

INFLUENCE OF MATERIAL UNCERTAINTY ON THE DAMAGE RESISTANCE AND TOLERANCE OF STIFFENED COMPOSITE PANELS

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Abstract

Composite materials exhibit complex phenomena associated with damage onset and propagation. In particular damages induced by low velocity impacts represent a main concern in aircraft structures, which may be subjected to hail impact, tools dropped during maintenance, or foreign object damage during landing and takeoff. The heterogeneous and anisotropic nature of composites allows for several modes of failure, however it has been observed that the most dangerous and unpredictable one is the delamination, which can considerably reduce the in-plane compressive strength of the laminates. The development of damage resistance and damage tolerance design methodologies have allowed a more efficient management of the damage in composites. However these methodologies strongly depend on composite material properties, which are affected by an intrinsic uncertainty. In this paper the effect of the material properties scatter on the damage resistance and the damage tolerance of composites will be assessed by using a probabilistic approach.

1 Introduction

Composite materials, due to their better performances in terms of corrosion resistance and fatigue behavior with respect to metals, prove to be particularly suitable for the design of lighter aircraft structural components. However, at the present, their employment is restrained to secondary structures because of their vulnerability to damage induced during service and of the complex phenomena involving the damage evolution within the structure. Four major modes of failure can be induced in the structure as a consequence of a low velocity impact: matrix cracking, delamination, fiber breaking and penetration (the impactor completely perforates the

surface). It has been observed that delaminations, which can considerably reduce the structural carrying capabilities under compressive loads, represent the most detrimental damage in fiber-reinforced laminates caused by low-velocity impact. This is a particular problem in aircraft structures which maybe subject to hail impact, tools dropped during maintenance, or foreign object damage during landing and takeoff.

In order to take into account these complex phenomena, in the last years, different methodologies, oriented both to the prediction of the impact induced damage extent [1-3] and to the prediction of the residual strength after impact [4-7], have been developed. The main objective is to design panels with no induced delaminations after low velocity impacts (damage resistant panels) or panels with induced delaminations still able to sustain the design loads (damage tolerant panels).

These capabilities, which can be determinant in order to avoid over-dimensioning of composite structures, can strongly depend on material properties. This is the reason why it is worth to investigate the influence of material properties uncertainty on the damage resistance and tolerance of Composite structures.

In this paper a probabilistic approach has been adopted in order to understand the effects of material properties scatter on the damage resistance and damage tolerance characteristics of stiffened composite panels.

Two simplified linear procedures have been adopted for the evaluation of the damage resistance [9] and of the damage tolerance [8] of a stiffened panel. In particular the Force threshold method [1] is adopted to evaluate, by means of linear analyses, the threshold impact energy needed for the delamination onset. This energy value provides an effective indication of the damage resistance of the panel at a specific impact location. On the other side, the linear

delamination growth method [8] is adopted to evaluate, by means of linearized buckling and linear analyses, the delamination growth initiation load which gives an idea of the damage tolerance of the panel for a specific delamination size and position.

Such methodologies have been implemented in the PDS (Probabilistic Design System) module of ANSYS FEM code. The D Sampling form of the Monte Carlo method has been chosen for the samples generation.

The results in terms of probability distribution and cumulative distribution functions of the impact threshold energy and of the delamination radius will be analyzed and critically assessed in order to understand how and how much they are affected by the uncertainty in material properties chosen as random input variables. The higher probability material properties and delamination radius will be used as input parameters for the determination of the delamination growth initiation load.

Finally, the scatter plots, the sample histories and the sensitivities will be used as checks for the verification of the significance of the performed probabilistic analysis.

3 Damage resistance and tolerance

As previously said, different approaches can be adopted for the design of composite structures subjected to low velocity impacts. In this context we will focus the attention on damage resistant and damage tolerant design methodologies.

The former is aimed to design structures with no induced delaminations after low velocity impacts, the latter admits the presence of delaminations in composite structures as long as they are able to sustain the design loads.

In particular the impact damage resistance is the ability of a composite structure to withstand a damage caused by a foreign object impact and is represented by a threshold impact energy for the onset of a delamination. This is an important parameter determining the resistance of a composite structure to impact. If the impact energy is below this threshold level the impact does not cause delamination in the laminate. When the impact energy is just above this level, the delaminations take place instantaneously, by a sudden jump of the area from zero to a certain value.

The impact damage tolerance on the other hand is the ability of the material/structure system to still withstand the in-service loads also in presence of an impact damage and can be identified by the external compressive load needed to initiate the delamination

growth. In the next subsections two linearized procedures for the determination of the damage resistance and tolerance will be illustrated.

