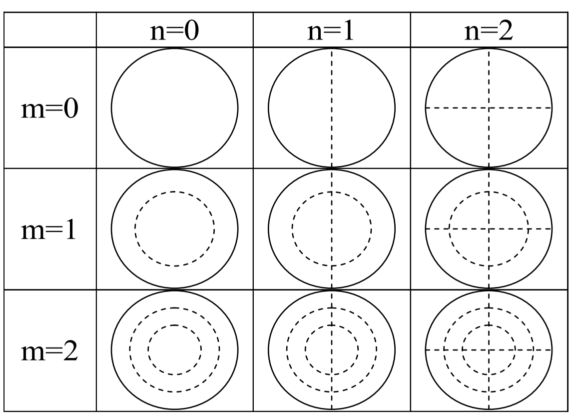


Fig. 1. Theoretical node lines for an isotropic circular plate of constant thickness (m indicates the number of circumferential node lines, while n indicates the number of diametral node lines)

The two equations are mathematically equivalent, but have different physical interpretations. Equation 4 describes the mode shape and allows for one amplitude degree of freedom, A, and one rotational degree of freedom, represented by the arbitrary angle, ϕ. Equation 5 is a linear combination of two identical mode shapes which are offset by a constant phase angle of $\pi/2$ radians. In Equation 5, the degrees of freedom are represented by the amplitudes, B and C.

The node lines of a mode shape are the collection of points for which u_3 is always zero. Figure 1 shows the location of node lines for several mode shapes of circular, isotropic plates. Two types of node lines exist for isotropic circular plates. Circumferential node lines are parallel to the plate boundary, while diametral node lines are straight lines which pass through the center of the plate.

2.2 Anisotropic Plates

In general, the solutions to the equation of motion become impractical to solve analytically when boundary conditions and material properties of a plate do not contain the same type of symmetry [3]. Solutions have been derived for plates with Cartesian orthotropy and rectangular boundary conditions [4], and for plates with cylindrical orthotropy and circular boundary conditions [5], but very little work has been done with plates containing conflicting symmetries [6].

Of particular importance to transverse deflections in thin plates is flexural rigidity. Each ply in the laminate contributes to the flexural rigidity of the resulting plate. If a bending moment is generated in the plate, one side is placed in tension, and the other side is placed in compression while the center of the plate is subjected to relatively low stress levels. For this reason, the flexural rigidity of a laminated CFRP plate is highly dependent upon the properties of the laminate near each surface. Although the $[0\ \pm45\ 90]_s$ stacking sequence in quasi-isotropic in the plane of the laminate, its out-of-plane bending properties are anisotropic. The plate exhibits approximately five times the flexural rigidity when bending in the zero degree direction as opposed to bending in an orientation closer to the ninety degrees. Fig. 2 shows the anisotropic nature of a laminate can be minimized by increasing ply dispersion, even if the total laminate thickness does not change. With an eight-ply laminate, the flexural rigidity varies by ±66% of the mean. At 32 plies, and the same laminate thickness, the flexural rigidity varies by only ±5% of the mean.

It is interesting to note that the angles of maximum and minimum flexural rigidity are perpendicular to each other. Also, it is important to notice that the minimum and maximum angles are not a zero and 90 degrees, but slightly rotated to angles of approximately 10 and 100 degrees. This has occurred because of the secondary effect of having +45 degree plies closer to the outer plate surfaces than the -45 degree plies. The proximity of the +45 degree plies to the plate surface has shifted the peak flexural rigidity slightly clockwise. In a similar manner, the -45 degree plies have shifted the angle of minimum flexural rigidity clockwise.

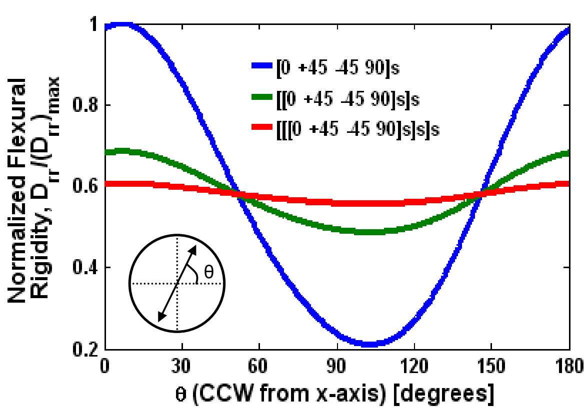


Fig. 2. Theoretical flexural rigidity of a laminated composite plate with respect to bending orientation angle (all three laminates have the same total thickness)

The difficulties with analytical solutions to mismatched symmetry problems have led to the current use of FEM for analysis of CFRP laminated plates subjected to a circular boundary condition.

