DELAMINATION PROPAGATION ANALYSIS OF COMPOSITE LAMINATE USING X-FEM

Toshio Nagashima, Hiroshi Suemasu
Sophia University

Keywords: *Extended FEM; Composite laminate; Delamination; Energy release rate*

Abstract

The extended finite element method (X-FEM), which utilizes interpolation functions based on the concept of the partition of unity, has been applied to evaluate fracture parameters such as stress intensity factors and crack extension simulations. X-FEM can simplify the modeling of continua containing several cracks, and can thus be used to perform effective stress analyses related to fracture mechanics. X-FEM is applied to model interlaminar delamination, which may propagate in composite laminate. The present paper describes a numerical procedure for modeling the delamination and the propagation of composite laminate. As numerical examples, three-dimensional stress analyses for DCB test specimens were performed by X-FEM, and fatigue delamination propagation analysis was demonstrated.

1 Introduction

The extended finite element method (X-FEM) [1][2], which utilizes interpolation functions based on the concept of the partition of unity [3], has been applied to evaluate fracture parameters such as stress intensity factors and crack extension simulations. The X-FEM can simplify the modeling of continua containing several cracks, and thus can be used to perform effective stress analyses related to fracture mechanics. In the X-FEM, the basis functions, which can approximate the properties of the displacement field, are added locally to the interpolation functions used in the conventional FEM. Such an added function is referred to as an enrichment function. Stress analyses of three-dimensional bodies containing planar and/or non-planar cracks having complex geometries [4-6] were performed more effectively in conjunction with the level set method [7][8]. Therefore, to effectively perform design analyses and damage evaluation of composite structures, we have applied the X-FEM to model interlaminar delamination, which may propagate in composite materials [9][10]. The authors performed X-FEM analysis of the specimens used in double cantilever beam (DCB) and end notched flexure (ENF) fracture tests and evaluated the energy release rate at the front tip of the delamination in a composite laminate [11]. As delamination can be modeled independently of the finite element mesh in the X-FEM analysis, the X-FEM can be used to perform stress analysis of a composite laminate with interlaminar delamination much easier than the conventional FEM in conjunction with the automatic mesh generation method. The present paper describes a numerical procedure for analyzing the delamination propagation of a composite laminate and presents numerical results for the DCB test specimen.

2 X-FEM

2.1 Interpolation function

In the present study, interlaminar planar delamination in a composite laminate is assumed. As shown in **Fig. 1**, the analyzed domain is defined using Cartesian coordinates (x, y, z), and the planar delamination is assumed to be in any plane parallel to the x-y plane. In the X-FEM, the approximate displacement function $\boldsymbol{u}^{\mathrm{h}}$ of the distributed displacement $\boldsymbol{u}$ near a delamination is expressed as follows:

$$\mathbf{u}^h(\mathbf{x}) = \sum_{I=1}^{8} N_I(\mathbf{x})\mathbf{u}_I + \sum_{I\in\mathbf{C}} N_I(\mathbf{x})\sum_{k=1}^{4}\gamma_k(\bar{f}(\mathbf{x}),z)\mathbf{a}_I^k + \sum_{I\in\mathbf{J}} N_I(\mathbf{x})H(z)\mathbf{b}_I \tag{1.1}$$

$$\bar{f}(\mathbf{x}) = \sum_{I=1}^{8} N_I(\mathbf{x}) f(\mathbf{x}_I) \tag{1.2}$$

where N_I is the interpolation function used in the formulation of the conventional FEM, $\boldsymbol{C}$ and $\boldsymbol{J}$ denote the node set considering the asymptotic solution and the discontinuity of displacement near a crack, respectively, and $\boldsymbol{u}_I$, $\boldsymbol{a}_I^k$, and $\boldsymbol{b}_I$ denote the vectors of freedoms assigned to each node. Here, $\mathbf{C} \cap \mathbf{J} = \phi$ is satisfied. In addition, γ_i (i = 1,2,3,4) are the near-tip functions, which consider the discontinuity near the crack tip, $H(z)$ is the Heaviside function used to express the discontinuity of the displacement on a delamination, and $f(\mathbf{x})$ is a level set function introduced in order to express the shape of the front tip of the delamination using nodal information. The level set function $f(\mathbf{x})$ is described below.