3.1 Davies procedure

The damage resistance of a composite structure can be evaluated by following different approaches. In this context the Davies procedure [9] has been adopted to predict the threshold impact energy for the onset of a delamination in fiber-reinforced quasi-isotropic laminates under low-velocity impact. It is based upon two models: the energy-balance model which equates the kinetic energy of the impactor with the static deformation energy of the laminate and the force-driven model previously proposed by Davies and Zhang [1]. The combination of these two models forms a simplified and straightforward procedure for the prediction of the threshold impact energy for the onset of delamination without relying on a time-consuming dynamic analysis.

The procedure can be divided into two steps: the evaluation of the maximum impact force for any impact energy and the prediction of the threshold impact energy.

The maximum impact force for any impact energy is calculated by combining the quasi-static energy-balance model proposed by Shivakumar [10] with a nonlinear finite element static analysis.

The quasi-static energy-balance model is based on the energy conservation principle by assuming that the structural response of the composite under low-velocity impact is the same exhibited under static loading. Also the hypothesis that the kinetic energy of the impactor is equal to the sum of the energies due to contact between the impactor and the plate and the overall deformation of the plate itself, has been made. The energy losses for material damping, surface friction, etc., are neglected. The quasi-static energy-balance model is expressed by Eq. 1:

$$\frac{1}{2}MV^2 = E_c + E_l \quad (1)$$

where M is the mass of the impactor and V the impact velocity. E_c and E_l are the energies absorbed by the local contact deformation and the overall deformation of the laminate respectively.

The contact energy E_c is given by Eq. 2:

$$E_c = \int_0^{\alpha_m} Pd\alpha \Rightarrow E_c = \frac{k}{n+1} \left[\frac{P_m}{k} \right]^{\frac{n+1}{n}} \quad (2)$$

where $P = k\alpha^n$ is a general accepted contact law for composite laminates, k is the contact stiffness, α is the indentation and $n=1.5$ in the classical Hertz problem. For carbon epoxy systems the value of k is typically $130 \text{ MN/m}^{3/2}$. P_m is the maximum impact force.

The energy E_l due to the overall deformation of the composite laminate can be evaluated as the area below the force/displacement curve obtained by a non-linear static finite element analysis and is given by the Eq. 3.

$$E_l = \int_0^{\delta_{\max}} Pd\delta \quad (3)$$

By substituting Eq. 2 and Eq. 3 into Eq. 1 we get

$$\frac{1}{2}MV^2 = \frac{k}{n+1} \left[\frac{P_m}{k} \right]^{\frac{n+1}{n}} + \int_0^{\delta_{\max}} Pd\delta \quad (4)$$

that, solved for P_m , provides the maximum impact force for any given impact energy $MV^2/2$.

Davies procedure for the evaluation of the threshold impact energy consists of the three following steps:

- Evaluation of the threshold impact force by using the force-driven model proposed by Davies and Zhang

$$P_c^2 = \frac{8\pi^2 Et^3 G_{IIc}}{9(1-\nu^2)} \quad (5)$$

where P_c is the threshold force for the onset of delamination in a quasi-isotropic laminate, E and ν are the equivalent Young's modulus and Poisson's ratio of the quasi-isotropic laminate, t is the thickness of the laminate. G_{IIc} is the mode II critical inter-laminar energy release rate of the material.

- Perform a non linear static analysis to get the force/displacement relation.
- Evaluation of the threshold impact energy by substituting P_c and the force/displacement relation into Eq. 4:

$$W_{cr} = \frac{k}{n+1} \left[\frac{P_c}{k} \right]^{\frac{n+1}{n}} + \sum_0^{\delta(P_c)} Pd\delta \quad (6)$$

Once the threshold impact energy for delamination onset has been evaluated, the circular damage area of delamination can be easily calculated as [1]:

$$A = \frac{9}{16\pi^2} \left(\frac{P}{\tau} \right)^2 \quad (7)$$

3.2 Linearized procedure for damage tolerance

A novel approach for the simulation of the delamination growth in delaminated composite panel has been proposed in [8]. In order to design damage tolerant structures, it is apparent the importance of predicting when delaminations start to propagate

The proposed approach represents a fast and inexpensive numerical methodology, particularly suitable for preliminary design purpose, based on a limited number of linear analyses, aimed to support an effective damage tolerant design of composite structures.

The first analysis is a linearized buckling needed to obtain the delamination buckling shape and load. An additional linearized buckling analysis together with two linear static analyses are aimed to the evaluation of the energy amount released during the delamination propagation by means of the elastic energy balance application.

The procedure proposed is detailed in the following steps:

- Perform a linearized buckling analysis to get the local buckled shape of the thinner sub-laminate
- Perform a linearized buckling analysis together with two linear static analyses to calculate energy released by the structure during the delamination propagation
- Redistribution of the global released energy along the delamination front
- Evaluation of the load value at which the local ERR (Energy Release Rate) has been attained

The most important assumption which the method is based on, is that the delamination front propagates only in consequence of the delamination buckling and following the first opening mode (see Fig.1). This assumption is valid when the buckled sub-laminate thickness is considerably smaller than the total laminate thickness.