3 Modeling Methods

All FEM modeling was completed using version 6.5 of the *ABAQUS* software package. The four-node, doubly curved, linear shell element selected for the models can be found in the *ABAQUS* element library. Shell elements were chosen over three dimensional elements in order to maintain acceptable element aspect ratios with a reasonable mesh density.

The boundary conditions of the circular plate are perfectly clamped. The nodes at the perimeter of the plate are constrained from all translational and rotational motion. While a perfectly clamped boundary condition in the model may be nearly impossible to recreate in an experiment, it provides an excellent opportunity to compare the results of the model to analytical results.

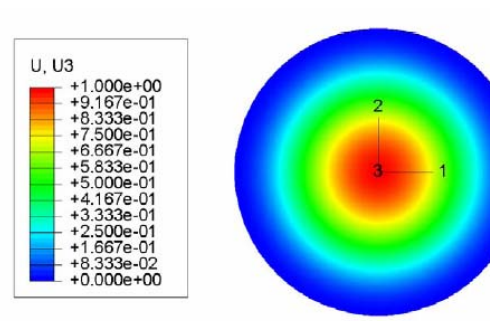
No damping was included in the model. Experimental characterization of the damping properties of each of the plate types is beyond the scope of the current study. While frequency dependent damping may influence the relative amplitudes of the modes when a plate is excited, low to moderate levels of damping have little influence on mode shapes and resonant frequencies.

There are three types of plates that were modeled over the course of this study: isotropic plates, CFRP laminated plates, and stiffened CFRP laminated plates. Each type of plate required slightly different modeling techniques. Isotropic plates were the simplest model to construct. Each shell element was assigned the same thickness and the material properties are the same in every direction. The isotropic plates are 0.889 mm thick. In order to model the CFRP laminated plates, the thicknesses, material properties, orientations, through-thickness locations of each ply must be defined. The $[0 \pm 45\ 90]_s$ laminate is made up of eight unidirectional plies, each of which is approximately 0.09 mm thick, for a total laminate thickness of 0.72 mm. The CFRP plates with stiffeners were modeled in the same way as the unstiffened CFRP plates, except for the changes in plate thickness in the stiffened regions. The stiffeners were assumed to be perfectly attached and were simply modeled as localized regions of the plate with extra unidirectional plies. All of the stiffeners in this study are situated with the fibers aligned with the zero degree plies on the outer most surfaces of the laminate.

4 FEM Results

4.1 Isotropic Plates

The FEM results for isotropic plates agree very well with mode shapes and frequencies predicted by the analytical work. The first four modes, as predicted by the model are shown in Fig. 3.



(a)

