RELIABILITY MODELING OF IMPACTED COMPOSITE MATERIALS FOR RAILWAYS

Guillaumat L*, Dau F*, Cocheteux F , Chauvin T****
***LAMEFIP – ENSAM**
Esplanade des Arts et Metiers
33405 Talence cedex
****Agence d'Essais Ferroviaire**
21, avenue du Président Allende
94407 Vitry sur Seine cedex

Keywords: impact, damage, reliability, genetic algorithm

Abstract

This paper deals with a reliability approach for designing small composite railways structures under low velocity impact loading.

Impacted composite plates in bending configuration are considered in this work. Mass of projectile, height of fall and distance between supports are the three variable parameters considered. The limit state function G is defined by the impact force compared to a critical one. The probability of failure Pf is evaluated for several values of the critical force using different methods to obtain the reliability factor β; genetic algorithms are investigated in this paper. Results issued from different methods are finally compared to those issued from Monte Carlo reference simulations.

1 Introduction

Fatigue is known to be responsible for the majority of failures of structural components in transportation applications but impact loading is quite dangerous because the damage induced is not always visible. Analyzing the behavior of small composite structures under low velocity impact loading is the main subject of this research. The question is how to take into account impact damage for designing railways vehicles. This work can be roughly divided into two parts. The first one deals with the impact tests on composite structures using experimental design. The second part is focused on mechanical reliability models used to predict the probability of failure taking into account the variables uncertainties.

Impact tests have been carried out to understand the main damage mechanisms and to determine the different mechanical responses of the impacted structure. For instance, the contact force between the striker and the sample, the contact duration and the out of plane displacement are registered. Previous mechanical responses have been represented using the Response Surface Methodology (RSM). Each investigated response is then modelized by empirical polynomials issued from an experimental design [1,2].

In this study, the projectile mass (m), the height fall (h) and the span of the simply supported structure (p) are considered as mechanical uncertain input variables. Variability of output mechanical responses are then analyzed using the reliability tools in order to evaluate a failure probability P_f . In this purpose, direct Monte Carlo method and approximate FORM method are especially investigated.

2 Experimental stage

2.1 Impact device

The impact tests have been performed using a dropping mass set-up designed in our laboratory to simulate accidental falls on a structure, Fig. 1. The contact force history between the striker and the composite specimen is measured by a piezoelectric sensor. A first laser sensor is placed just underneath the center of the specimen to provide the out-of-plane displacement history. A second laser sensor measures the striker displacement versus time so that the velocity of the dropping mass can easily be assessed.

2.2 Composite sample

The material sample is an eight layers carbon/epoxy laminated composite. It was elaborated by hand lay-up technique and was cured under vacuum. Samples were cut from a large plate.

Each sample is simply supported at its edges and impacted on its center. Sample dimensions are chosen such as *l/p* ratio is a constant value (about 2,7), Fig. 1.

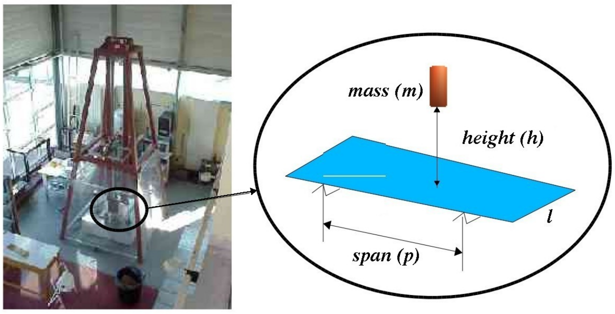

Fig. 1 Drop tower and composite sample

2.3 Impact tests

All tests are suggested by an experimental design [3]. Codification is presented in Table 1.

Variable	Name	Level +1	Level 0	Level -1
m (kg)	x_1	3,50	2,75	2
h (m)	x_2	1,0	0,6	0,2
p (mm)	x_3	480	305	130

Table 1. Impact test parameters.

Force versus time registered curves are representative of classical vibratory response. The damage is mainly represented by localized cracks and delamination always inside the cracked zone.