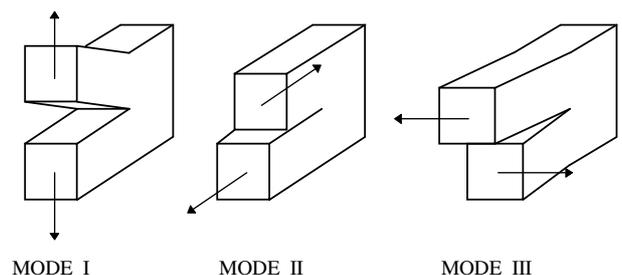


Fig.1. Basic fracture modes

The first analysis is then aimed to carry out the first buckling load and mode and at each position i along the delamination front, it is possible to univocally determine the normalized out-of-plane displacements $(\Delta u_i/\Delta u_1)(i=1\dots N)$, where N is the total number of location considered along the delamination front (see Fig.2).

After the delamination buckling event, characterized by a critical load F^{cr} , it can be assumed that the buckled thinner sub-laminate does not contribute to the global residual stiffness. This means that the stiffness K^A of the post-buckled structure with delamination size A and $K^{A+\Delta A}$ with delamination size $A+\Delta A$ can be calculated by removing the buckled sub-laminate.

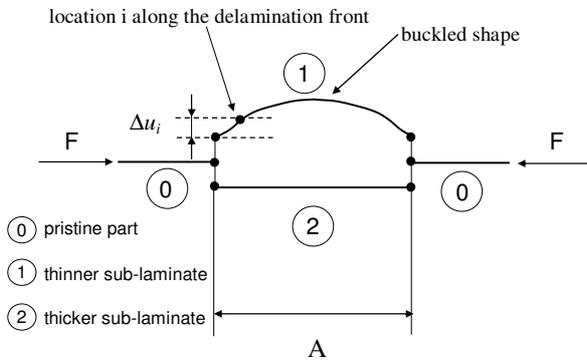


Fig. 2. Locally delaminated structure

In Fig.3 the evolution of the global structural stiffness, starting from the pre-buckling stiffness K^0 , is reported.

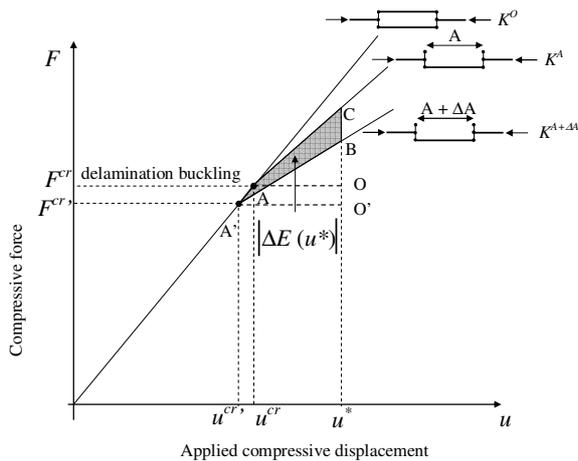


Fig. 3. Stiffness evolution of the structure

For each value of compressive displacement u^* the overall energy loss $\Delta E(u^*)$ (area $A'ACB$ in Fig.3), associated with the delamination size increment is given by

$$\begin{aligned} \Delta E(u^*) &= E^A(u^*) - E^{A+\Delta A}(u^*) = \\ &= \text{area}(A'ACO) - \text{area}(A'BO') = \\ &= \text{area}(ACO) + \text{area}(A'A'O'O') - \text{area}(A'BO') = \\ &= \frac{1}{2} K^A (u^* - u^{cr})^2 + \frac{1}{2} K^0 (2u^* - u^{cr} - u^{cr})(u^{cr} - u^{cr'}) - \\ &= \frac{1}{2} K^{A+\Delta A} (u^* - u^{cr'})^2 \end{aligned} \quad (8)$$

where $E^A(u^*)$ and $E^{A+\Delta A}(u^*)$ are the elastic energies absorbed for effect of the applied displacement u^* with delamination sizes A and $A+\Delta A$ respectively. The local energy release rate for each location i along the delamination front, for simple geometries and for equidistant locations along the front, is given by

$$G^i(u^*) = - \frac{\Delta E^i(u^*)}{\Delta A/N} \quad (9)$$

where $\Delta E^i(u^*)$ is the local energy loss associated with the location i along the delamination front and $\Delta A/N$ is the fraction of delamination increment associated with the same location. The overall energy loss is then given by

$$\begin{aligned} \sum_N \Delta E^i(u^*) &= \Delta E(u^*) \Rightarrow \\ \Rightarrow \sum_N G^i &= - \frac{\Delta E(u^*)}{\Delta A/N} \end{aligned} \quad (10)$$