In the present study, the level set function $f(\mathbf{x})$ used in Eq. (1.2) is defined as follows:

$$f(\mathbf{x}) = \min_{\bar{\mathbf{x}} \in \Gamma} \|\mathbf{x} - \bar{\mathbf{x}}\| sign(\mathbf{n}(\bar{\mathbf{x}})^T (\mathbf{x} - \bar{\mathbf{x}})) \qquad (2)$$

where Γ represents the curved front line of the delamination, $\bar{\mathbf{x}}$ is a point on the curved line Γ, and $\mathbf{n}(\bar{\mathbf{x}})$ denotes the vector orthogonal to the curved line Γ at point $\bar{\mathbf{x}}$.

This function is called the signed distance function, and the absolute value of f is the distance between the point and Γ.

In the present study, the near-tip function γ_i, which is determined from the asymptotic solution of a crack in a homogeneous isotropic material, is defined as follows:

$$\begin{aligned} &\gamma_1 = \sqrt{r}\cos\left(\frac{\theta}{2}\right), \gamma_2 = \sqrt{r}\sin\left(\frac{\theta}{2}\right), \\ &\gamma_3 = \sqrt{r}\sin\left(\frac{\theta}{2}\right)\sin\theta, \gamma_4 = \sqrt{r}\cos\left(\frac{\theta}{2}\right)\sin\theta \end{aligned} \qquad (3)$$

where r and θ are the polar coordinates in a plane near the crack tip, and

$$r = \sqrt{f^2 + z^2} \qquad (4.1)$$

$$\theta = \arctan(z/f) \qquad (4.2)$$

The near-tip function given in Eq. (3) can be used in the X-FEM analysis for a crack in a homogeneous isotropic elastic material. Since γ_2 in Eq. (3) is discontinuous on the crack line ($\theta = \pm\pi$), it can express the discontinuity displacement on the crack line.

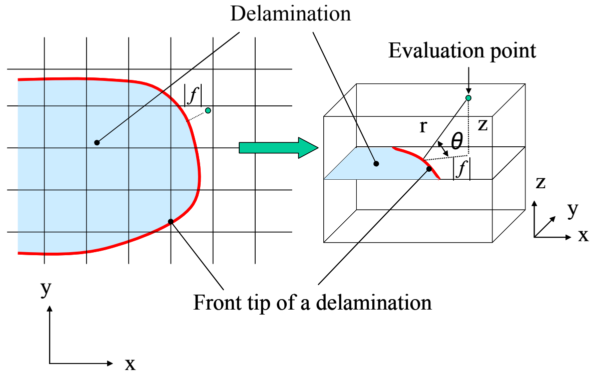


Fig. 1 X-FEM modeling of a delamination.

2.2 Nodal property

Using the X-FEM, a delamination can be modeled independently of the finite element mesh much easier than would be possible using the conventional FEM. An example distribution of the enriched node used for three-dimensional X-FEM analyses is illustrated in **Fig. 2**. In the figure, the node labeled C has an additional freedom for the near-tip function, and the node labeled J has an additional freedom for the Heaviside function, as shown, respectively, by the second and third terms on the right-hand side of Eq. (1.1).

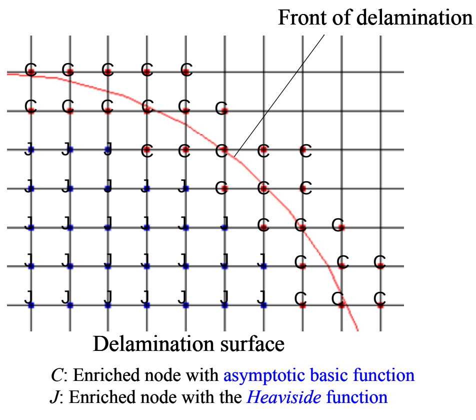


Fig. 2 Distribution of enriched nodes for modeling of delamination.

2.3 Numerical integration

Since the interpolation function in the element, which is constructed from enriched nodes, includes high-order functions, the normal order of the Gauss numerical integration may be inadequate. In the X-FEM analysis, if an element includes a discontinuity displacement field such as a crack surface, then the element is partitioned into multiple domains, and numerical integration is performed for each domain [4-6].However, in the present study, a planar delamination is assumed to be located just on the element surface, and an element never contains a discontinuous surface. Therefore, a higher order Gauss numerical integration is used rather than the partitioned domain method. A higher order of integration is adopted in elements with enriched nodes.