3 Reliability approach

3.1 General concept

The reliability approach needs the definition of a performance function, *G*, Equ. 1, issued from the mechanical phenomenon to be considered [4]. In this study, the maximum contact force $F_{contact}$ is compared to an ultimate force $F_{ultimate}$ such as (1):

$$G=F_{ultimate}-F_{contact} \qquad (1)$$

Putting down $R=F_{ultimate}$ and $S=F_{contact}$, it is classically denoted *(R,S)* approach.

The limit state is given by Equ. 2 and represents the boundary between “safety area” (pale area) and “failure area” (dark area), Fig. 2.

$$G=0 \qquad (2)$$

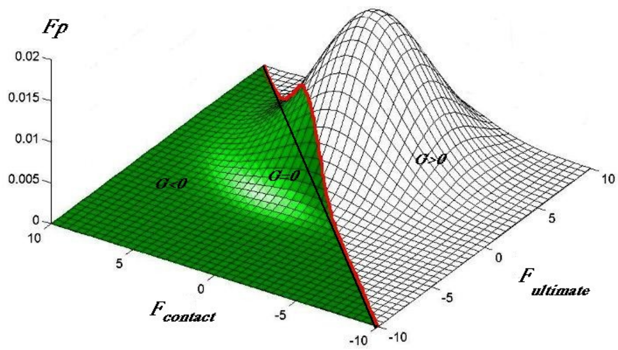

Fig. 2 Joint density of probability

The next step consists in representing the distribution of uncertain quantities; the mass *(m)*, the height *(h)*, and the span *(p)* are such quantities in this study. They will be described by a normal distribution.

3.2 Tools for reliability analysis

In a general way, two kinds of method can be distinguished for reliability analysis:

- direct methods; this is Monte-Carlo method and derivated,
- approximated methods; First and Second Order Reliability Method, respectively denoted *FORM* and *SORM* method, are such methods.

In Monte-Carlo approach, all the variables are randomly sampled according to their statistical distribution in order to create a random vector. For each trial (a set of data) the *G* function (1) is then calculated. Finally, the number of situations giving *G* negative are counted to obtain the failure probability P_f, Equ. 3.

$$P_f=\frac{number \ \ of \ \ situations \ \ G\leq 0}{total \ \ number \ \ of \ \ simulations} \qquad (3)$$

Using this method, an error depending on the trials number and the estimated probability level can advantageously be obtained, Equ. 4.

$$\%\quad error=200\sqrt{\frac{1-P_f}{nP_f}} \qquad (4)$$

On the opposite, this method can be very time consuming when limit state function calculation is complex and needs for instance the use of finite elements method (FEM). In this work, a modelisation of G function by response surface technique is advantageously used to avoid previous disavantage. (see section 4.1)

The FORM and SORM method consist in an analytical approximation of the failure probability by calculating a reliability index, β [5]. It is necessary in this case to formulate the limit state Equ. 2 in a reduced variable space (standard space) where each variable has a zero mean and unit standard deviation. The transformation for a given gaussian distribution from physical space, x_i variables, to standard space, u_i variables, is called isoprobabilistic transformation, Equ. 5.

$$u_i=\frac{x_i-m_i}{\sigma_i} \qquad (5)$$

m_i and σ_i stand for average and standard deviation of x_i .
In the previous standard space, the reliability index β represents the shortest distance from the limit state surface to the origin. Optimisation algorithms are then needed to determine this index. Genetic algorithms are investigated in this purpose looking for the best value of this index (global minimum) [6].

Then, knowing β index, the failure probability can be easily determined using the standard normal cumulative distribution function (CDF) Φ .
For instance, for FORM method used in this work, it is expressed as:

$$P_f=\Phi(-\beta)=1-\Phi(\beta) \qquad (6)$$

Thus, the failure probability estimation can be suitable provided that β issued from optimization algorithms is sufficiently accurate: its determination is of first importance.