By further assuming that the geometrical and material configurations along the delamination front do not differ considerably each other, the local delamination bending stiffness can be considered nearly constant:

$$G^i = \left(\frac{\Delta u_i}{\Delta u_{i-1}} \right)^2 G^{i-1} \quad (11)$$

Combining Eq.10 and Eq.11 we get the expression of the local energy release rate associated with the first position along the delamination front:

$$G^1(u^*) = - \frac{\Delta E(u^*)}{(\Delta A/N) \sum_N \left(\frac{\Delta u_i}{\Delta u_1} \right)^2} \quad (12)$$

By increasing the compressive displacement u^* , the first location whose ERR $G^i(u^*)$ exceeds the critical energy release rate G_{IC} , can be easily calculated. At that location the delamination propagation is supposed to take place and the corresponding displacement u^* becomes the critical displacement u^{del} at which the delamination starts to growth. The external compressive load needed for the initiation of the delamination growth is given by:

$$F^{del} = F^{cr} + K^A (u^{del} - u^{cr}) \quad (13)$$

3 Probabilistic design approach

Any finite element analysis needs a fixed set of input data given by material properties, geometrical parameters, load and boundary conditions. Starting from such data, the finite element code provides the output data in terms of displacements, temperatures, stresses and so on. However the input parameters are in general subjected to a scatter due to either variability of the mechanical properties or inaccuracies during the manufacturing process. These uncertainties can be well described by suitable statistical distribution functions. A probabilistic design approach, based on probabilistic analyses, is aimed to quantify the uncertainties in the output variables corresponding to given statistical distributions of the input parameters and to provide the probability that a given design criterion based on the output variables is no longer fulfilled. An additional sensitivity analysis can provide useful indication about the input variables that most affect the uncertainty of the output.

The methods adopted to perform probabilistic analyses (stochastic methods) are all based on the processing of the results obtained by solving a deterministic problem several times. Each deterministic solution (simulation loop) is obtained by generating a different set of values for the random variable which simulates a particular set of input data. Probabilistic methods differ each other in the way they choose the set of values (sampling points) to be processed for the deterministic analyses. In this context the Monte Carlo Simulation method has been adopted. It is the most commonly used method to perform probabilistic analyses and is also called sampling method since the inputs are randomly generated from probabilistic distributions in order to simulate the process of sampling from an actual population. The sampling points are then located at random locations in the space of random input variables. Monte Carlo method makes use of two different techniques to evaluate the random

locations of the sampling points. The first and the most traditional one is called Direct Sampling and has no memory, namely it may happen that more sampling points are very close each other. This kind of sampling, although could lead to inefficient sampling, well describes the natural intrinsic variability of the random parameters. The more advanced and efficient form of the Monte Carlo method is the Latin Hypercube Sampling. This technique has memory of the samples evaluated before and then avoids clustering samples. The range of all random parameters is divided into n intervals with equal probability and, for each variable, in each interval only one sampling point can be caught. It is clear that this technique requires fewer simulation loops than the direct method to obtain the same results with the same accuracy.

4 Finite element implementation

The procedures illustrated in previous sections have been implemented in the finite element code Ansys version 10.0 according to the flowchart reported in Fig.4.

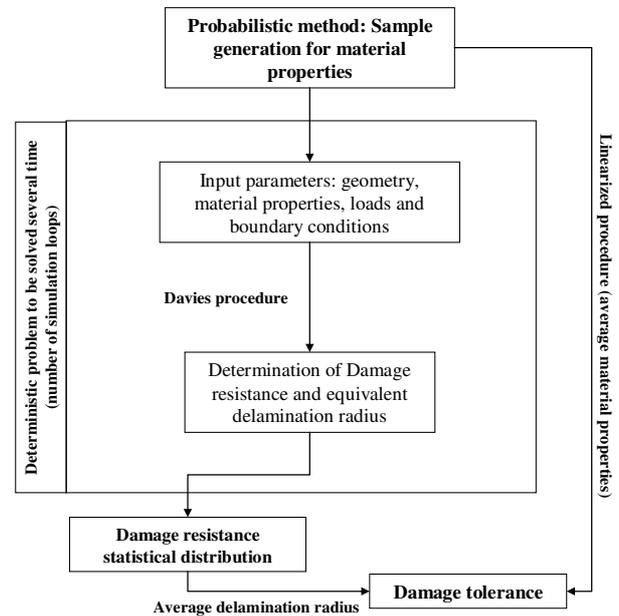


Fig. 4. Flowchart of the implemented procedure

The PDS (Probabilistic Design System) module of Ansys has been used to perform the probabilistic analyses. The probabilistic method adopted (Direct Sampling form of the Monte Carlo method) generates the material properties samples according to the corresponding statistical distributions and to the number of simulation loops entered. The random input parameters together with

the deterministic ones (geometry, loads and boundary conditions) are used to determine the statistical distributions of the damage resistance of the structure (Davies procedure) and of the delamination radius corresponding to an impact energy greater than its critical value. The average value of the delamination radius distribution, together with the average material properties, are the inputs for the linearized procedure illustrated in section 2.2. In this way it is possible to determine the damage tolerance, in terms of delamination growth force, corresponding to the most probable input parameters.