2.4 Evaluation of energy release rate

In the finite element models obtained by X-FEM analysis, the geometry of the crack front does not always match the element boundary. Therefore, the virtual crack closure method, which is used in conjunction with conventional FEM analysis, cannot be applied to evaluate the energy release rate. The domain form of the energy release rate [12] can be used to compute the energy release rate in conjunction with X-FEM. If we define the local Cartesian coordinates $\tilde{x}$, $\tilde{y}$, and z such that the direction of $\tilde{x}$ is parallel with the crack extension, then the domain form of the energy release rate G in three-dimensional problems can be expressed as follows:

$$\overline{G}=\int_V\Big[(\sigma_{\tilde{x}}\frac{\partial u_{\tilde{x}}}{\partial\tilde{x}}+\tau_{\tilde{x}\tilde{y}}\frac{\partial u_{\tilde{y}}}{\partial\tilde{x}}+\tau_{\tilde{x}z}\frac{\partial u_z}{\partial\tilde{x}}-w)\frac{\partial q}{\partial\tilde{x}}+(\tau_{\tilde{x}\tilde{y}}\frac{\partial u_{\tilde{x}}}{\partial\tilde{x}}+\sigma_{\tilde{y}}\frac{\partial u_{\tilde{y}}}{\partial\tilde{x}}+\tau_{\tilde{y}z}\frac{\partial u_z}{\partial\tilde{x}})\frac{\partial q}{\partial\tilde{y}}+(\tau_{\tilde{x}z}\frac{\partial u_{\tilde{x}}}{\partial\tilde{x}}+\tau_{\tilde{y}z}\frac{\partial u_{\tilde{y}}}{\partial\tilde{x}}+\sigma_z\frac{\partial u_z}{\partial\tilde{x}})\frac{\partial q}{\partial z}\Big]dV \tag{5.1}$$

$$G=\frac{\overline{G}}{\int_{L_C}\alpha ds} \tag{5.2}$$

where V is the volume, including the evaluation point on the front tip of the delamination, α is a scalar field that is unity at the evaluation point and vanishes at the surface of the volume V, L_c is the delamination front, and σ_{ij}, u_j, and w are the stress, displacement, and strain energy density, respectively. The cylindrical region can be used as an integrated domain for the conventional FEM, and this region can be used for special cases of the X-FEM analysis, where the geometry of the front tip matches the element boundary. However, in the X-FEM analysis, the geometry of the front tip does not always match the element boundary. In such a case, a rectangular parallelepiped region defined in local Cartesian coordinates ($\tilde{x},\tilde{y},z$), as shown in **Fig. 3**, is used, the center of which corresponds to the evaluation point at the front tip [4-6]. The energy release rate G is calculated using the stress, strain, and displacement, which are evaluated at integration points in the domain. The weight function q takes the value of one at the center of the rectangular parallelepiped and vanishes at the surface. The weight function q is expressed as follows:

$$q(\tilde{x},\tilde{y},z)=\left(1-\frac{2\tilde{x}}{L_{\tilde{x}}}\right)\left(1-\frac{2\tilde{y}}{L_{\tilde{y}}}\right)\left(1-\frac{2z}{L_z}\right) \tag{6}$$

where $L_{\tilde{x}}, L_{\tilde{y}}$, and L_z are the lengths of the edges of parallelepiped in the $\tilde{x},\tilde{y}$,and z directions, respectively.

Equation (5.1) can be integrated numerically using several integration points inside the rectangular parallelepiped. Moreover, Eq. (5.2) can be evaluated as follows:

$$G=\frac{\overline{G}}{\int_{L_c}\alpha dC}\cong\frac{\overline{G}}{L_{\tilde{y}}/2} \tag{7}$$

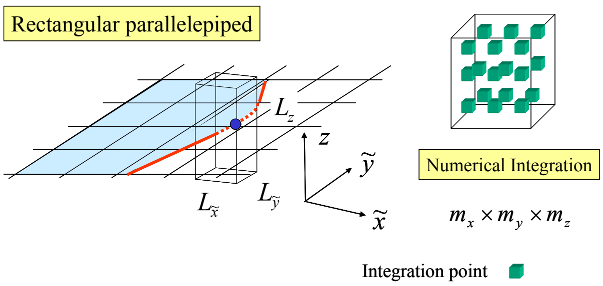


Fig. 3 Volume for the domain integral used to evaluate the energy release rate in 3D X-FEM analysis.