Using optimization algorithms, x_i variables corresponding to β index can be obtained, which is of prime importance for design.
Moreover, if only two independent gaussian variables, $R=F_{ultimate}$ and $S=F_{contact}$ are considered, β can be obtained analytically by Equ. 7.

$$\beta=\frac{\bar{R}-\bar{S}}{\sqrt{\sigma_R^2+\sigma_S^2}} \qquad (7)$$

where $\bar{R}$ and $\bar{S}$ respectively denotes R and S average, σ_R and σ_S , R and S standard deviation.
With FORM (or SORM) method, no error estimation is available contrary to Monte Carlo approach.

In general way, coupling several methods for 'reliable' failure probability estimation is preferable.

4 Methodology for this work

4.1 Response surface

In order to avoid consuming time by heavy calculations (FE) or numerous experimental tests, a response surface is elaborated for $F_{contact}$. So, the limit state function G can be easily evaluated and Monte Carlo approach can be used.

$F_{contact}$ response surface is based on a minimum experimental tests number defined by an experimental design, see Table 1. A second order polynomia is elaborated, Equ. 8. Polynomial coefficients are estimated using a multilinear regression, Equ. 9.

$$\begin{aligned}F_{contact}=&1272+171x_1+485x_2-516x_3\\&-20x_1x_2+71x_1x_3-79x_2x_3\\&-97x_1^2-49x_2^2+262x_3^2\end{aligned} \qquad (8)$$

$$\hat{a}=(x^Tx)^{-1}x^Ty \qquad (9)$$

where $\hat{a}$ is the column of estimated polynomial coefficients.

Variance analysis [7] has shown that $F_{contact}$ response variation was really involved by variation of parameters x_1 , x_2 , x_3 and not due to experimental noise.

Using Equ. 8., $F_{contact}$ response, represented on Fig. 3, can be then simply evaluated with minimum cost.

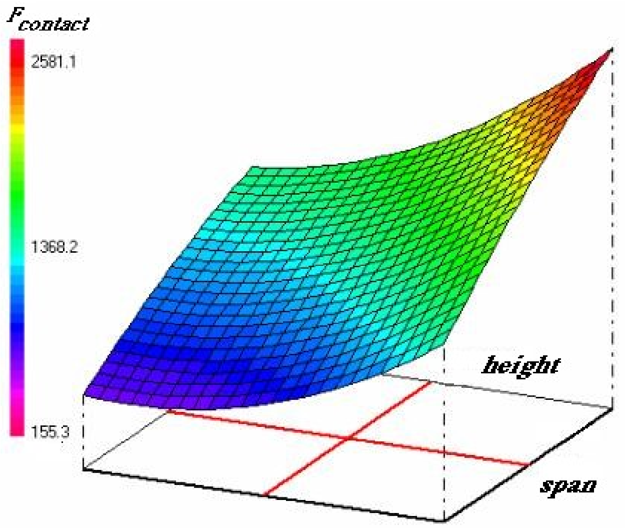

Fig. 3 Responses surface of the contact force during impact

4.2 Reliability tools

Based on previous $F_{contact}$ approximation response, Monte Carlo approach is performed with minimum cost. Results issued from this 'direct method' will be considered later as reference.

. On the other hand, FORM method is performed with different way for reliability index β determination as explained in the following section.

4.3 β determination

FORM method used in this work is based on β index obtained:

- either analytically using Equ. 7. In this case, $R=F_{ultimate}$ and $S=F_{contact}$ variables need to be considered as independent gaussian variables, see section 3.2.
- or numerically using genetic algorithms (GA). In this case, x_1 (mass), x_2 (height) and x_3 (span) are considered as independent gaussian variables.

Genetic algorithms (GA) are here preferred to deterministic algorithms (DA) to improve global minimum research: the best β value is expected in our case. GA can reveal themselves more efficient than DA ones even for simple function. Such a case is illustrated Fig. 4 where a paraboloid function is considered. Critical points P_1* or P_2* (feasible exact solutions) are systematically found with GA whereas point P is often obtained using DA; DA are influenced by gradient descent method and does not converge to the right optimum in this case!
Several strategies of research have been investigated (coupling GA and DA, GA 'Islands' strategy **[8]**, ...) and carefully evaluated on reference tests to obtain reliable value of β for present problem. In this work , 'Islands' strategy is retained for its robustness.