5 Geometry, boundary conditions and material properties

The probabilistic analysis has been performed on a stiffened panel with three stringers. The geometrical parameters of the panel are reported in Fig.5.

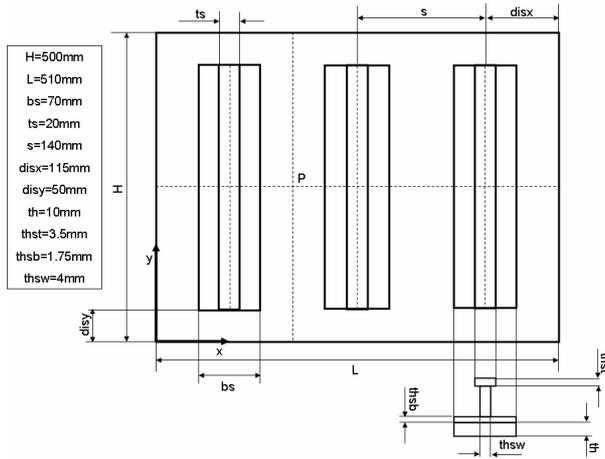


Fig. 5. Stiffened panel geometry

The boundary and load conditions imposed for the first linearized buckling analysis are the following:

$u_x=u_y=u_z=rot_x=rot_y=rot_z=0$	$0 < x < disx/4, L-disx/4 < x < L$
	$0 < y < disy/2, H-disy/2 < y < H$
$F_z = -1$	$x = disx + s/2, y = H/2$

The boundary conditions for the subsequent linear analyses are the following:

$u_x=u_y=u_z=rot_x=rot_y=rot_z=0$	$0 < y < disy$
$u_x=u_z=rot_x=rot_y=rot_z=0$	$H-disy < y < H$

2.2 Stacking sequences

The stacking sequences adopted for the different parts of the panel are reported in Table 1.

Table 1. Stacking sequences of the panel

Skin	$[-45^\circ/45^\circ/0^\circ/90^\circ/-45^\circ/45^\circ/0^\circ/-45^\circ/90^\circ/45^\circ/0^\circ/-45^\circ/90^\circ/45^\circ/0^\circ/-45^\circ/90^\circ/45^\circ/0^\circ/45^\circ]_s$
Stringer base	$[45^\circ/-45^\circ/0^\circ/90^\circ/0^\circ/-45^\circ/45^\circ]$
Stringer top	$[45^\circ/-45^\circ/0^\circ/90^\circ/0^\circ/-45^\circ/45^\circ]_s$
Stringer web	$[45^\circ/-45^\circ/0^\circ/90^\circ/0^\circ/-45^\circ/45^\circ/0^\circ]_s$

The choice of quasi-isotropic lay-up for each part of the panel has been made in order to respect the main assumption which the linear procedures adopted are based on.

2.2 Material properties

Matrix and fiber components of the composite laminate are constituted by an epoxy resin (Hexcel 8552) and carbon fibers (IM7). The material properties of the ply are reported in Table.2

Table 2. Material properties

E_1 (GPa)	158.66
$E_2 = E_3$ (GPa)	8.76
$G_{12} = G_{13} = G_{23}$ (GPa)	5.135
$\nu_{12} = \nu_{13} = \nu_{23}$	0.27
G_{Ic} (J/m ²)	262
G_{IIc} (J/m ²)	543
τ (MPa)	50
σ_1 (MPa)	2548.33
σ_2 (MPa)	66.88

where G_{Ic} and G_{IIc} are the critical energy release rates corresponding to the first and to the second failure modes respectively, τ is the allowable interlaminar shear stress, σ_1 and σ_2 are the in-plane tensile strengths.

The material properties considered as random input parameters are the in-plane elastic moduli and the shear modulus. Their statistical distributions have been assumed to be Gaussian (normal), characterized by two distribution parameters, namely the mean value μ and the standard deviation σ . Such values have been obtained by experimental data and are reported in Table 3

Table 3. Material properties

Property	Mean Value (GPa)	Standard Dev. (MPa)
E_1	158.66	1154
E_2	8.76	884
G_{12}	5.135	618

8 Numerical results

The dependence of the damage resistance and of the delamination dimension on material properties scattering has been determined by performing a probabilistic analysis with 1000 simulation loops, generated by the Direct Sampling form of the Monte Carlo probabilistic method..