3. Development system

3.1 X-FEM Solver

A structural analysis program based on the X-FEM and denoted as XSOLID has been developed. XSOLID is outlined in **Table 1**. We can easily perform stress analysis of composite laminate with a delamination using XSOLID. The procedure used to perform X-FEM analysis in the present study is shown in **Fig. 4**. First, a three-dimensional finite element model for a composite laminate that has no delaminations is generated using mesh generation software. In addition, the geometry of a planar delamination in the laminate is modeled by defining lines, which can be obtained by connecting the points on the front tip of the delamination. Second, the enrichment type for each node near the delamination is determined automatically using development software. Consequently, the finite element model data for X-FEM analysis is prepared. The elastostatic X-FEM analysis is performed, and the energy release rate at the delamination front is computed. Moreover, the crack extension is evaluated for each point on the delamination front using the crack extension law, and the geometry of the delamination front is updated. This procedure provides the delamination propagation analysis.

Table 1 Outline of the development code XSOLID.

Code name	XSOLID
Development Language	C
Discretization method	Extended Finite Element Method
Analysis type	Elastostatic, Natural frequency, Buckling
Element type	8-node hexahedral element
Method to solve system equation	Direct method: ●Skyline method (in-house) ●PARDISO (Intel Math Kernel Library) Iterative method: Conjugate Gradient Method
Eigenvalue analysis scheme	Subspace method

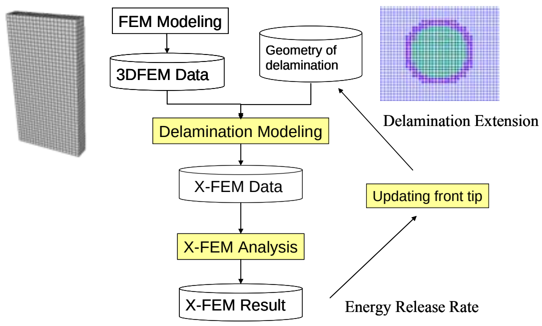


Fig. 4 Procedures for the stress analysis of a delaminated composite laminate by X-FEM.

3.2 Crack propagation analysis system

In order to perform the procedure described in the previous section, the control program CRACK3D, which controls the XSOLID code, has been developed (see **Fig. 5**). In the development system, both the mesh data file denoted as “ORG” and the control data file denoted as “CNT” are required. The “ORG” data file contains the finite element mesh information without the crack geometry, and the “CNT” data file describes the crack geometry and analysis conditions. The user can perform the stress analysis and the crack propagation analysis easily by preparing “ORG” and “CNT” data files. The control program CRACK3D executes the XSOLID code as another process and obtains the value of the energy release rate at the delamination front from the log file denoted as “LOG”, which is output file from the XSOLID code. Using this system, we can perform delamination propagation analysis more efficiently than with the conventional FEM using mesh generation software.

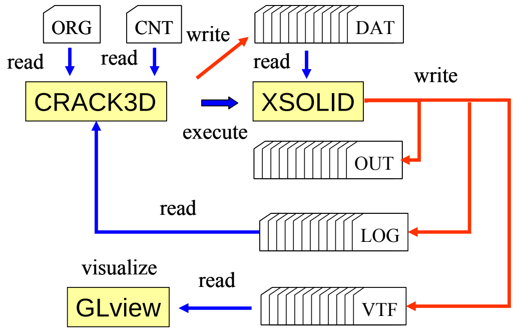


Fig. 5 Development system for delamination propagation simulation using X-FEM.

