VIBRATION AND TRANSFER FUNCTION CHARACTERISTIC OF COMPOSITE SANDWICH PLATES WITH DAMAGE

Man Wang *, Ruixiang Bai, Haoran Chen** [Man Wang#]: wangman3669@sina.com**
***Mechanical Engineering & Automation Department, Dalian Institute of Light Industry;**
**** State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Mechanical Engineering, Dalian University of Technology, Dalian, 116024, China**

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Abstract

Based on Zig-Zag deformation assumption and Mindlin first order shear deformation theory, a finite element method for vibration and transfer function analysis of composite sandwich plates has been developed. Considering the viscoelasticity of face and core, a modal damping model on basis of Adams' strain energy method (MSE) is established, and the formulas of stiffness, mass and damping matrix for the perfect and damaged composite sandwich plate with face/core debonding. According to the theories and models given above, the governing equations for numerical analysis are constructed. By some typical examples, the characteristics of nature frequency, modal and transfer function are investigated. The conclusions obtained can be useful for the damage identification of such structures.

1 Introduction

Most composite sandwich laminates structures working under dynamic loadings, therefore, the study of dynamic characteristics and damage behavior of delaminated sandwich laminates will provide regulations for profound understanding durability and allowable value of sandwich structures. Deformation characteristics and damage behavior of delaminated sandwich laminates are extremely complicated, moreover quite analytical difficulties in theory and experiment aspect relating to nonlinear dynamics problems of anisotropic material, such as frequency and modal analysis with delamination between face/core under constraint deformation and damage generation are the leading edge topic in vibration of damage sandwich laminates. With the formation of numerical analytical theories and models, an overview for those theories and models is systemic appreciated in reference [1-2], in which Reissner and Qing-hua Du theory are the most common sandwich plate theory. However, quite few research works has been reported on the dynamic characteristics of composite sandwich structures with face/core interface debonding. For such damage structure, the analytic method has to face many difficulties, while the numerical simulation provides a quite effective way as well as experimental study. The finite element analysis is a quite effective method for mechanical property of complex composite sandwich laminates.

The paper deduced the finite element formulations of vibration and frequency response function based on Zig-zag deformation assumption and Mindlin theory related to the toothed feature and the shear effect of the face and the core for sandwich laminates. Considering the viscoelasticity of face and core, a modal damping model on basis of Adams' strain energy method (MSE) is established, and the formulas of stiffness, mass and damping matrix for the perfect and damaged composite sandwich plate with face/core debonding. According to the theories and models given above, the governing equations for numerical analysis are constructed. By some numerical examples, the natural frequencies and modes are discussed and the effects of delaminations between face/core on frequency response function are considered, so the dynamic behavior of delamination between face/core got solved for sandwich laminates in frequency domain.

2 Formulations

2.1 Displacement of Sandwich Plates[3]

Based on Zig-zag deformation assumption and Mindlin first order shear deformation theory, the expressions of the displacement field of the sandwich composite laminates can be written as follows:

The upper face

$$\begin{cases} u_1 = u_0 + (h_1\theta_{y1} + h_2\theta_{y2})/2 + z_1\theta_{y1} \\ v_1 = v_0 - (h_1\theta_{x1} + h_2\theta_{x2})/2 - z_1\theta_{x1} \\ w_1 = w_0 \end{cases} \quad (1)$$

The core

$$\begin{cases} u_2 = u_0 + z_2\theta_{y2} \\ v_2 = v_0 - z_2\theta_{x2} \\ w_2 = w_0 \end{cases} \quad (2)$$

The lower face

$$\begin{cases} u_3 = u_0 - (h_2\theta_{y2} + h_3\theta_{y3})/2 + z_3\theta_{y3} \\ v_3 = v_0 + (h_2\theta_{x2} + h_3\theta_{x3})/2 - z_3\theta_{x3} \\ w_3 = w_0 \end{cases} \quad (3)$$

In which, u_0、v_0、w_0 are the displacement component of core middle plane of the core; θ_{y1}、θ_{x1}、θ_{y2}、θ_{x2}、θ_{y3}、θ_{x3} are the rotational components of the upper laminates, the core and the lower laminates, respectively. Adopting the eight-nodes isoperimetric element discrete, each node with nine degrees on freedom（u_0，v_0，w_0，θ_{x1}，θ_{y1}，θ_{x2}，θ_{y2}，θ_{x3}，θ_{y3}）.

For the damaged sandwich laminates plate between the laminates and the core, the plate can be divided into three parts. In accordance with the requirement of displacement continuity at the nodes along the delaminated front between perfect and delamination regions, the additional continuity conditions must be imposed in Ref.[4].

2.2 Stiffness Matrix of Delaminated Sandwich Laminates Plate

Adopting the item-isoperimetric interpolation to construct the eight-node isoperimetric element, according to the variation principle, the stiffness matrices can be given as

$$\boldsymbol{K}^e = \sum_{n=i}^{j} \int_{-1}^{1}\int_{-1}^{1} \boldsymbol{B}_{(n)}^{\mathrm{T}} \boldsymbol{D}_{(n)} \boldsymbol{B}_{(n)} \boldsymbol{J} d\xi d\zeta \quad (4)$$

While $i=1$ $j=3$, $i=1$, $j=1$ and $i=2$, $j=3$, the Eq.(4) represent the element stiffness matrix of the core, the upper and the lower, respectively. In which, $\boldsymbol{B}_{(n)}$ and $\boldsymbol{D}_{(n)}$ are strain-node displacement matrix and elasticity matrix, respectively.