8.1 Sample history

The first and fundamental step in screening the results of the analysis is the review of the simulation loop results (mean, minimum, maximum values or standard deviation) as a function of the number of simulation loops. In particular the trends of the mean values and of the standard deviation history could provide very helpful information in verifying the effectiveness of the simulation loops number. The convergence of such values, achieved if the corresponding plots approach a plateau, guarantees that the number of simulation loops is sufficient to obtain a reliable probabilistic analysis. In Fig.6 and Fig.7 the sampling histories of the longitudinal Young modulus mean value and standard deviation are reported respectively:

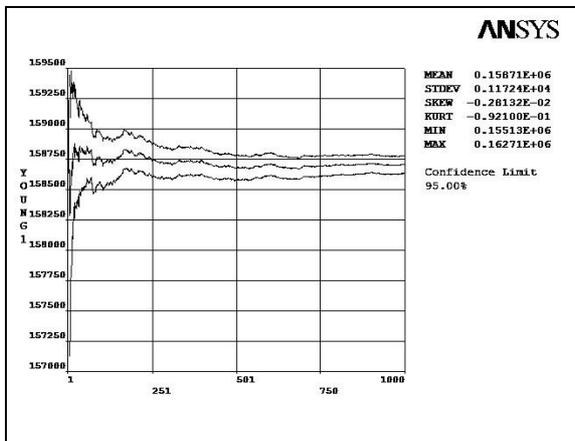


Fig. 6. Sampling history of the Young1 mean value

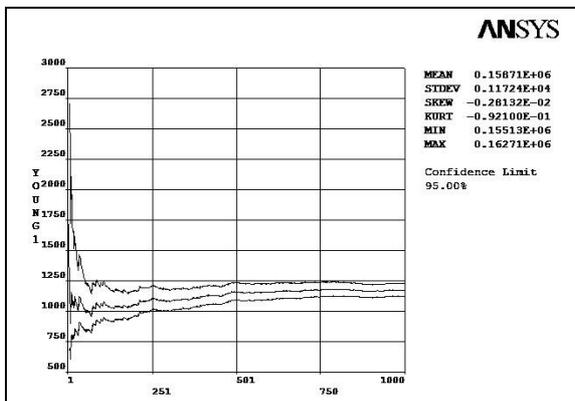


Fig. 7. Sampling history of the Young1 std dev

Both the mean value and the standard deviation approach a plateau at about 200 simulation loop. This means that the sampling process on the longitudinal Young modulus is effective to obtain a reliable analysis. The same results have been found for the other input and output random variables.

8.2 Histograms

In Fig.8 and Fig.9 the histograms corresponding to the critical impact energy and to the delamination radius are shown.

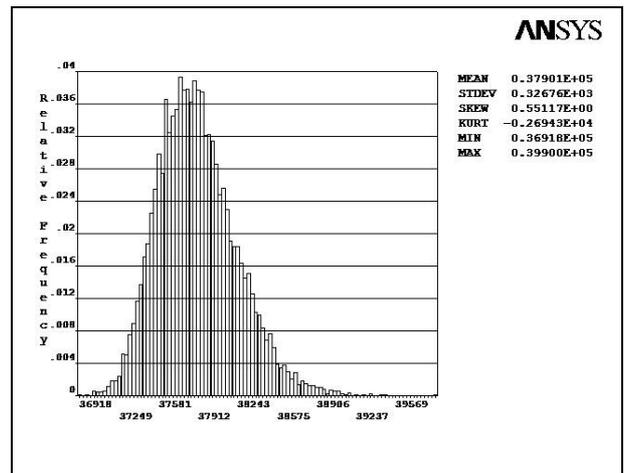


Fig. 8. Histogram of the critical energy

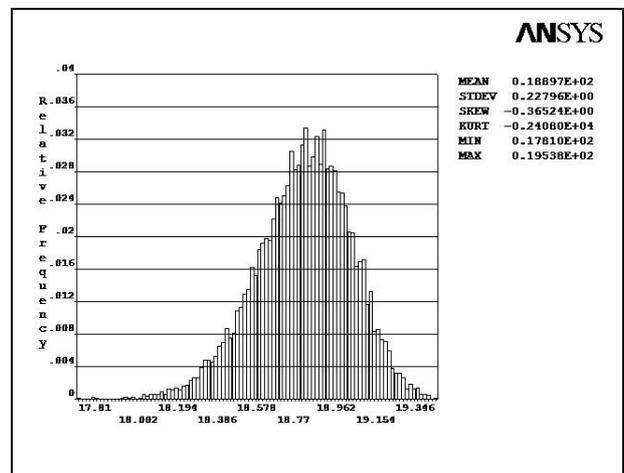


Fig. 9. Histogram of the delamination radius

The mean value and the standard deviation of the critical impact energy are 37901 J and 326 J respectively, 18.89 mm and 0.22 mm for the delamination radius. Since about the 99% of a distribution values are within 3 standard deviation of the mean, this means that the 99% of the critical energy values can differ from the mean value of about 2.5% and the 99% of the radius values can moves away from the mean of about 3.4%.