4 Numerical Examples

As a numerical example, elastostatic and fatigue delamination propagation analyses of a DCB specimen, as shown in **Fig. 6**, were performed. The DCB specimen analyzed herein has a length of L = 150 mm, a width of B = 25 mm, a thickness of H = 3 mm, and an initial delamination. The employed finite element mesh, which is shown in **Fig. 7**, has 150 divisions in the length direction, 25 divisions in the width direction, and eight divisions in the thickness direction. The fine mesh division was used near the zone, where the delamination front exists. The material of the DCB specimen is assumed to be fiber-reinforced, where the elastic properties of the unidirectional ply of the fiber reinforced material are E_L = 142 GPa, E_T = 10.8 GPa, G_{LT} = 5.49 GPa, G_{TT} = 3.71 GPa, ν_{LT} = 0.3, and ν_{TT} = 0.45. In the present calculation, an eight-node hexahedral isoparametric linear element was used. The 6th-order Gauss integration is adopted for the evaluation of the local stiffness containing any enriched elements.

4.1 Elastostatic analysis

The DCB test [13] [14], as shown in **Fig. 6**, is the standard experiment used to evaluate the mode-I interlaminar toughness of carbon fiber-reinforced plastic composite materials. In the DCB test, the delamination provided as the starter crack is extended by displacing the opposite sides of the two beams, and the relationship between the load and the opening displacement between the two beams is measured. The energy release rate is evaluated using the domain integral method as post-processing of numerical stress analysis by X-FEM and FEM. The 3 mm x 3 mm x 2.25 mm parallelepiped region with 50 x 50 x 50 equally distributed numerical integration points are used for domain integration. In the present analysis, delamination with either a linear or curved front tip, as shown in **Fig. 8**, is modeled. The curved front tip is assumed to be a parabolic curve. The distribution of the energy release rate at the linear front tip is shown in **Fig. 9**, as compared with that obtained by the conventional FEM and the reference solution provided by the beam theory. Comparison with the reference value reveals that the evaluated distribution of the energy release rate at the front tip gives appropriate results. Next, the analysis of the delamination with the curved front tip was performed. The conventional FEM analysis using the double node was also performed for comparison. The distribution of the energy release rate was evaluated as shown in **Fig. 10**. The distribution of the energy release rate for the curved front is somewhat flatter than that for the linear front. Although the results obtained by the X-FEM and the conventional FEM differ slightly, they are similar.

4.2 Fatigue delamination propagation analysis

The fatigue crack propagation law for the delamination front in the composite laminate is assumed as follows:

$$da / dN = C(\Delta G)^m \tag{8}$$

where a is the crack extension and N is the load cycle, ΔG is the range of the energy release rate, and C and m are empirical parameters, which are constant and depend on the material and the environment.

The delamination propagation analysis for the DCB specimen is performed using the development system. The parameters C and m for the crack propagation law in Eq. (8) are set as C = 28100 and m = 8.8, respectively [15]. The elastostatic analysis using the X-FEM was performed several times in succession. The calculations are performed for various increments of the load cycle ΔN. The relationship between the load cycle and the maximum extension of the delamination are shown in **Fig. 11**. Both the initial geometry and the final geometry of the delamination fronts in the composite laminate in case of ΔN=1000 are depicted in **Fig. 12**. It was shown that the developed system using the X-FEM can adequately solve the crack propagation analysis.

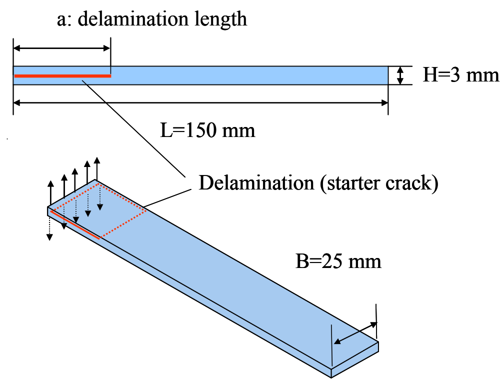


Fig.6 DCB (Double Cantilever Beam) test

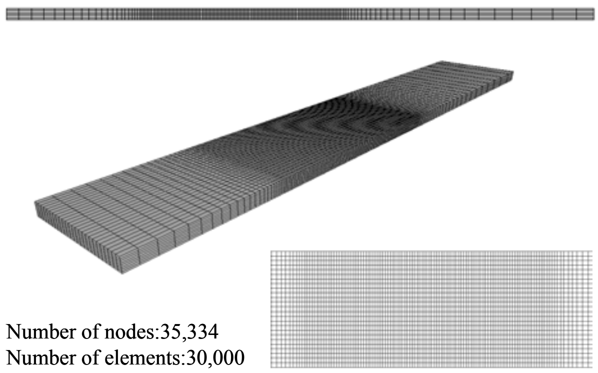


Fig.7 Finite element meshes utilized in X-FEM analyses.

