NONLINEAR DYNAMIC STABILITY ANALYSIS FOR PIEZOELECTRIC COMPOSITE LAMINATES WITH DEBONDED INTERFACES

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Abstract

Nonlinear instability associated with piezoelectric composite laminates with debonded interfaces under dynamic loads has been studied. On basis of Reddy's simple higher order shear deformation theory a dynamic instability equation, called Methieu equation, was deduced, which was considered nonlinear elastic, damping, axial inertia force and electromechanical interaction effects，and the corresponding analytical solution was given.. From some typical examples, it is clear that the effects of interfacial debonding length and feed-back control on the instability regions, axial and transverse resonance frequency and the maximum deepness of "traction" are significant. Some typical examples indicate that the dynamic instability behavior of the laminates gradually decreases with increasing interfacial debonding length; and the effects of feed-back control for the case of laminates with short interfacial debonding damage are rather weaker than that for the case of laminates with long length interfacial debonding damage. For the latter, feed-back control on the laminates causes decreasing the interaction regions between axial and transverse parameter vibrations with increasing debonding length.

1 Introduction

Engineering structure components are frequently affected by the dynamical load in a flat surface, and the load bearing abilities of those are often destroyed by the buckling damage resulting from the dynamical effect. Bolotin and Tang Wenyong comprehensively summarized some progresses in the research of the dynamic stability for isotropy and/or composite materials respectively in Ref.s [1] and [2]. Recently, with the development of intelligent materials, people have attached increasingly attention to the research of using sensor and driving device made by intelligent materials to control the vibration of the structure. Most of intelligent material controlling is applied to the controlling of modality, damping and vibration. Therefore, accurate models of the electro-mechanical interaction between the structure and piezoelectric materials are required to design an active system. L W Chen[3] *et.al* considered a slender laminated composite beam with piezoelectric layers subjected to axial periodic compressive loads. G H Qing[4] *et.al* proposed an effective numerical measure to analyze the dynamic property of piezoelectric composite laminates; S C Choi *et.al* [,5] made a research in the vibration control of pre-twisted rotating composite thin-walled beams containing piezoelectric layer. Whereas most of these studies focused on perfecting the analysis of complete structure, not on dynamic stability of piezoelectric composite laminates with interfacial debonding damage.

This article has studied on dynamic stability of piezoelectric composite laminates with interfacial debonding damage, and discussed the effect of interfacial debonding damage and feed-back stresses on dynamic instability regions of laminates according to the principle of changing the state of driving components to control piezoelectric composite laminates with interfacial debonding damage. The work would provide valuable analysis measure and designing suggestion for the analyzer and designer of intelligent composite material structure.

2 Analysis model for piezoelectric composite laminates with interfacial debonding damage based on Reddy's theory

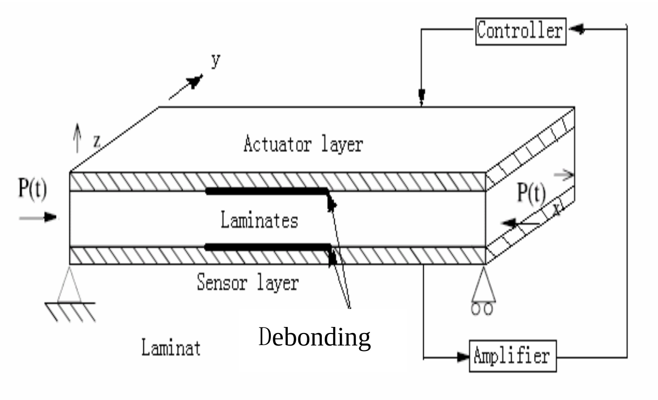


Fig.1. Sketch of analysis model for piezoelectric composite laminates with interfacial debonding damage

Fig.1 shows a composed plate consisting of a composite laminates as the main structure and two piezoelectric ceramics adhered on upper and lower surface, which act respectively as the driving device and sensor for the whole structure. Because of the big difference in the properties of materials between intelligent material and main structure, it is very easy to get interfacial debonding damage in the stick interface between driving device, sensor and main structure. Suppose a couple of through-width interfacial debonding damages with same length are located at the upper and lower stick interface and symmetrically distributed in mid-span of the plate, respectively, which divide the whole structure into five parts, as shown in Fig.1, and outside of the plate is installed the control system. Its main working principle is: the whole plate will be vibrated under the periodicity load in plate axial direction, and as a result of the change in the plate of sensing component; some data collected is input into outer control system, that forces the driving component to act correspondingly, so that the whole structure will be changed and controlled.

