OPTIMUM STRUCTURAL DESIGN OF COMPOSITE XYLOPHONE BARS

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Abstract

In this study the fiberglass/vinylester composite material was used to replace the rosewood as a new material for xylophone bars. The optimization technique combined with the finite element analysis was used to calculate the natural frequencies of the first three bending modes of transverse motion for designing composite xylophone bars. The optimization techniques used were the subproblem approximation method and the first order method respectively. After comparing the computational efficiency and accuracy of the two optimization methods, the subproblem approximation method is a more efficient method, while the first order method is a more accurate method.

1 Introduction

Composite materials have advantages of high specific stiffness and high specific strength and are suitable to replace the traditional materials. The xylophone is a kind of percussion instruments struck by a mallet on the bars to vibrate. The rosewood, a common material used for xylophone bars, becomes rare recently and the composite material is a good substitute for xylophone bars. Bork [1] investigated the effect of the position of undercut on the natural frequencies of xylophone bars; he provided a method for tuning three partials in order to improve the tonal quality of the xylophone. Orduna-Bustamante [2] addressed the problem of optimal undercut and found that the frequencies of modes 2 and 3 of transverse motion were harmonically related with that of the fundamental one (mode 1). Brancheriau et al. [3] studied the frequency shift phenomenon in acoustic resonance of xylophone bars during their tuning operation using the finite element method and by experiments. Petrolito and Legge [4] discussed the application of numerical optimization techniques to xylophone bars. Yeh et al. [5] studied the vibration analysis of composite xylophone bars.

In this study the fiberglass/vinylester composite material was used as a new material for xylophone bars to replace the rosewood. We compared the computational efficiency and accuracy between the subproblem approximation method and the first order method in order to decrease the time consumed in designing xylophone bars.

2 Finite Element Optimization Analysis

The natural frequencies of xylophone bars are often calculated by the modal analysis. The optimization technique is needed to obtain the optimal undercut of xylophone bars. In this study the modal analysis and two kinds of optimization method were used to design the undercut for xylophone bars.

2.1 Modal Analysis

In order to discuss the tone of xylophone bars, the modal analysis [6] was used to calculate the first three natural frequencies of bending modes for designing composite xylophone bars. Neglecting the damping effect and the external force, the governing equation can be written as

$$[M]\{\ddot{Q}\}+[K]\{Q\}=0 \tag{1}$$

where $[M]$ is the global mass matrix, $[K]$ is the global stiffness matrix, and $\{Q\}$ is the global nodal displacement vector. In the linear system for free vibration, the nodal displacement vector of system can be expressed by the harmonic form as

$$\{Q\}=\{\phi\}_i \cos\omega_i t \tag{2}$$

where $\{\phi\}_i$ is the vector of i^{th} mode shape, ω_i is the i^{th} natural frequency and t is time. Substituting the modal vector of the system, Eq. 1 becomes

$$\left(-\omega_i^2[M]+[K]\right)\{\phi\}_i=\{0\} \qquad (3)$$

For nontrivial solutions, the determinant of $\left(-\omega_i^2[M]+[K]\right)$ is set to zero.

$$\left|\,[K]-\omega_i^2[M]\right|=0 \qquad (4)$$

From Eq. 4, the eigenvalue ω_i, corresponding to the ith natural frequency of the system, and the eigenvector $\{\phi\}_i$, corresponding to the ith mode shape, can be obtained.

2.2 Optimization Methods

The optimization technique combined with the finite element analysis was used to calculate the natural frequencies of the first three bending modes for designing composite xylophone bars. The optimization techniques used are the subproblem approximation method and the first order method. The following paragraph will describe the optimization problem and optimization methods [6].

2.2.1 The Optimization Problem

The optimization analysis is a method to find out the best design in the problem. The independent variables in an optimization analysis are the design variables. The vector of design variables is denoted by

$$X=[x_1,x_2...,x_n] \qquad (5)$$

The design variables lie between the upper and lower limits, expressed by upper and lower bars.

$$\underline{x}_i \le x_i \le \overline{x}_i \quad ; \quad i=1,2,...,n \qquad (6)$$

where n is the number of design variables.

