ON IMPROPER FOUNDATION MODELS FOR THE DCB SPECIMEN

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Keywords: *Beam, DCB, Delamination, Double cantilever beam, Foundation, Mode I*

Abstract

This paper critically reviews available models for the DCB specimen. One of the most popular models incorporates a two-parameter Pasternak spring foundation. This paper argues that the Pasternak foundation model is physically incorrect for modelling the DCB specimen as it accounts for the shear stiffness ahead of the crack tip twice and thereby provides an overly stiff model. Numerical examples are presented to support the arguments. The paper advocates use of Timoshenko beams on Winkler tensile spring foundations or use of energy approaches to incorporate the crack tip compliance.

1 Introduction

The Double Cantilever Beam (DCB) specimen is the most popular specimen for testing fracture toughness of adhesives and interlaminar toughness of composites. Numerous structural models have been published to facilitate DCB specimen design and data evaluation. A comparison of beam, plate and 3D solutions for the DCB specimen can be found in [1].

Complete 3D or 2D elasticity models are obviously the most accurate although such models are impractical for test evaluation. 3D models are the only models able to cope with free edge effects, but such effects are confined to a very small region close to the free edge and do not significantly affect the crack growth in the wide specimens commonly used for composites [2]. 2D elasticity solutions are limited by the assumption of either plane stress conditions (specimens of negligible width) or plane strain conditions (infinitely wide specimens) but cannot capture the anticlastic (saddle-shaped) deformation associated with plate bending.

Thus, when a bending moment is applied to an unconstrained plate the coupling via Poisson's ratio also causes a curvature perpendicular to the applied moment. This contributes to a width-wise variation of the strain energy release rate which is far greater than the variation associated with the transition from plane stress to plane strain as illustrated by the comparison of a DCB specimen with a middle cracked (MC) specimen of equal width [2].

Beam models are the most common and can easily be generalised to wide plates for specimens with a small crack length to width. Beam models include the Euler-Bernoulli beam, with infinite shear stiffness, and the Timoshenko beam, which also includes shear compliance. Improved beam models of the DCB specimen usually involve a beam on an elastic foundation, Fig. 1. The most commonly used foundation models are the Winkler foundation, which only includes an out-of-plane normal stiffness, k_w, and the two-parameter Pasternak foundation, which also includes shear stiffness, k_s.

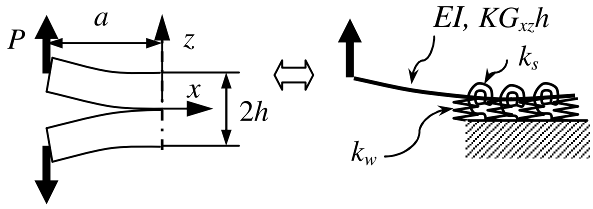


Fig. 1. Elementary model of the DCB specimen.

2 Overview of models for the DCB specimen

3D solutions for the DCB specimen have only been obtained by use of FE models, e.g. [2]. A 2D plane strain elasticity solution for the stress intensity factor of an isotropic DCB specimen was derived by Chang et al [3] but explicit compliance expressions were not provided.

The important effect of anticlastic curvature was studied by Davidson and Shapery [4], who used a Raleigh-Ritz plate model to study the transition from a uniaxial strain solution at relatively small crack length-to-width ratios to a uniaxial stress solution at relatively large ratios. The former case is well described by cylindrical bending of a plate, while the latter case is described by a beam solution. The difference between these asymptotic cases is small for unidirectional laminates, moderate for isotropic specimens and large for ±45° laminates.

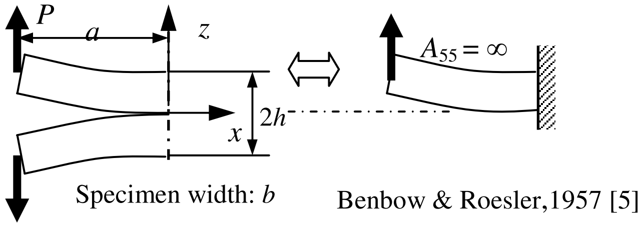
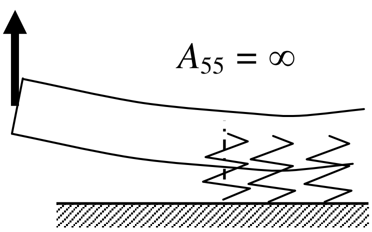


Fig. 2. Elementary model of the DCB specimen.