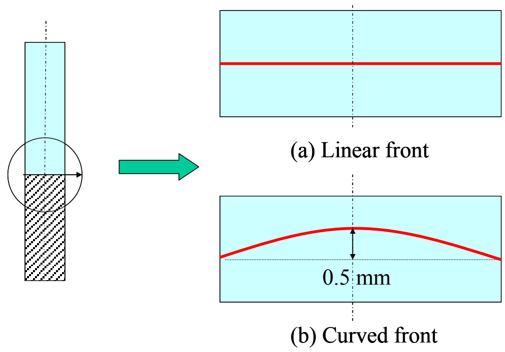


Fig.8 Delamination front tip with various geometry.

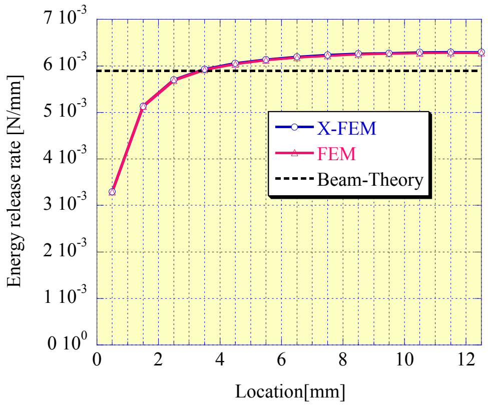


Fig.9 Distribution of energy release rate at the linear front tip of DCB test specimen.

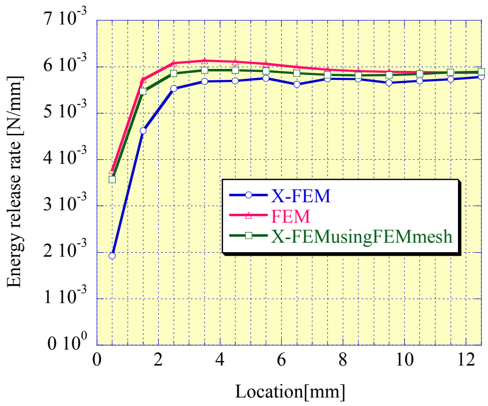


Fig. 10 Distribution of energy release rate at the curved front tip of DCB test specimen.

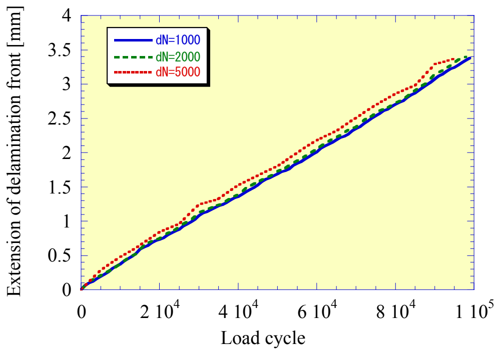
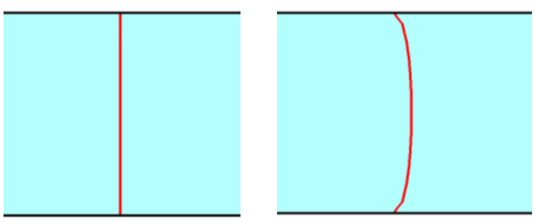


Fig.11 Extension of delamination front vs. applied load cycle.



(a) (b)

Fig. 12 Geometry of delamination front in the propagation simulation (a) initial state (b) final state (ΔN=1000).

5 Concluding remarks

The present paper described the numerical procedure based on the X-FEM in order to perform the delamination propagation analysis of composite laminates and presented numerical results for the DCB specimen with interlaminar planar delamination. In the future, the method proposed herein will be applied to composite laminates with more realistic delaminations and cracks. Moreover, the near-tip functions for orthotropic materials should be improved because the near-tip functions used in the present analysis cannot reconstruct the asymptotic displacement solutions of a crack in an orthotropic material.

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