A constrained minimization problem is established to minimize the objective function, as shown in Eq. 7, with the side constraints shown from Eq. 8 to Eq. 11.

$$\text{Minimize} \qquad V=v(X) \qquad (7)$$

Subject to

$$\underline{x}_i \le x_i \le \overline{x}_i \quad ; \quad i=1,2,...,n \qquad (8)$$

$$g_j(X) \le \overline{g}_j+\alpha_j \quad ; \quad j=1,2,...,m_1 \qquad (9)$$

$$h_k(X) \ge \underline{h}_k-\beta_k \quad ; \quad k=1,2,...,m_2 \qquad (10)$$

$$\underline{w}_r-\gamma_r \le w_r(X) \le \overline{w}_r+\gamma_r; \quad r=1,2,...,m_3 \qquad (11)$$

where V is the objective function, x_i are design variables, g_j, h_k and w_r are the state variables, with underbar and overbar representing the lower and upper bounds, m_1 m_2 and m_3 are the number of side constraints and α_j, β_k and γ_r are tolerances of state variables.

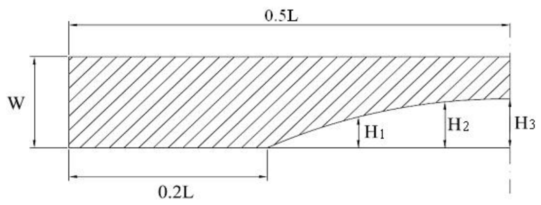
This study used the optimization techniques in the finite element analysis software ANSYS to design the optimal undercuts for xylophone bars. The xylophone bars are assumed homogeneous and have no defects inside, with all fibers arranging in a uni-direction. The material properties of fiberglass reinforced vinylester specimens were obtained from the tensile test and the results were used in the finite element analysis.

The procedure of optimization analysis first defines the objective function and design variables. Then the inequality constraints, the convergence criterion and the initial conditions of design variables were given to start the optimization technique. In the beginning, the program first used the modal analysis to obtain the solution from the initial model and checked whether the solutions are optimal results or not. If the solutions are not an optimal design, the program will do the iterations until the optimal design is found.

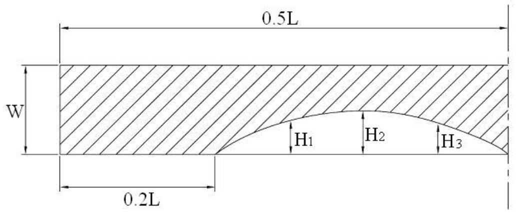
The objective function is to find out the minimum volume of xylophone bar, and inequality constraints are the frequencies of the first three bending modes (f_1, f_2 and f_3) and design variables (H_1, H_2 and H_3). Three design variables are the heights of the undercut, as shown as Fig. 1, in which types A and B represent the xylophone bar with one and two undercuts. In order to make sure the geometry of xylophone bars is symmetric, we used a spline curve to fit one half of the undercut, and the distance between each design point is the same in the horizontal direction. Table 1 shows the constraints of frequency bounds of the first three bending modes [5]. The optimal design is called a feasible design, when all the frequencies are in the feasible region. The mathematical model of composite xylophone bars is shown below

$$\begin{array}{lll} \text{Minimize} & \text{Volume} & \\ \text{Subject to} & 0 \le H_i \le \overline{H}_i & i=1,2,3 \\ & \underline{f}_j \le f_j \le \overline{f}_j & j=1,2,3 \end{array} \qquad (12)$$

where H_i are design variables, and f_j are state variables, with underbar and overbar representing the lower and upper bounds.



(a) Type A



(b) Type B

Fig. 1. Design variables of two types of xylophone bars

Table 1. Frequency bounds of the first three bending modes

	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
Type A	336.2±0.25	1267.1±5	2774.8±10
Type B	397.2±0.25	1199.0±5	2777.6±10

2.2.2 Subproblem Approximation Method

The subproblem approximation method [6] can be described as an advanced zero order method. This technique requires an objective function and state variables, and there is no need to calculate their derivatives.