3 Methieu equation of nonlinear dynamic stability analysis for piezoelectric composite laminates with debonded interfaces

Using Reddy's simple higher order shear deformation theory and considering the constitutive relations of laminae and ceramics, and based on Hamilton's variational principle (as shown in equation 1)

$$\delta\int\left[T-U+W\right]dt=0 \qquad (1)$$

the following dynamic equations for each sub-laminates in the domain Ω_i (i= 1~5), can be deduced as

$$\partial N_{1i}/\partial x+\partial N_{6i}/\partial y+\sum_{k=1}^{m}e_{31}^{k}\left(\sum_{j=1}^{n}\beta_1^{jk}\,\partial\phi_j^x/\partial x\right)=0$$

$$\partial N_{6i}/\partial x+\partial N_{2i}/\partial y+\sum_{k=1}^{m}e_{32}^{k}\left(\sum_{j=1}^{n}\beta_1^{jk}\,\partial\phi_j^x/\partial y\right)=0$$

$$\partial M_{1i}/\partial x+\partial M_{6i}/\partial y-Q_{1i}+4/h_i^2\,R_{1i}$$
$$-4/3h_i^2\left(\partial P_{1i}/\partial x+\partial P_{6i}/\partial y\right)$$
$$+\left[\sum_{k=1}^{m}e_{31}^{k}\left(\sum_{j=1}^{n}\beta_2^{jk}\,\partial\phi_j^x/\partial x\right)-\sum_{k=1}^{m}e_{15}^{k}\left(\sum_{j=1}^{n}\alpha_4^{jk}\,\partial\phi_j^x/\partial x\right)\right]=0$$

$$\partial M_{2i}/\partial y+\partial M_{6i}/\partial x-Q_{2i}+4/h_i^2\,R_2$$
$$-4/3h_i^2\left(\partial P_{2i}/\partial y+\partial P_{6i}/\partial x\right)$$
$$+\left[\sum_{k=1}^{m}e_{32}^{k}\left(\sum_{j=1}^{n}\beta_2^{jk}\,\partial\phi_j^x/\partial y\right)-\sum_{k=1}^{m}e_{24}^{k}\left(\sum_{j=1}^{n}\alpha_4^{jk}\,\partial\phi_j^x/\partial y\right)\right]=0$$

$$\partial Q_{2i}/\partial y+\partial Q_{1i}/\partial x-4/h_i^2\left(\partial R_{2i}/\partial y+\partial R_{1i}/\partial x\right)$$
$$+4/3h_i^2\left(\partial^2P_{xi}/\partial x^2+\partial^2P_{yi}/\partial y^2+2\partial^2P_{xy}/\partial x\partial y\right)$$
$$+N_{xi}\,\partial^2w_{0i}/\partial x^2+2N_{xyi}\,\partial^2w_{0i}/\partial x\partial y+N_{yi}\,\partial^2w_{0i}/\partial y^2-\rho\ddot{w}_{0i}$$
$$+\left[\sum_{k=1}^{m}e_{31}^{k}\left(\sum_{j=1}^{n}\beta_3^{jk}\,\partial^2\phi_j^x/\partial x^2\right)+\sum_{k=1}^{m}e_{32}^{k}\left(\sum_{j=1}^{n}\beta_3^{jk}\,\partial^2\phi_j^x/\partial y^2\right)\right]$$
$$+\left[\sum_{k=1}^{m}e_{15}^{k}\left(\sum_{j=1}^{n}\alpha_4^{jk}\,\partial^2\phi_j^x/\partial x^2\right)+\sum_{k=1}^{m}e_{24}^{k}\left(\sum_{j=1}^{n}\alpha_4^{jk}\,\partial^2\phi_j^x/\partial y^2\right)\right]=0$$

(2)

As the same way in authors early work[6,7], the nonlinear dynamic stability equation, called Methieu equation, can be given as:

$$f''(t)+\omega^2f(t)-\omega^2/N_{cr}\left(N_0+N_t\cos\theta t\right)f(t)$$
$$+rf^3(t)=0$$

(3)

Because the space is limited, the deducing procedure and formulae of analytical solution corresponding above equation are omitted here; however the readers can find them in Ref.s [6, 7].