In the standard space, the optimization problem is formulated as:

Finding u_i which minimize the distance *d* from the limit state surface to the origin

$$d=\sqrt{(\sum_i u_i^2)} \qquad (10)$$

and satisfying

$$H(u_i)=0 \qquad (11)$$

Function H in Equ. 10 represents the performance function expressed in the standard space.

Generally known to be time consuming when they are coupled to finite element analysis, GA are interesting when the formulation is quite explicit like in this study.

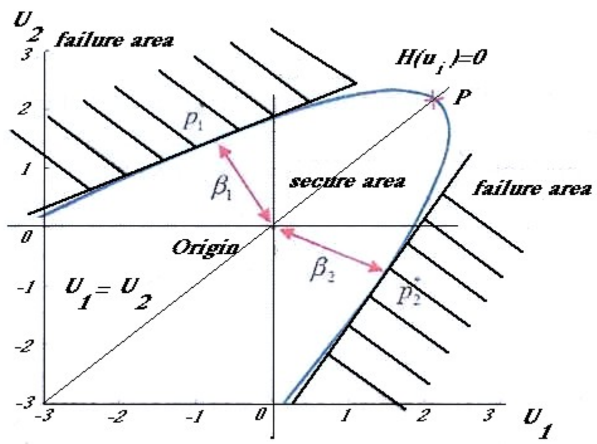

Fig. 4 Paraboloid state limit function

4.4 Simulations

For Monte Carlo approach, the performance function G is directly evaluated by the approximation given in Equ. 8 for $F_{contact}$. All variables distributions are supposed to be gaussian, so completely defined by the average and the standard deviation.

Concerning $F_{ultimate}$, the mean value varies from 600 N to 2100 N by increment of 30 N. A variance coefficient of 5% is used. This variable is considered as a parameter for the designer.

$F_{ultimate}$ variation is justified by the $F_{contact}$ mean value *1271 N,* constant value in Equ. 8. A zero mean value and 0.2 standard deviation are retained for each variable x_1, x_2 and x_3. In that way, complete distribution between [-1,+1] as suggested by experimental design (see section 2.3) are considered.

A first serie of Monte Carlo simulations, denoted *MC1,* is performed. It consists in one million of trials for each variable. For each variable set, function G is evaluated using Equ. 8. and Equ. 1. The failure probability is finally estimated by Equ. 3.

A second serie of Monte Carlo simulations, *MC2*, is next performed. For this one, only linear terms in Equ. 8 are considered to evaluate $F_{contact}$ and then G function.

Concerning FORM method, it is based on:

- an analytical calculation for β issued from Equ. 7. $F_{ultimate}$ and $F_{contact}$ are then considered as independent gaussian variables, as explained in section 4.3. Results are denoted *FORM1-Anal* when complete expression given in Equ. 8 is used and *FORM2-Anal* when only first order terms are conserved.

- a numerical calculation for β using genetic algorithms. x_1 (mass), x_2 (height) and x_3 (span) are then considered as independent gaussian variables. Results are denoted *FORM1-GA* when complete expression given in Equ. 8 is used and *FORM2-GA* when only first order terms are conserved.

5 Results

First of all, a comparison between *MC1* and *FORM1-Anal* is shown on Fig. 5. A gap is obvious beyond $F_{ultimate} = 1500\,N$. FORM approximation considering a second order polynomia, Equ. 8, is a first explanation for this gap. Secondly, second order polynomia induces a non gaussian distribution for $F_{ultimate}$, see Fig. 6. Even if the distribution seems to be not so far from a gaussian distribution, it is not. This point is significantly revealed by Henry test [7], see plots on Fig. 7. The non gaussian distribution is traduced by the non linear curve. Indeed, this curve obtained by Henry test deviates from the straight line beyond $F_{ultimate} \sim 1500\,N$. At this stage, it means that FORM method is not applicable because.it needs gaussian distribution to be used. An isoprobabilistic transformation is then needed to obtain a gaussian transformation.