The state variables are first replaced with approximations by means of least squares fitting. This method uses the penalty functions to convert from a constrained minimization problem to an unconstrained problem, and uses the sequential unconstrained minimization technique (SUMT) to solve the problem in each iteration. Finally, the optimal results can be obtained in several iterations.

The iterations in the subproblem approximation method continue until the convergence is achieved. The convergence criterion of this method is based on the tolerance of the objective function and design variables.

2.2.3 First Order Method

The first order method [6] uses the steepest decent method and the conjugate gradient method to find out the optimal search direction and gradient in design space. Each iteration is composed of sub-iterations that include the computations for the searching direction and gradient. One iteration in the first order method includes several analysis loops. This method also uses the penalty functions to combine the objective function and constraints, and the constrained optimization problem is converted to an unconstrained optimization problem with sub-iterations in each iteration.

This method searches the design space in each iteration, and it contracts the size of the searching space by the golden-section algorithm and the local quadratic fitting technique. The iterations in the first order method continue until the convergence is achieved. The convergence criterion of this method is based on the tolerance of the objective function.

3 Experiment

The experiments in this study included the fabrication, the mechanical properties testing, and the density measurement of glassfiber/vinylester composite materials. The experimental results were used in the finite element optimization analysis to optimize the undercut of composite xylophone bars.

3.1 Fabrication of Composite Specimens

The composite specimens used in this study were fabricated by the hot-press method. Fig. 2 shows the procedure about the fabrication of composite material specimens. The matrix used in this study is a thermosetting type vinylester, and the reinforcement is glass fiber. The suggested curing temperature of the vinylester is 140^{o}C, and the curing time is 30 min. The specimens were laminated with four prepreg layers and after hot-pressing a cutting machine was used to cut the specimen to the size desired.

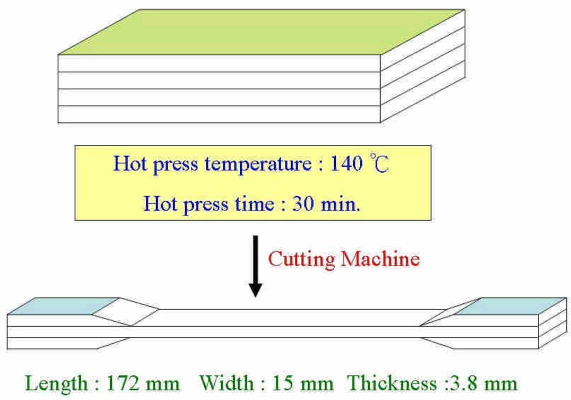


Fig. 2. Fabrication of glassfiber/vinylester composite specimens

3.2 Mechanical Properties Testing

The glassfiber/vinylester composite specimens were tested on a tensile test machine to measure the mechanical properties according to ASTM 3039-76 [7] standard and ASTM D3518-76 [8] standard. The tensile strength, Young's modulus, Poisson's ratio, and shear modulus were obtained. The tensile test system includes a tensile test machine, strain amplifiers, an AD/DA card and a computer.

In the principle of mechanics of composite materials [9], the uni-direction fiber reinforced polymer can be considered as a specially orthotropic material. There are five independent material properties E_{11}, E_{22}, G_{12}, ν_{12} and ν_{23} needed to be measured. The $[0^o]_4$ fiberglass/vinylester composite specimens can be used to measure E_{11} and ν_{12}. The $[90^o]_4$ specimens can be used to measure E_{22} and the $[\pm45^o]_s$ specimens can be used to measure G_{12}. For specially orthotropic material [9], E_{22} is equal to E_{33}, ν_{12} is equal to ν_{13}, and G_{12} is equal to G_{13}. G_{23} can be calculated from $E_{22}/2(1+\nu_{23})$. Since ν_{23} is not easy to measure, it can be assumed equal to ν_{12} according to the previous reports [10,11]. Yeh et al. [5] showed that the influence of the variation of ν_{23} is not obvious in vibration analysis of composite xylophone bars. The experimental results of mechanical properties of fiberglass/vinylester composites are listed in Table 2.