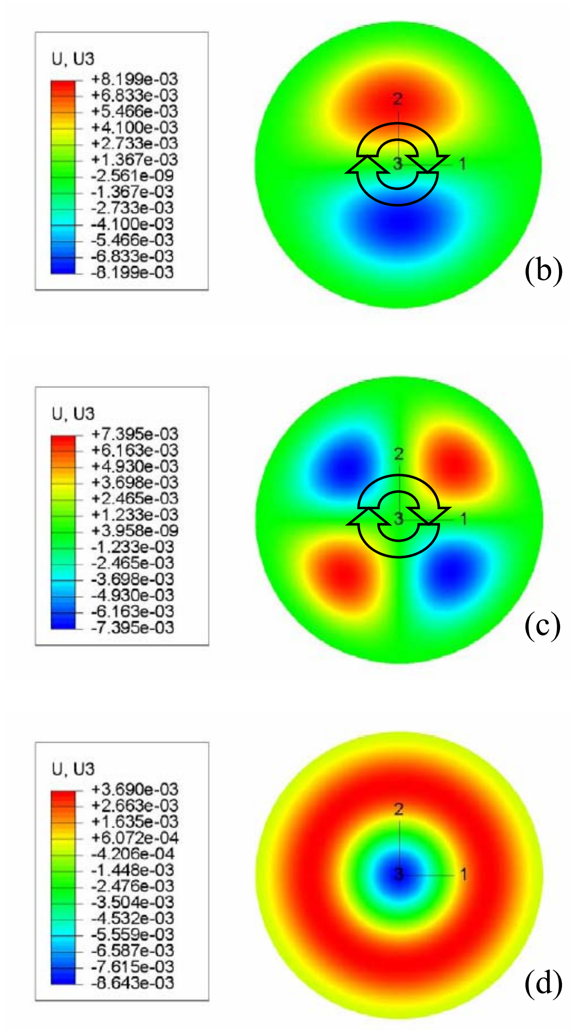


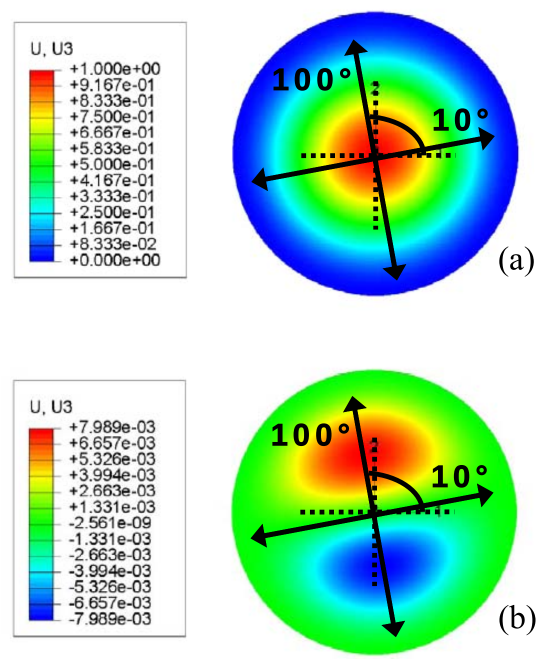
Fig. 3. FEM results showing the (a) m,n=0,0 mode at 2194 Hz, (b) m,n=0,1 mode at 4563 Hz, (c) m,n=0,2 at 7466 Hz, and (d) m,n=1,0 mode at 8457 Hz for the titanium plate (transverse displacement values have been normalized)

The FEM confirms the analytical solutions for the mode shapes and frequencies. All of the mode shapes in the isotropic model are free to rotate in the plane of the plate about the center of the circular boundary. The FEM results for the resonant frequencies and mode shapes of isotropic plates match very well with the analytical solutions described above. Mode shapes and resonant frequencies for isotropic plates are easily predicted using either method.

4.2 CFRP Laminated Plates

The cylindrical symmetry found in the isotropic plates is disrupted by the anisotropic flexural rigidity in the $[0 \pm 45\ 90]_s$ CFRP laminated plate (see Fig. 1). The lack of symmetry has a two-fold effect on the natural modes of the CFRP plates. The most obvious effect is that new mode shapes are created. Shapes like CFRP mode 4 (see Fig. 4d) cannot exist in an isotropic plate of uniform thickness.

The second effect is the splitting of the double modes found in the isotropic plates. At first glance, the CFRP modes 2 (Fig. 4b) and 3 (Fig. 4c) both resemble the m,n=0,1 mode for the isotropic plates. Both mode shapes have one diametral node line and zero circumferential node lines. However, there are some important differences. In the CFRP plate, mode 2 and mode 3 are two completely separate modes with distinct resonant frequencies. Also, the anisotropic flexural rigidity of the CFRP plate requires that every mode shape be fixed in space. As can be seen in Fig. 4, each mode shape is symmetric about the axes of lowest and highest flexural rigidity, which are approximately 100 degrees and 10 degrees, respectively. Mode shapes which generate large bending moments in locations and directions of high flexural rigidity will occur at higher frequencies than mode shapes which do not match regions of high flexural rigidity with the locations subjected to large bending moments.