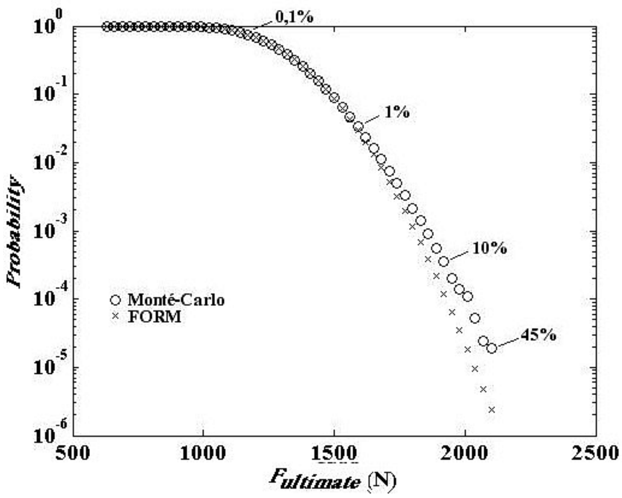

Fig. 5. Probability evolution estimated by *MC1* and *FORM1-Anal*

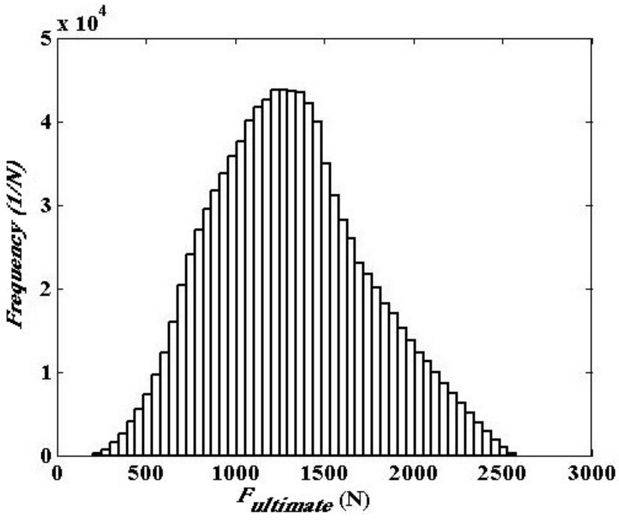

Fig. 6 $F_{ultimate}$ non gaussian distribution.

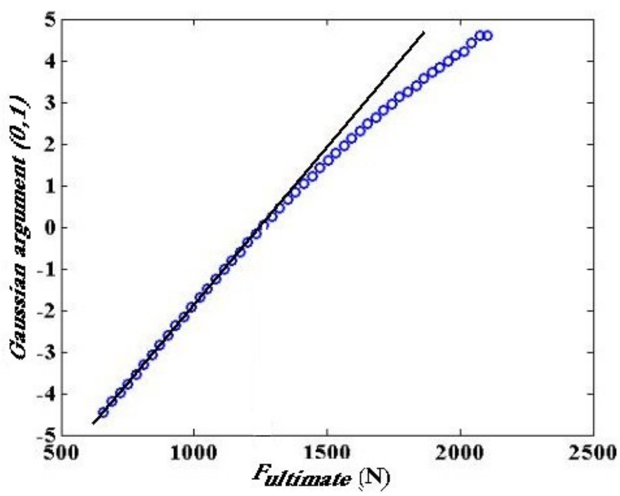

Fig. 7 Henry test for a non gaussian distribution

Percentage of error obtained with Monte Carlo method is mentionned for different values of $F_{ultimate}$, see Fig. 5. It indicates that Monte Carlo results are probably reliable until $F_{ultimate} \sim 1900\,N$, 10% error is reached at this level. Beyond this value, % error rapidly increases till 45 % for $F_{ultimate} \sim 2100\,N$: number of simulations is finally unsufficient in the range $[1900\,N - 2100\,N]$.