Table 2. Mechanical properties of fiberglass/vinylester composites

Young's Modulus (GPa)	Shear Modulus (GPa)	Poisson's Ratio
$E_{11}=49.133\ ^{+1.089}_{-0.948}$	$G_{12}=6.836\ ^{+0.621}_{-0.628}$	$\nu_{12}=0.256\ ^{+0.028}_{-0.014}$
$E_{22}=14.240\ ^{+2.760}_{-1.866}$	$G_{13}=6.836\ ^{+0.621}_{-0.628}$	$\nu_{13}=0.256\ ^{+0.028}_{-0.014}$
$E_{33}=14.240\ ^{+2.760}_{-1.866}$	$G_{23}=5.672\ ^{+1.166}_{-0.691}$	$\nu_{23}=0.256\ ^{+0.028}_{-0.014}$

3.3 Density Measurement of Composite Materials

The density measurement of composite materials was performed according to ASTM D792-00 [12] standard test method for density. First, a scale was used to measure the weight of specimens in air and in water respectively. The difference divided by the water density was calculated to find the volume of specimen. The density was found by dividing the weight of specimen in air over its volume. The average density of fiberglass/vinylester composite specimens was found to be 1963.2 (kg/m^3).

4 Results and discussion

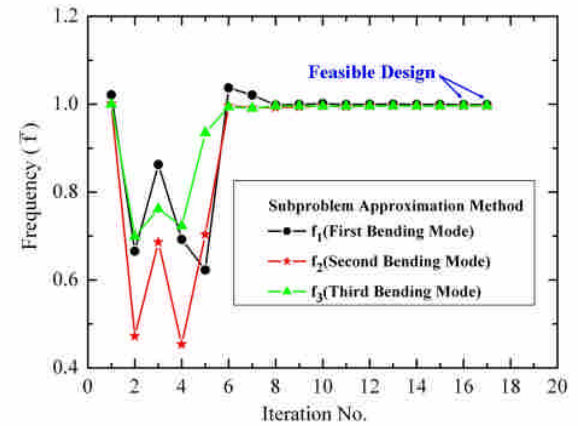
After substituting the experimental mechanical properties in the optimization analysis, the best design of composite xylophone bars can be found. The results for two types of xylophone bars by two kinds of optimization methods are described in the following paragraphs.

4.1 Type A Xylophone Bar

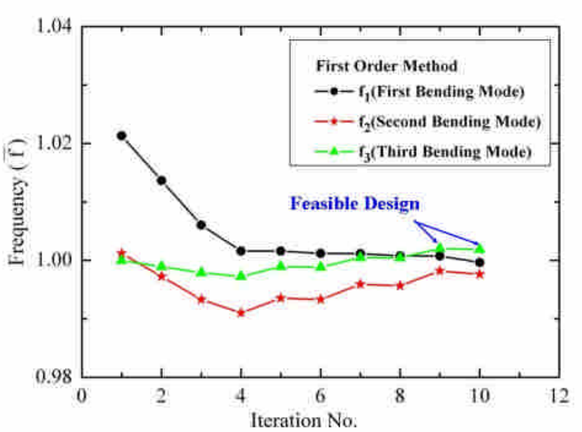
The optimization results of the subproblem approximation method for Type A xylophone bar are shown in Fig. 3(a). The frequencies of the first three bending modes reached the feasible design after 16 iterations. The feasible design means that the results satisfy all the constraints within the tolerances. The optimization results of the first order method for Type A xylophone bar are shown in Fig. 3(b). $\bar{f}$ is the natural frequency normalized by the corresponding natural frequency.

The frequencies of the first three bending modes reached the feasible design after 9 iterations. According to the finite element analysis, the computational time of the first order method is 5.14 times that of the subproblem approximation method, because each iteration in the first order method includes several sub-iterations in calculating the searching direction and gradient. Therefore the subproblem approximation method is more efficient than the first order method.