4 Analysis and discussion of the numerical results

4.1 The variations of closed-circle feedback control stress with interfacial debonding lengths

Fig.2 illustrates the variations of closed-circle feedback control stress vs. l and plate length L.

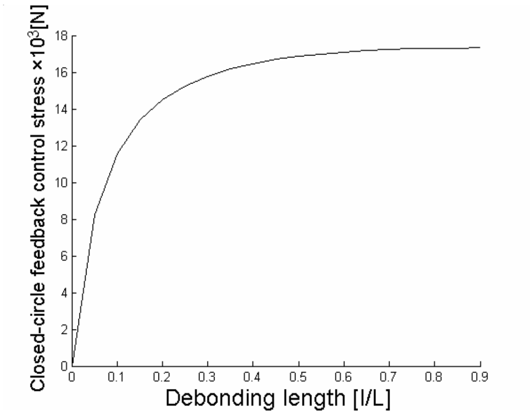
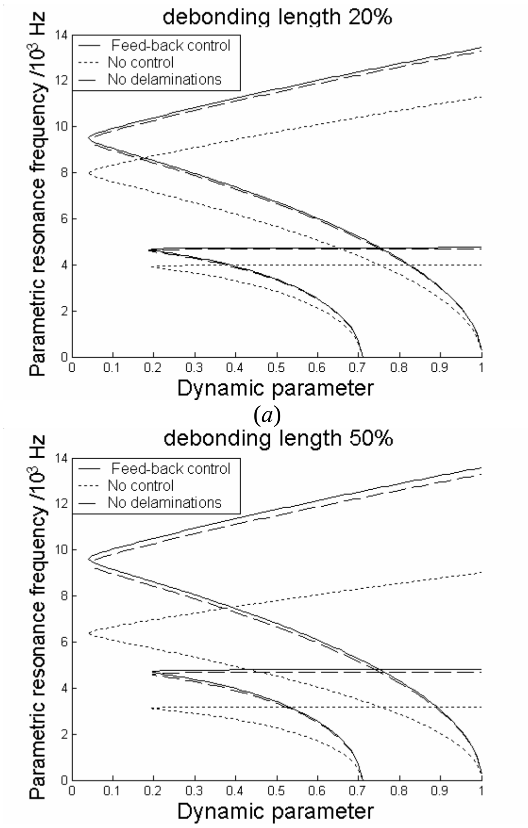


Fig.2 The variations of closed-circle feedback control stress vs. *l*/L

From figure 2, it can be seen that feedback control stress increases with increasing the value of *l/L*. However, for *l/L* < 20%, even *l/L* increases a little, the feedback control stress increase rapidly; for *l/L* > 40%, the variety rate of feedback control stress vs. *l/L*. is rather slow, whereas for *l/L* > 80%, the driving machine will not provide enough feedback control stress, in order to bring the structure back to the former state.

4.2 The variations of dynamic instability regions with interfacial debonding lengths considering damping effect

The first and second dynamic instability regions for the plates with *l*/L = 20% and 50% in the non-conservative system are shown in Fig.3, respectively. The curves in double line represents the results for the case of perfect plate under control, in dotted line for the case of plates with debonded interfaces without control, while in the solid line for the case of plates with debonded interfaces under active control. From the Figure, it indicates: (1) the interface debonding induces the variety of dynamic instability regions of the plate, and the first dynamic instability region is larger than the second dynamic instability region; (2) the interface debonding induces locations of the first and second dynamic instability region move down and corresponding areas decrease, hence, the value of parameter resonance frequency of the plate decreases; (3) Compared the curves of *l*/L =20% and *l*/L =50% shows the variety of dynamic instability region for the plate with small length debonded interface is more significant than for the one with large debonded interface.