8.3 Sensitivities

Probabilistic sensitivities measure how much the range of scatter of a response parameter is influenced by the scatter of the random input variables. After a preliminary test of significance for each input variable, the PDS will plot only the sensitivities of the random input variables that are found to be significant. This information reveals to be of primary importance when one wants to make a component more reliable or improve its quality. Indeed, once sensitivities of the input variables has been verified, one can act on the most significant ones if they are controllable.

In Fig.10 and Fig.11 the sensitivities of the critical energy and of the delamination radius to the random input parameters are shown as pie charts.

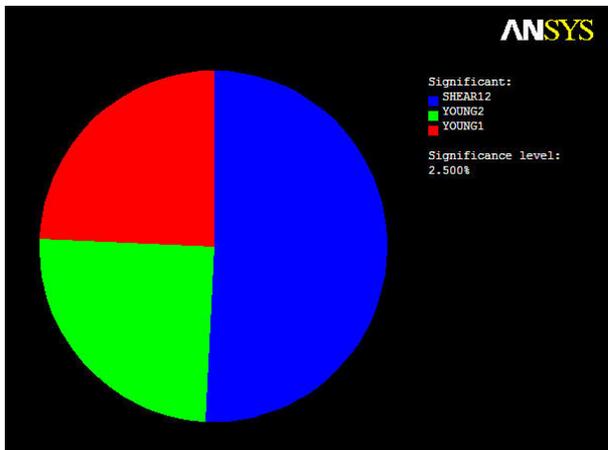


Fig. 10. Sensitivity of the critical energy

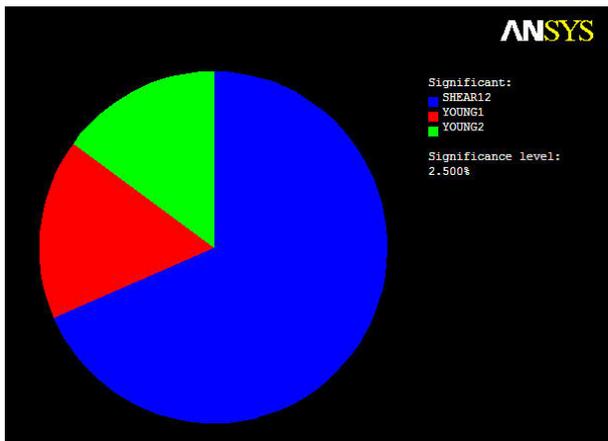


Fig. 11. Sensitivity of the delamination radius

Although all the random input variables affect the output distributions, the in-plane shear modulus scattering (blue piece of the pie) has a more significant impact on the scattering of the results.

8.4 Scatter plot

While the sensitivity analysis suggests you which random input variables mostly affect the scattering of the output results, scatter plots provide a better understanding on how and how far the input variables can be modified. To this end the plot of the output parameters as a function of the most significant input variables can be very helpful. Fig.12 and Fig.13 show the trends of the two output variables as a function of the in-plane shear modulus scattering. The shear modulus is in fact the input random parameter that mostly affect the output.