2.3 Mass Matrix of Delaminated Sandwich Laminates Plate

The kinetic energy of each part of sandwich can be expressed as

$$T_i = \frac{1}{2}\int_{\Omega} [\rho_{0i}[(\dot{u}_{0i}^2 + \dot{v}_{0i}^2 + \dot{w}_{0i}^2) + 2\rho_{1i}(\dot{u}_{0i}\dot{\theta}_{yi} + \dot{v}_{0i}\dot{\theta}_{xi}) + \rho_{2i}(\dot{\theta}_{xi}^2 + \dot{\theta}_{yi}^2)]d\Omega \quad (5)$$

In which，subscript I=1, 2, 3 represent the upper laminates, the core and the lower laminates, respectively. The mass matrix according to variation principle can be written as

$$\boldsymbol{M} = \int_{\Omega} \boldsymbol{N}^T \boldsymbol{R} \boldsymbol{N} d\Omega \quad (6)$$

Here N is the shape functional. Density matrix $\boldsymbol{R}$ can be expressed as $\boldsymbol{R}_I$, $\boldsymbol{R}_{II}$ and $\boldsymbol{R}_{III}$ for the upper, lower laminates and the core, respectively. (See Ref.[3-4].)

2.4 Damping Matrix of Delaminated Sandwich Laminates Plate

According to Adams' strain energy theory of composite laminates plate[5], the specific damping capacity ψ_j corresponding to j-th mode can be determined by the following formulation:

$$\psi_j = \frac{\Delta U_j}{U_j} = \frac{\sum_{e=1}^{n}\int_v \boldsymbol{\varphi}_j^T \sum_{i=m}^{n} \boldsymbol{K}_{di}^e \boldsymbol{\varphi}_j dv_e}{\sum_{e=1}^{n}\int_v \boldsymbol{\varphi}_j^T \sum_{i=m}^{n} \boldsymbol{K}_i^e \boldsymbol{\varphi}_j dv_e} \quad (7)$$

Here, {ϕB$_{jB}$} is the *j-th* modal shape of the structures, $\boldsymbol{K}_i^e$ and $\boldsymbol{K}_{di}^e$ denote the structural stiffness and the damped structural stiffness matrix, respectively. For the upper laminates: m=n=1, for the lower laminates: m=2， n=3 and for the perfect base laminates: m=1， n=3. If ξ_j is defined as the modal damping ratio for the *j*-th mode, thus

$$\xi_j = \psi_j / 4\pi \qquad (8)$$

Furthermore, the damping matrix [C] can be given by Raleigh damping model

$$\boldsymbol{C} = \alpha \boldsymbol{M} + \beta \boldsymbol{K} \qquad (9)$$

3 Motion Wquations and Solution

According to Hamilton's principle, the undamped-free vibration equation and the motion equation of sandwich plate can be written in terms of relation

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = 0 \qquad (10)$$

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{f} \qquad (11)$$

The solution for solution Eq. (10) is subspace iterative method. To ensure integral stability and accuracy in time domain[6], the PD-S form precise time-integration iteration formulation is adopted in Eq. (11).

Assumption that the linear displacement response conform to the expression $\boldsymbol{q}(t) = \boldsymbol{q}e^{iwt}$ under simple harmonic excitation, and substitute it into Eq.(11). The first to N order modes are conversed into regularized modes according to the normal property of vibrating. Then after derivation, the frequency response function *H* can be written as

$$H_{mn}(\omega) = \sum_{j=1}^{N} \frac{\boldsymbol{\Phi}_{jm}\boldsymbol{\Phi}_{jn}}{\omega_j^2 - \omega^2 + 2i\xi_j\omega_j\omega} \qquad (12)$$

The amplitude and phase frequency response of point *m* excited by unit simple harmonic loading in point *n* can be expressed by real part $H_{mn}^R(\omega)$ and imaginary part $H_{mn}^I(\omega)$ shown as

$$A_{mn}(\omega) = \sqrt{H_{mn}^R(\omega)^2 + H_{mn}^I(\omega)^2} \qquad (13)$$

$$\vartheta(\omega) = actg^{-1}\left(\frac{H_{mn}^I(\omega)}{H_{mn}^R(\omega)}\right) \qquad (14)$$