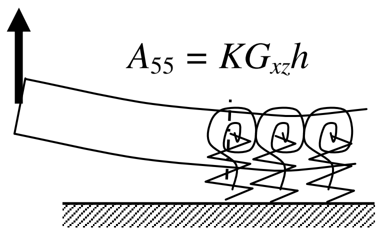
An elementary model of the DCB specimen was suggested by Benbow & Roesler [5] who prevented rotation at the loaded end, effectively doubling the crack length and displacement of the modern DCB specimen. They modelled each member as a rigidly clamped Euler-Bernoulli beam, which has infinite shear stiffness ($A_{55}= \infty$), Fig. 2. This model neglects shear deformations in the cracked part of the specimen and the compliance of the uncracked part. These effects are particularly important in fibre composites, which have low shear stiffness and tensile stiffness transverse to the fibres.

The compliance of the uncracked part of the specimen is reflected in a non-zero slope at the crack tip, and has been demonstrated experimentally by use of optical methods [6] although the suggested structural model was questionable. A discussion on the experimental observations in [6] and their agreement with the suggested model as well as with various other DCB models was provided in [7].

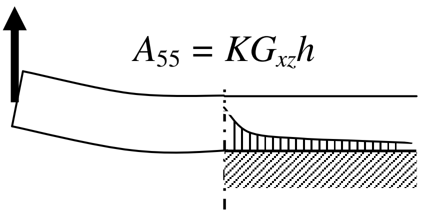
Kanninen [8] originally modelled the specimen as an isotropic Euler-Bernoulli beam on a Winkler foundation, which models the uncracked part as a tensile spring foundation, Fig. 3a. An equivalent model for orthotropic specimens was provided in [9]. More accurate models have allowed for shear deformations by replacing the Euler-Bernoulli beam with a Timoshenko beam, which has finite shear stiffness ($A_{55}=KG_{xz}h<\infty$). Here $K\approx5/6$ is the shear factor of the section. In a subsequent paper Kanninen [10] modelled the specimen as an isotropic Timoshenko beam on a Pasternak foundation, where the uncracked part is a combined tensile/shear spring foundation, Fig. 3b. This model was later extended to orthotropic specimens by Williams [11]. This model has been used extensively in standards development and recently by Szekrényes and Uj [12] to model mixed mode specimens.



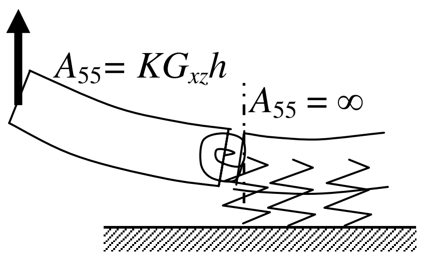
a) Kanninen, 1973 [8]
Ozdil & Carlsson, 1999 [9]



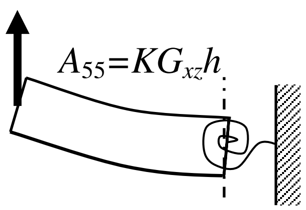
b) Kanninen, 1974 [10]
Williams, 1989 [11]



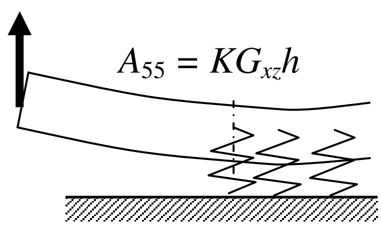
c) Whitney, 1985 [13]



d) Olsson, 1992 [1]



e) Balendran, 1994 [14]



f) Kondo, 1995 [15]

Fig. 3. Overview of improved DCB models

Whitney [13] used higher order plate theory to obtain a solution by an assumed normal stress distribution in the uncracked part of the specimen, Fig. 3c. The model assumed cylindrical bending (plane strain) but can easily be compared to beam solutions by setting ν_{xy} in the plate stiffness to zero. Similarly beam models can easily be applied to the case of cylindrical bending of a wide plate by replacing E_xI by D_{11}.

Olsson [1] used a Timoshenko beam for the cracked part but modelled the uncracked part as an Euler-Bernoulli beam on a Winkler foundation (k_s=0) and obtained the additional shear rotation by an energy approach combined with Saint Venant stress decay rates, Fig. 3d. Balendran [14] found the rotation at the crack tip by use of Reissner's mixed variational approach, but did not allow for transverse normal stresses and the associated displacement, Fig. 3e. Kondo [15] used a more straight forward approach and modelled the entire specimen as a Timoshenko beam on a Winkler foundation, Fig. 3f. An identical model for isotropic specimens was recently presented by Shahani and Forqani [16] as a part of a more general dynamic solution.

3 Theoretical discussion

It is essential to realise that the beam-foundation model is actually an abstraction, where the foundation (with out-of-plane normal stiffness k_w and shear stiffness k_s) and beam represent different aspects of the *same* piece of material, Fig.1. The beam represents the mid-plane of the upper specimen half, and the ability of this mid-plane to deform in shearing and bending. The foundation represents the material *between* this mid-plane and the specimen symmetry plane, and the resistance of the mid-plane to translate.