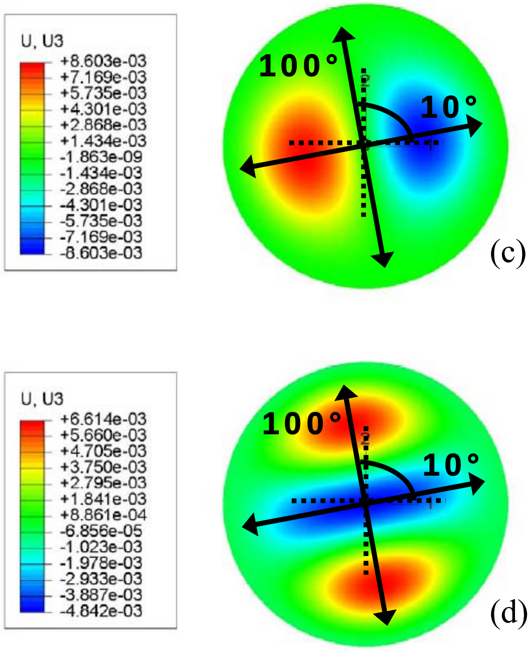


Fig. 4. FEM results for (a) mode 1 at 1936 Hz, (b) mode 2 at 3302 Hz, (c) mode 3 at 4592 Hz, and (d) mode 4 at 5198 Hz for the $[0 \pm 45\ 90]_s$ CFRP laminated plate (transverse displacement values have been normalized)

4.3 Stiffened CFRP Laminated Plates

In an effort to purposefully change the dynamic properties of the CFRP plate, discrete stiffeners were added to the model (see Fig. 5).

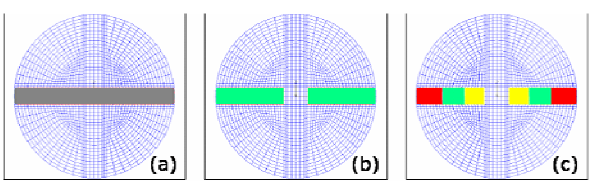


Fig. 5. Diagram representing stiffener locations for the D-2, D-4, and D-6 stiffeners (a), the T-4 stiffener (b) and the T-6-4-2 stiffener (c). Red regions are six plies thick, green regions are four plies thick, and yellow regions are two plies thick. The grey region is two plies thick for D-2, four plies thick for D-4 and six plies thick for the D-6 configuration.

Two types of stiffeners were used in this study. Diametral stiffeners are the stiffeners that traverse the entire span of the plate through its center. They have a constant thickness and width. The diameter stiffeners are designated D-2, D-4 and D-6 for the two, four and six ply diameter stiffeners. The tapered stiffeners are stiffeners which do not fully traverse the span of the plate. Instead, they stop short and are thickest near the plate boundary. The tapered stiffeners are designated T-4 for the four-ply stiffener, and T-6-4-2 for the stiffener that gradually decreases in thickness from six plies to four plies to two plies (see Fig. 5). All of the stiffeners are 6.35mm wide and consist of unidirectional plies which run parallel to the outermost zero degree plies in the laminate.

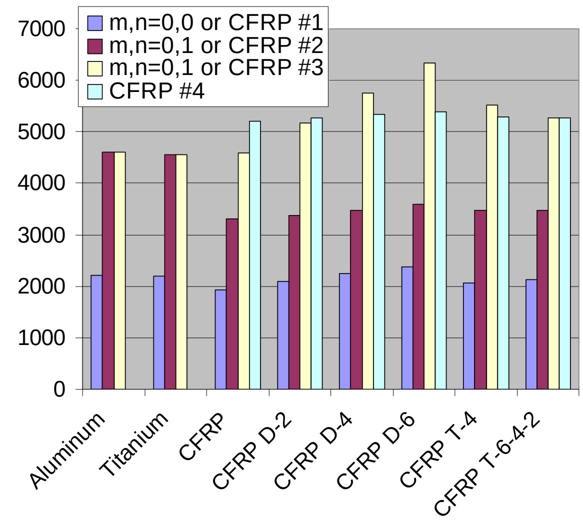


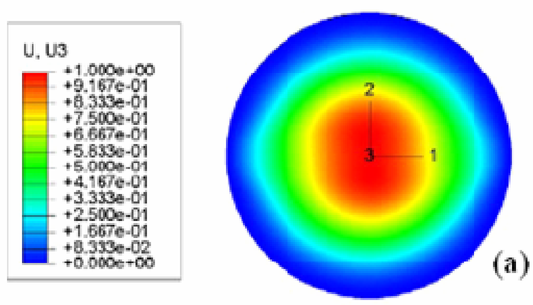
Fig. 6. FEM results for resonant frequencies

The results of the FEM work are summarized in Fig. 6. The diameter stiffeners show an increase in each resonant frequency for each mode shape stiffener thickness increases. However, it is clear that some mode shapes are more sensitive to these types of stiffeners than others. CFRP mode 3, in particular, is very sensitive to the thickness of the diameter stiffeners. In fact, CFRP mode 3 is so sensitive to the diameter stiffeners that it has a higher resonant frequency than CFRP mode 4 in the D-4 and D-6 configurations. In other words, the mode shapes have switched positions in frequency space. See Fig. 8 for the mode shapes of the D-6 stiffened plate. The mode shapes have been distorted, but still posses the defining characteristics found in the mode shapes of the unstiffened CFRP plate.