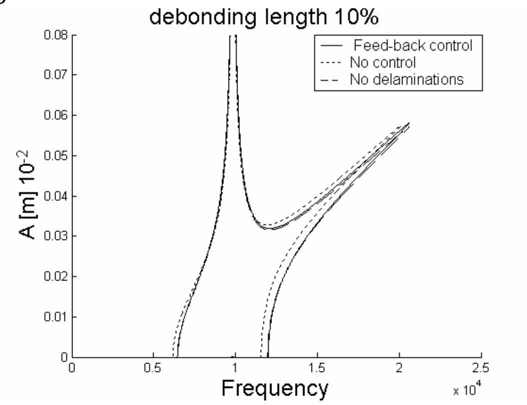
(*a*)

(*b*)

Fig.3 The variations of the first and second dynamic instability regions with closed-circle feedback control stress considering damping effect

4.3 The variations of amplitude of parameter vibration with interfacial debonding lengths considering axial inertial force and damping effects

Fig.4 gives the variations of amplitude of parameter vibration with interfacial debonding lengths considering axial inertial force and damping effects. From the Figure， It can be seen: (1) with increasing the debonding length, the frequency values of axial parameter resonance and normal inertial vibration of the plate decreases, simultaneously, and the former decrease more dramatically than the later， on contrary， the influence of the biggest “traction” depth[6,7] on frequency values the later more significant than the former; (2) feedback control force can promote the biggest “traction” depth of normal parameter resonance, efficiently; however it can’t increases the frequency values of parameter resonance; (3) the effects of feed-back control for the case of laminates with short interfacial debonding are rather weaker than that for the case of laminates with long length interfacial debonding. For the latter, feed-back control on the laminates causes decreasing the interaction regions between axial and transverse parameter vibrations with increasing debonding length



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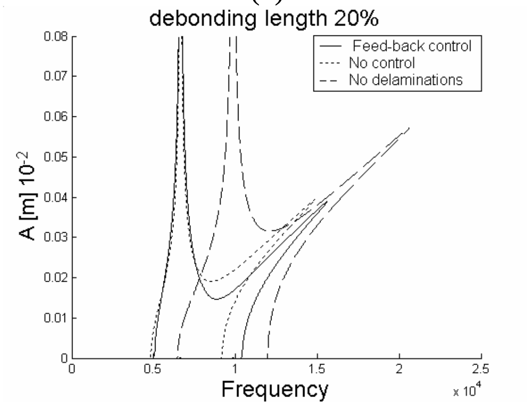

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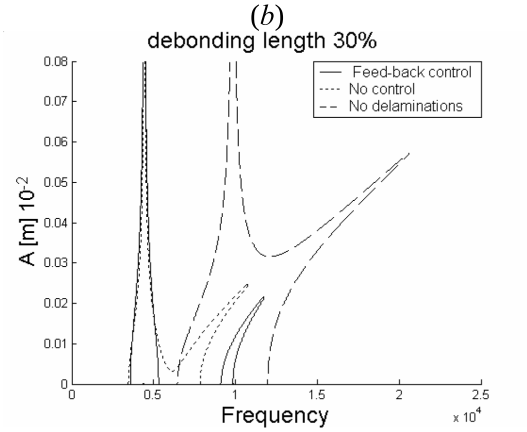

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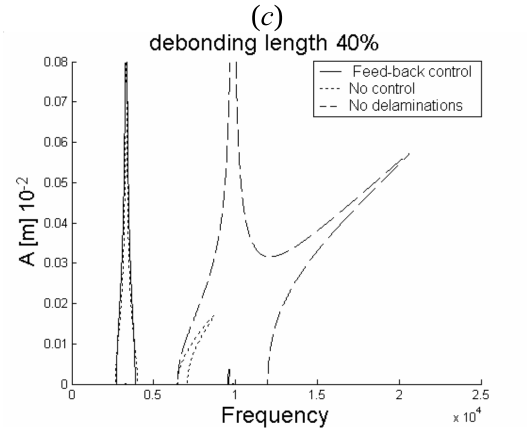

(*d*)

Fig.4 Parameter vibration of the plates with different debonding lengths considering damping and axial inertia force effects

5 Conclusion

(1) The character of stability of the piezoelectric composite laminates with debonded interfaces can be effectively promoted by reasonably adjusting the state of the driving piezoelectric component.

(2) The analysis method and conclusions provided are reference values for engineers to get better understanding of inefficiency of the intelligent composite plates

Acknowledgements

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