An Euler-Bernoulli beam has infinite shear stiffness ($G_{rz}=\infty$) and thus a Timoshenko beam is clearly necessary to incorporate through-thickness shear deformations in beam models. A Timoshenko beam has, however, infinite stiffness in through-thickness tension/compression. The compliance due to out-of-plane tension/compression may be incorporated by a Winkler spring foundation. The use of a Pasternak foundation (k_s>0) introduces an additional shear stiffness of the material ahead of the crack front. In general improved beam models result in the following type of expressions for the specimen compliance [1,5-12]:

$$w/P = C_0\left(\bar{a}^3 + a_2\bar{a}^2 + a_1\bar{a} + a_0\right) \qquad (1)$$

where $\bar{a} = a/h$

The cubic term corresponds to the elementary DCB model with a rigidly clamped Euler-Bernoulli beam. The square term accounts for crack tip rotation and includes contributions from both normal and shear compliance ahead of the crack.

Crack length corrections $\bar{\Delta}$ are frequently used in the elementary DCB model to approximately account for the increased compliance caused by shear deformation and crack tip rotation:

$$\begin{aligned} w/P &\approx C_0\left(\bar{a} + \bar{\Delta}\right)^3 \\ &= C_0\left(\bar{a}^3 + 3\bar{\Delta}\bar{a}^2 + 3\bar{\Delta}^2\bar{a} + \bar{\Delta}^3\right) \end{aligned} \qquad (2)$$

It is evident that the three constants a_2, a_1 and a_0 in Eq. 1 cannot be correctly fitted by the single correction term $\bar{\Delta}$ in Eq. 2, although the approximation $\bar{\Delta}$ = $a_2/3$ usually only results in negligible errors. A comparison of theoretical crack length correction terms for different models was provided in [17]. For evaluation of experiments it is more reliable to use the empirical crack length correction method suggested by Hashemi et al. [18].

4 Comparison of models

Figure 4 gives a comparison of the compliance of a typical unidirectional carbon/epoxy specimen (cases 1-4 in Table 1) as obtained by a plane stress FE analysis in [11] and by the beam models of Williams [11], Olsson [1] and Kondo [15]. In this comparison Williams is represented by the theoretically derived compliance, rather than by the compliance obtained using the FE based empirical correction suggested in [11].

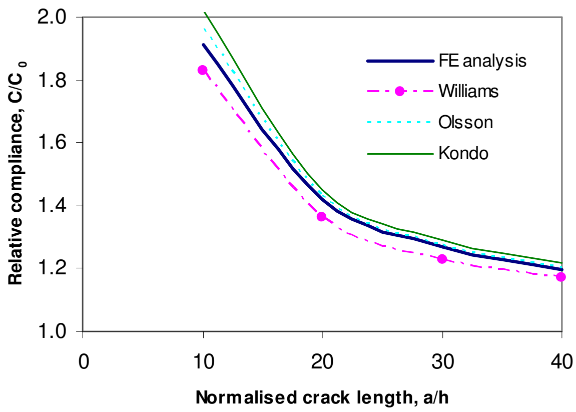


Fig. 4. Comparison between compliance for FE analysis and some improved beam models

It is noted that the Pasternak foundation model adopted by Kanninen/Williams/Szekrényes persistently underestimates the specimen compliance. Further studies are required to clarify if the small discrepancy between the FE analysis and the other models is due to insufficient mesh refinement or due to limitations of the beam models.

Table 1 gives a comparison of the compliance of the different models for a number of test cases. This table includes the compliance of a plane stress FE analysis [11] normalised by the rigidly clamped Euler-Bernoulli beam model, and the deviation of each model with respect to the FE results. To allow a comparison with the beam models the Poisson's ratio ν_{xy} was set to zero in the plate model by Whitney [13].

The models by Olsson [1], Whitney [13] and Kondo [15] are generally more compliant than the FE solution and provide small errors for most cases, although the model by Olsson [1] is less reliable for some extreme cases. The deviation from the exact solution is likely to be even smaller since discrete models like FEM tend to be stiffer than the actual continuous structure. The models by Ozdil & Carlsson [9], Kanninen [10]/Williams [11] and Balendran [14] are persistently stiffer than the FE solution, which in itself is slightly stiffer than the exact solution.

The model by Ozdil & Carlsson [9], which is based on the Euler-Bernoulli beam severely underestimates the specimen compliance. This highlights the importance of including transverse shear deformation, particularly for short beams and materials with low shear modulus.

The deviation of the solution by Balendran is explained by the neglect of the transverse compliance at the beam root. Not surprisingly the model by Balendran is relatively inaccurate when the transverse modulus is small (Case 11) but it is worth noting that it is extremely accurate when the modulus is large (Case 12). It is likely that an extension of the approach by Balendran to include transverse normal stresses and displacements in the uncracked specimen part would yield a highly accurate solution.