FEM Resonant Frequencies [Hz]				
Isotropic Mode	m,n=0,0	m,n=0,1		N/A
CFRP Mode	Mode 1	Mode 2	Mode 3	Mode 4
Aluminum	2214	4604	4604	N/A
Titanium	2194	4563	4563	N/A
CFRP	1936	3302	4592	5198
CFRP D-2	2089	3382	5163	5273
CFRP D-4	2239	3481	5754	5344
CFRP D-6	2381	3597	6336	5381
CFRP T-4	2054	3476	5523	5291
CFRP T-6-4-2	2132	3480	5269	5279

Fig. 7. Table of FEM resonant frequencies.

An efficient stiffener should be able to increase the resonant frequency of a natural mode with a minimum amount of added mass. The modeling results for the diametral stiffeners suggest that regions of high strain are particularly sensitive to changes in stiffness. The tapered stiffeners are designed to add the most flexural rigidity to the regions of the plate subjected to the largest strains for CFRP mode 1. The D-6-4-2 stiffener is thickest near the plate boundary, where the largest strains in the plate occur during mode 1 vibration. The T-4 stiffener adds the same amount of mass as the D-6-4-2 configuration, but its allocation of additional plies to high strain areas is slightly different. According to the FEM results (see Fig. 7) the T-6-4-2 stiffener is a more efficient configuration. With the same amount of material, the T-6-4-2 stiffener increased the mode 1 frequency by 10.1%, while the T-4 stiffener increased the mode 1 frequency by 6.1%. By carefully choosing the location and fiber orientations of the stiffeners, selected modes can be efficiently shifted in frequency space.



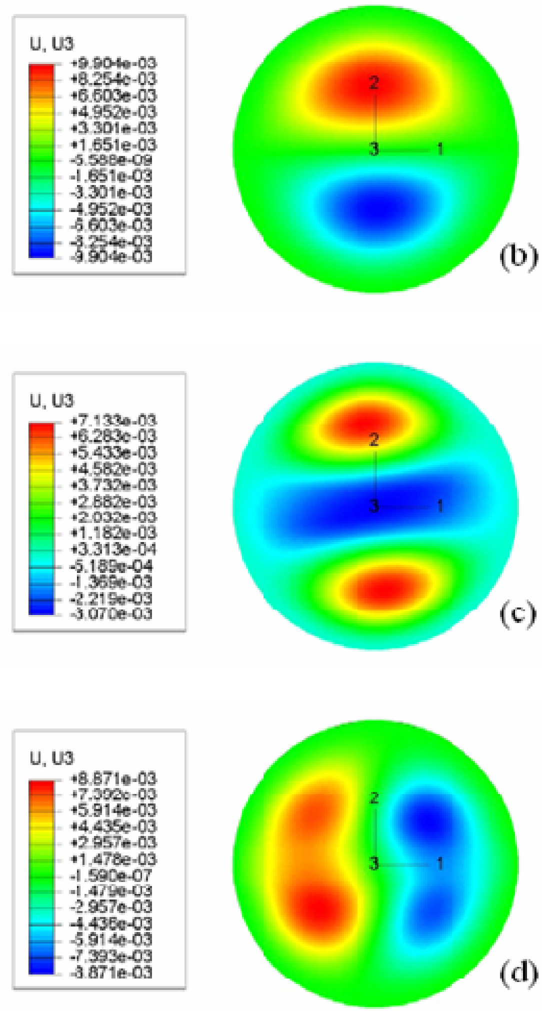


Fig. 8. FEM results showing (a) mode 1 at 2381 Hz, (b) mode 2 at 3597 Hz, (c) mode 4 at 5381 Hz, and (d) mode 3 at 6336 Hz for the D-6 stiffened $[0 \pm 45\ 90]_s$ CFRP laminated plate (transverse displacement values have been normalized)

5 Acoustics

The acoustic emission of any structure is closely tied to its dynamical behavior. When a structure is vibrating, its surface applies pressure to the surrounding air (or other medium). The pressure waves in the surrounding air can be received by human ears where they are interpreted as sounds.