4 Numerical Examples and Discussion

Consider a $[(0/90)_s/\text{core}/(0/90)_s]$ sandwich laminated plates with length and width 600mm and 240mm as shown in Fig.1. A penetrated delamination area with varying length L_d is in width orientation of the middle span. The boundary conditions as: fixation at y=0, free at other sides. The application of exciting force lies in point 1 (x=180mm,y=600mm), the corresponding attribute points 2 and 3 (x=240mm, y=300mm) in upper and lower laminates after delamination between face/core. The thickness of the laminates is 0.125mm, the material property of the faces as: E_{11}=37.78 GPa , E_{22}=10.90 GPa , $G_{12}=G_{13}=G_{23}$=4.91GPa , ν_{12}=0.3 , ρ=1813.9Kg/m^3, ψ_1=0.1385%, ψ_2 = 0.8037%, $\psi_4 = \psi_5 = \psi_6$ = 1.0998% ; the thickness of the core is 5mm, and the material property of the core as: E_{11}=113.5MPa , E_{22}=3.27MPa , $G_{12}=G_{13}=G_{23}$=18.86MPa , ν_{12}=0.32 , ρ=130 Kg/m^3, ψ_1=0.1385%, ψ_2 = 0.8037%, $\psi_4 = \psi_5 = \psi_6$ = 1.0998% ; $\psi_1 = \psi_2$ = 2.88%, $\psi_4 = \psi_5 = \psi_6$ = 6.7%。

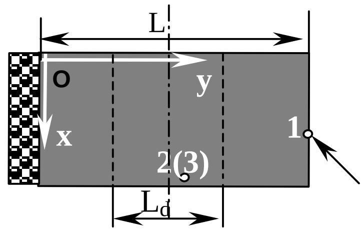


Fig.1 Cantilever composite sandwich plate

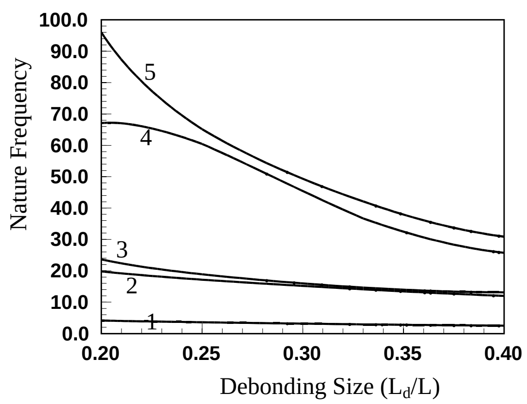
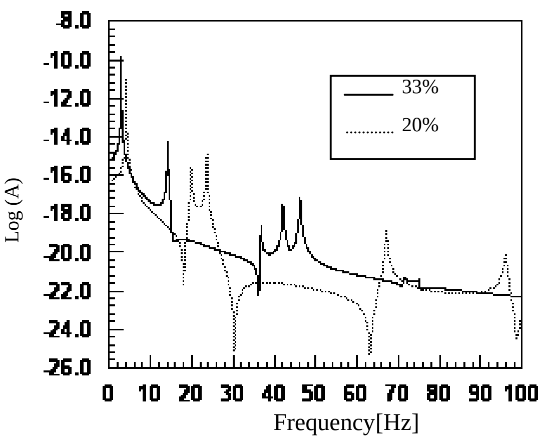
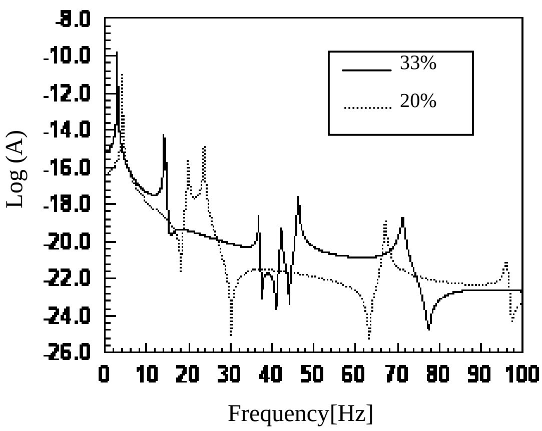


Fig.2 Effect of debonding size on natural frequency

The effects of face/core debonding size on the natural frequencies are illustrated in Fig.2 where the five curves coincide with the first five orders from the bottom to top. From the figure, it can be observed that each frequency presents decrease tendency associating with the increase of debonding length. The first three frequencies vary slightly and the fourth to fifth descend severely with the increase of debonding length lies in that the low order modes correlate with the sandwich structure integrity characteristics, but the high order ones depend on the local deformational features, namely, the debonding of face/fore induces the stiffness weakness of sandwich plates while the effect of debonding present relatively weak for the upper or lower sublaminates deform together with the base ones.



(a) FRF of point 2



(b) FRF of point 3

Fig.3 FRF of debonding size

5 Conclusions

The transfer function of debonding length 33% and 20% are shown in Fig.3. The excitation forcing point deviate the centerline of the sandwich plate in order to ensure the first ten modes deformation obviously. The location of resonance hump changed for the debonding between face/core, and the change drive to become obvious with the debonding area increase for the debonding induced the variation of high order frequencies.

A general framework has been developed for the vibration and transfer function analysis of debonding composite sandwich plate. From numerical results of some typical examples, the important observations have been concluded as follows: The debonding of face/core decrease the natural frequency of sandwich plates, but the extent is different depends on the order of frequency, namely, the influence on high order frequency is more obvious than low orders. The location of resonance hump changed for the debonding between face/core, and the debonding induced the variation of high order frequencies.

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