The overestimated stiffness of the models based on a Pasternak foundation is explained by the fact that the shear stiffness of the material ahead of the crack tip is accounted for twice, i.e. both in the beam model and in the elastic foundation. Williams [11] overcome this deficiency by an empirical correction factor based on FE results, which effectively reduced the shear stiffness in the model. A similar overestimation of the stiffness was observed in [12] and in [19]. Improved agreement with experimental data when replacing the Pasternak

Table 1. Compliance of FE model compared with elementary DCB model and various improved models

Case	E_x [GPa]	E_z [GPa]	G_{xz} [GPa]	a/h	FE C_{FE}/C_0 *	Ozdil & Carlsson	Williams	Balendran	Olsson	Whitney	Kondo
						[% deviation from FE results]					
1	147	7.81	2.76	40	1.197	-7.8	-2.3	-1.2	0.3	0.9	1.4
2	147	7.81	2.76	30	1.268	-10.4	-3.2	-1.5	0.4	1.2	1.8
3	147	7.81	2.76	20	1.418	-14.4	-3.9	-2.2	0.7	1.7	2.5
4	147	7.81	2.76	10	1.914	-23.9	-4.4	-3.2	2.6	4.1	5.5
5	73.5	7.81	2.76	20	1.295	-9.1	-3.0	-2.3	2.0	1.7	2.4
6	294	7.81	2.76	20	1.603	-21.6	-4.9	-2.1	1.2	1.8	2.6
7	147	7.81	0.69	20	1.860	-34.8	-6.1	-0.4	7.4	1.5	1.9
8	147	7.81	1.38	20	1.590	-23.7	-5.0	-1.3	3.2	1.5	2.1
9	147	7.81	5.25	20	1.309	-7.3	-3.1	-2.8	3.9	2.6	3.6
10	147	7.81	10.50	20	1.241	-2.2	-2.9	-4.3	5.8	3.1	4.1
11	147	0.78	2.76	20	1.499	-6.6	-4.6	-7.5	7.7	3.8	5.5
12	147	1000	2.76	20	1.380	-23.1	-4.1	0.4	7.6	0.8	0.9
13	147	147	56.5	40	1.049	0.0	-0.6	1.0	1.7	2.1	1.3
14	147	147	56.5	20	1.100	-0.1	-1.1	-2.0	3.3	4.1	2.5
15	147	147	56.5	10	1.208	-0.3	-2.0	-3.9	6.3	7.8	4.8

*) $C_0=8(a/h)^3/(E_x b)$, Compliance of elementary DCB model with two rigidly clamped Euler-Bernoulli beams of width b

foundation by a Winkler foundation was noted already by Gehlen et al. [19]. They argued that the foundation shear stiffness should vanish as shear stresses vanish on the symmetry plane of the specimen. This argument is not entirely relevant as the foundation model assumes constant stress through the foundation thickness, which should equal the *average* stress over the foundation thickness (h/2). The average shear stress in the beam and foundation are equal and non-zero although the shear stress is zero on both lateral surfaces.

The results in Table 1 clearly demonstrate the necessity to include the rotation and translation at the crack tip caused by shear deformations and through-thickness tension in the uncracked part of DCB specimens. This may be done by energy approaches or by combination of a Timoshenko beam with a Winkler foundation, which lacks shear stiffness. The additional shear stiffness of Pasternak foundations produces an overly stiff model by accounting for the shear stiffness twice. In most cases this only causes moderate errors, but the use of a Pasternak foundation is nevertheless physically incorrect. More appropriate models have been available for a long time but the choice of foundation model still appears to cause confusion in the composites community.

The strain energy release rate G is obtained by differentiating the compliance with respect to the crack length. By considering Eq. 1 it is obvious that any deficiencies in the DCB model will have a smaller effect on the strain energy release rate than on the compliance than. It should, however, be noted that values of G based on simplified models will be unconservative, i.e. too low.

It is evident that beam models are unable to capture phenomena such as finite width effects or crack front curvature, which require plate models or 3D models. However, there is still scope for further work on incorporating phenomena such as z-pins, fibre bridging and material or geometrical nonlinearity. This work should be based on the most accurate models currently available.

5 Conclusions

It has been demonstrated that the use of a Pasternak foundation, which exhibits both tensile and shear stiffness, produces an overly stiff and physically incorrect model of the DCB specimen, as it accounts for the shear stiffness of the uncracked part twice. DCB models based on shear deformable beams on a Winkler tensile foundation all provide acceptable results and should be the basis for future model development. Interesting alternative approaches include an extension of Reissner's mixed variational approach and the higher order plate theory accounting for through-thickness extension.

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