The concept of consonance and dissonance comes from music theory. When two pure tones are heard simultaneously, the pressure waves interact and can interfere with one another. The resulting sound wave is then interpreted by the human auditory system. When two tones are combined into a harmonious sound, the interaction is said to be consonant. When the interaction of two tones results in a non-harmonious sound, it is referred to as dissonant.

If the difference in frequency between two tones is very small, a phenomenon called beating occurs. Beating results in sound amplitude that oscillates with a frequency equal to the difference in frequency between the two tones. The pitch heard when two tones cause beating is equal to the average of the frequencies of the two pure tones. If the frequency difference in the two tones increases, the beat frequency increases. Dissonance occurs when the beat frequency is a substantial portion of the tonal frequency. When this occurs, the periodicity of the resulting sound wave is disrupted and a rough, non-harmonious sound is heard. Maximum dissonance occurs when two tones are separated by approximately ¼ of the critical bandwidth [7]. Over most of the audible range, the critical bandwidth is slightly less than ⅓ octave [7].

Consonant interactions occur whenever two pure tones are sufficiently separated in frequency space to avoid the non-harmonious interference phenomena that cause dissonance. In an effort to avoid dissonant interactions between resonances, discrete stiffeners can be added to a structure to shift resonant frequencies and increase the spacing between modes of interest.

6 Experiments

Preliminary experiments have produced mode shapes which match the FEM results very well, and resonant frequencies which are within 30% of the FEM results [8]. Improvements in experimental design need to be made for more accurate frequency results can be expected. Difficulties with quantifying material damping and creating ideal boundary conditions will be addressed in future work.

7 Conclusions

Although the $[0\ \pm 45\ 90]_s$ CFRP plate has a stacking sequence which creates quasi-isotropic in-plane material properties, the out-of-plane flexural rigidity is not isotropic. The anisotropic nature of the CFRP plate allows for additional modes to be generated which cannot be found in an isotropic plate. The extra modes change the structural dynamics and acoustical properties of the plate when compared to an isotropic plate of similar in-plane specific stiffness and dimensions.

More work needs to be done, but these preliminary results show that stiffeners could be used to efficiently change the resonant frequencies of a laminated CFRP plate. If done properly, selected resonances could be shifted away from other resonances in order to increase the likelihood of a consonant interaction between modes. Much of the work described in this study is based [9], which contains many details not included in this paper.

References

[1] Soedel, W. "*Vibrations of shells and plates*". Marcel Dekker, Inc., 2004.

[2] Blevins, R. D. "*Formulas for Natural Frequency and mode Shape*". Kreiger Publishing, 1979.

[3] Hoppmann, W.H. "Flexural vibration of orthogonally stiffened circular and elliptical plates". *Proceedings of The 3rd U.S. National Congress on Applied Mechanics*, New York, pp 181-187, 1958.

[4] Sundara Raja Iyenar, K. T. S.,& Jagadish, K. S. "Vibration of rectangular orthotropic plates". *Applied Scientific Research,* Vol. 13, No. 1, pp 37-42, 1964.

[5] Greenberg, J. B. & Stavsky, Y. "Flexural vibrations of certain full and annular composite orthotropic plates". *The Journal of the Acoustical Society of America,* Vol. 66, No. 2, pp 501-508, 1979.

[6] Leissa, A. W. "*Vibration of plates*". 1st edition, Office of Technology Utilization, National Aeronautics and Space Administration, 1969.

[7] Rossing, T. D. "*The science of sound*". 1st edition, Addison-Wesley Publishing Company, 1982.

[8] DeMille, B. D. & Kedward, K. T. "Experimental study of vibrations in CFRP laminates with discrete stiffeners". To be published in: *The SAMPE International Symposium and Exhibition Proceedings*, Baltimore, 2007.

[9] DeMille, B. D. "*Resonant Frequency Manipulation in Anisotropic Plates with a Circular Clamped Boundary Condition*". Masters of Science Thesis, University of California, Santa Barbara, 2006.