On the other hand, results issued from *MC2* and *FORM2-Anal* are compared on Fig. 8 and 9. Good agreement is observed. FORM approximated method gives an 'exact' estimation of the failure probability; this is due to linear polynomial response used for $F_{contact}$ in these simulations. Moreover, $F_{ultimate}$ gaussian distribution is revealed by Henry test in this case, Fig. 9.

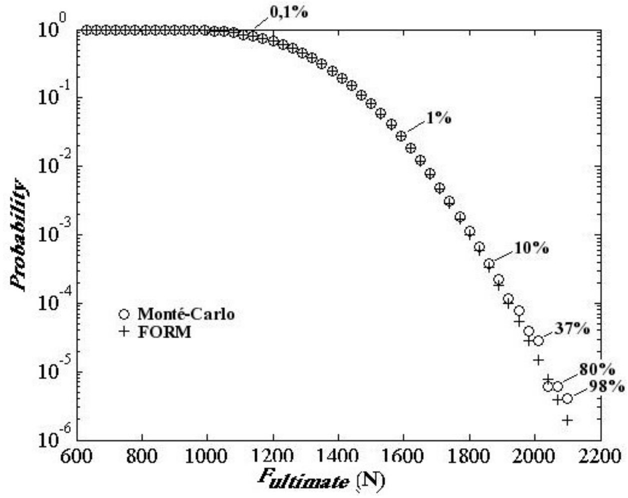

Fig. 8 Probability evolution evaluated by *MC2* and *FORM2-Anal*

Concerning Monte Carlo results, same remarks than previous ones for *MC1* can be done. Results are reliable as much as the number of simulation is in agreement with expected probability level. The limit of result validity is around $F_{ultimate} \sim 1850\,N$. Beyond this value, error is over 10% and rapidly increases. On the opposite, *FORM2-Anal* gives correct failure probability estimation without limitation. Indeed, as *FORM2-Anal* results are validated by *MC2* ones for high probability levels (till around $P_f = 3.10^{-4}$ corresponding to 10% error), the validation can be extended for small probabilities level as regards FORM properties in this case of linear limit state function.

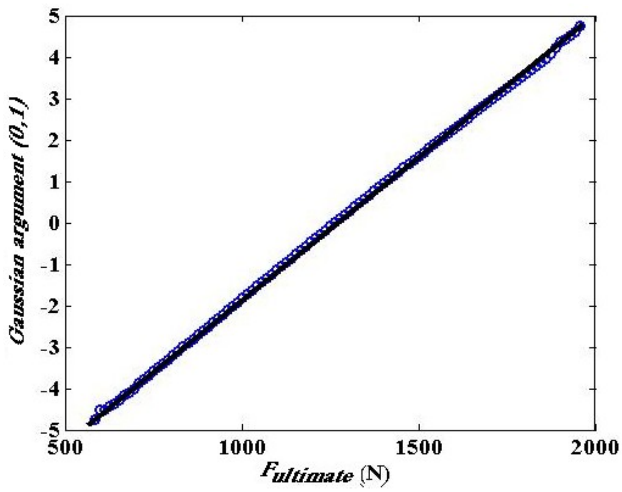

Fig. 9 Henry test for a gaussian distribution

Moreover, *MC1, FORM1-Anal* and *FORM1-GA* results are presented on Fig. 10. Those obtained by *FORM1-GA* are encouraging but need to be improved as regards discrepancies for low probability levels.