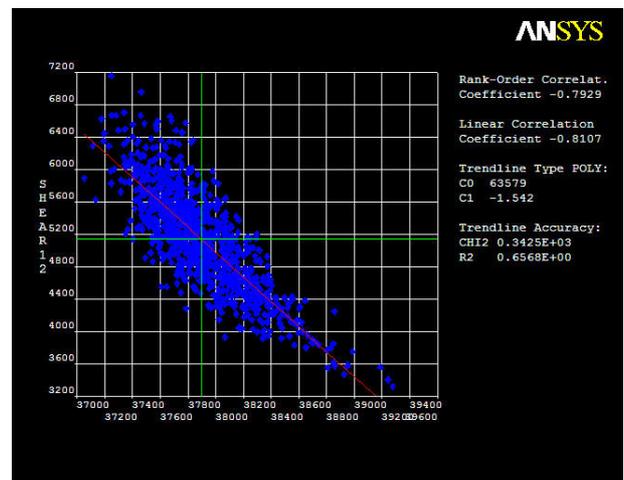


Fig. 12. Scatter of critical energy vs shear modulus

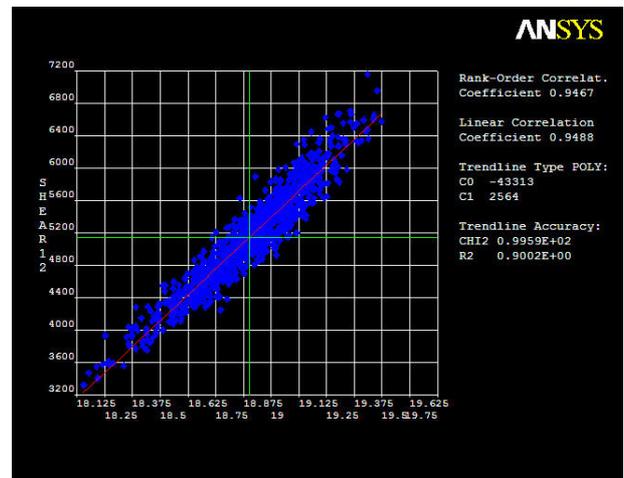


Fig. 13. Scatter of del. radius vs shear modulus

The scatter plots of the critical energy and of the delamination radius confirm the results of the sensitivity analysis. Both the output variables show an alignment, which is much more defined for the delamination radius, of the sampling points with an almost linear trendline. Such trendline is decreasing in the former case and increasing in the latter. The trendlines of the sampling points random spreading

as a function of the other two variables (longitudinal and transversal elastic moduli) are almost horizontal. This is the demonstration of a scarce effect of these parameters on the output.

8.5 Delamination growth initiation

Once the statistical distributions of the random output parameters have been carried out by performing a finite element probabilistic analysis, the most likely value of the delamination radius, together with the mean values of the random input material properties, has been entered the linearized procedure for the evaluation of the damage tolerance. The procedure provides, in addition to the compressive load to initiate the delamination growth, also the position along the delamination front where G_{Ic} (the mode I critical inter-laminar energy release rate of the material) attains its maximum value and then the delamination opening occurs.

As shown in section 8.2, the mean value of the delamination radius statistical distribution is 18.89 mm. In correspondence of this value, the compressive load needed to initiate the growth of the delamination has been found to be 1305645 N and the first point where the delamination opens is placed at 135° with respect to the x axis (the in-plane axis orthogonal to the stringers). At that point, in fact, the energy release rate corresponding to the first opening mode, reaches its critical value (262 J/m²).

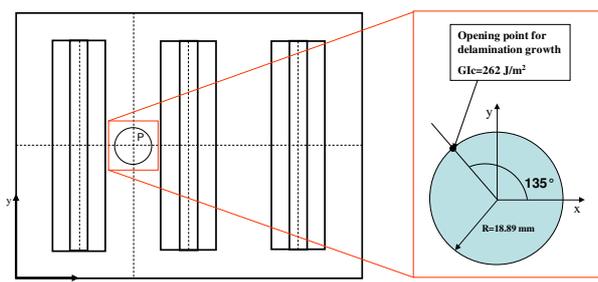


Fig. 14. Delamination opening point

10 Conclusions

Two linearized procedures have been adopted to determine the impact damage resistance of an integer stiffened panel and the damage tolerance of an impacted composite stiffened panel. The first procedure has been implemented within the probabilistic module of the finite element code Ansys version 10.0, in order to assess the

dependence of the critical impact energy and of the delamination radius on material properties considered as random input parameters. The results have first shown that the number of simulation loops is sufficient to obtain a reliable probabilistic analysis. The scatter of the material properties considered as random input parameters produces a maximum variability in the critical energy distribution of about 2.5% and of about 3.4% in the delamination radius distribution. The in-plane shear modulus variability is the one that mostly affects the scatter in the output random parameters. Indeed both the sensitivity analysis and the scatter plots have shown a strict dependence of the critical energy and of the delamination radius on the variability of the composite shear modulus. The delamination radius is more affected by that scattering of the input random parameters than the critical impact energy. Indeed the alignment of the sampling points with an almost linear trendline is much more defined; the sampling points corresponding to the critical energy are widely spread around the trendline.

The most likely value of delamination radius, together with the average values of the random material properties, has been entered the linearized procedure for the determination of the damage tolerance in terms of the compressive load needed to initiate the delamination growth. The same procedure has provided also the position along the delamination front where the opening of the delamination does occur, by comparing the energy release rate at the delamination front points with its critical value.

The choice of a simplified linear procedure for the evaluation of the damage tolerance is due to the high computational burden required by a non-linear analysis. Indeed the implementation of this procedure within an integrated probabilistic analysis will be a subsequent development of the present work in order to completely characterize the dependence of the delamination evolution (from the onset to the growth initiation) on the material properties scattering. To this end a non-linear procedure for the damage tolerance is not clearly feasible do to the high computational cost needed for simulation loops required by the probabilistic analysis.

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