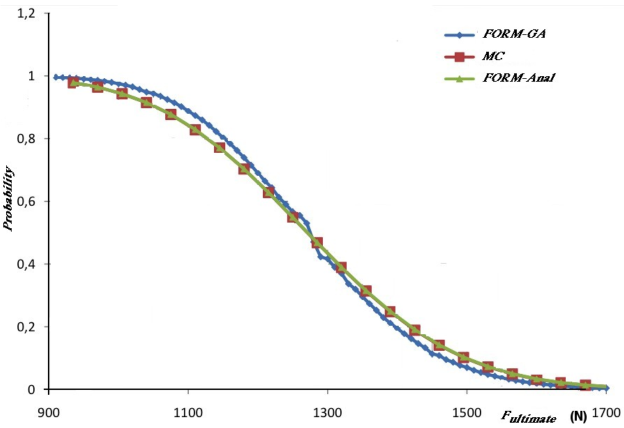

Fig. 10 Comparison between *FORM1-Anal*, FORM1-*GA* and *MC1*

Finally, two different statistical distributions were used to compare their influences on the probability level, Fig. 11. In one case a uniform distribution has been considered and in a second case a Gaussian one. Fig. 11 shows the strong variation between the two situations. Obviously, the Gaussian law provides a lower probability level because the values are more concentrated around the mean compared to the uniform law. Thus, the frequency of a value far from the mean is lower in a Gaussian case that implies a lower probability level.

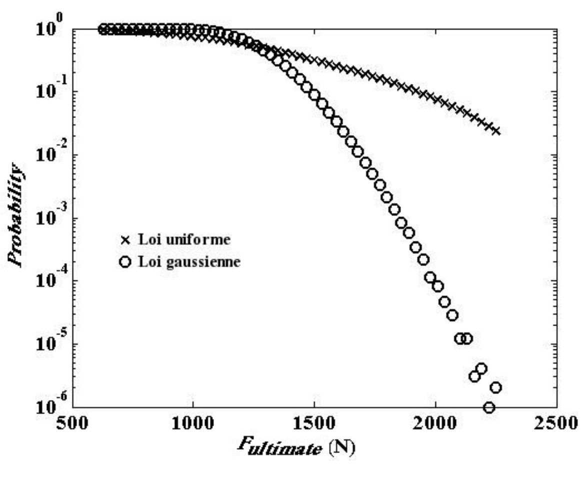

Fig. 11 Statistical distributions influence on probability levels

6 Conclusions

In this work, a reliability approach for designing small composite railways structure under low velocity impact loading is developped. A methodogy is presented for this approach. First of all, it consists in using experimental design to approximate correctly the interaction force $F_{contact}$ during the impact. Doing so, the limit state function G comparing $F_{contact}$ to a critical one $F_{ultimate}$ can be easily evaluated. The failure probability is then calculated by FORM based on two ways in obtaining the reliability factor β; the previous index is determinated either analytically or numerically. Genetic algorithms are principally investigated for the numerical way. Results issued from each technique are compared to Monte Carlo simulations standing for reference. Good agreement are observed. Feasibility of β determination using genetic algorithms is very satisfying. In this case of explicit problem formulation, previous algorithms happen to be interesting. Nevertheless, they may be used very precautionnally.

References

[1] Doehlert D.H.
"Uniform shell designs",
Applied Statistics, Vol 19, n°3 , pp. 231 239, 1970.

[2] Doehlert D.H. et Klee V.L.
"Experimental designs through level reduction of the d-dimensional cuboctahedron"
Discrete Mathematics, vol 2, pp. 309-334, 1972.

[3] Droesbeker J.J., Fine J., Saporta G.
"Experimental designs", *French Statistical Association*, *Technip Ed., 1997.*

[4] Lemaire M.
"Reliability and mechanical design",
Reliability engineering and system safety vol 55, Issue 2, pp 163-170 , 1997.

[5] Hasofer A.M., Lind N.C.
"Exact and invariant second moment code format",
Journ. Eng. Mechanics Div. vol 100, Pages 111-121, 1974.

[6] Paz E.C., Goldberg D.E.
"Efficient parallel genetic algorithms: theory and practice",
Comp. Meth. Appl. Mech. Eng. 186, pp 221-238, 2000.

[7] Saporta G.
"Probabilities – Data analysis and Statistic",
Ed. Technip, 1990.

[8] Maeda Y., Ishita M., Li Q.
"Fuzzy adaptative search method for parallel genetic algorithm with island combination"
International Journal of Approximate Reasoning, vol. 41, pp 59-73, 2006.