Table 3 shows the comparison between the results of the two optimization methods and objective frequencies for Type A xylophone bar. The errors from using the subproblem approximation method are no more than 0.395%, and the errors from using the first order method are no more than 0.237%. After comparing the results obtained from the two optimization methods, it is concluded that the subproblem approximation method is more efficient, while the first order method is a more accurate method for Type A xylophone bar.



(a) Subproblem approximation method



(b) First order method

Fig. 3. The optimization results of Type A xylophone bars

Table 3. Comparison of results between simulations and objective frequencies (Type A)

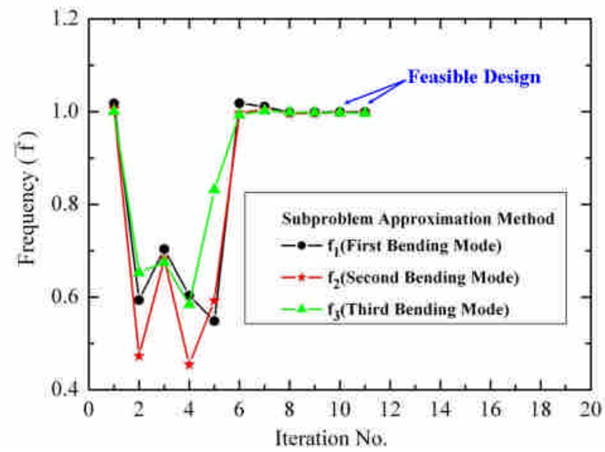
Type A	Objective (Hz)	SAM* (Hz)	FOM** (Hz)
f_1	336.2	336.0 (-0.059%)	336.1 (-0.030%)
f_2	1267.1	1262.1 (-0.395%)	1264.1 (-0.237%)
f_3	2774.8	2764.8 (-0.360%)	2780.0 (0.187%)

* Subproblem Approximation Method
** First Order Method

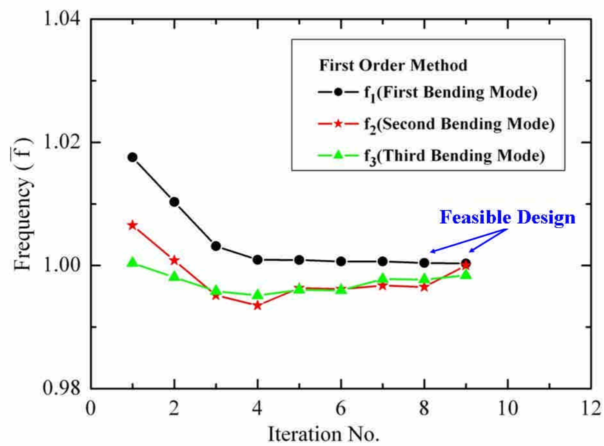
4.2 Type B Xylophone Bar

The optimization results of the subproblem approximation method for Type B xylophone bar are shown in Fig. 4(a). The frequencies of first three bending modes reached the feasible design after 10 iterations. The optimization results of the first order method for Type B xylophone bar are shown in Fig. 4(b). $\bar{f}$ is the natural frequency normalized by the corresponding objective frequency. The frequencies of first three bending modes reached feasible design after 8 iterations. Since Each iteration in the first order method includes several sub-iterations in calculating the searching direction and gradient, the computational time by the first order method is 6.42 times that of the subproblem approximation method in the finite element analysis. Therefore the subproblem approximation method is more efficient.

Table 4 shows the comparison between the results of the two optimization methods and objective frequencies for Type B xylophone bar. The errors from using the subproblem approximation method are no more than 0.338%, and the error of results by using the first order method are no more than 0.162%. After comparing the results of the two optimization methods, it is also concluded that the subproblem approximation method is more efficient, while the first order method is a more accurate method.



(a) Subproblem approximation method



(b) First order method

Fig. 4. The optimization results of Type B xylophone bars

Table 4. Comparison of results between simulations and objective frequencies (Type B)

Type B	Objective (Hz)	SAM* (Hz)	FOM** (Hz)
f_1	397.2	397.0 (-0.050%)	397.3 (0.025%)
f_2	1199.0	1196.5 (-0.209%)	1199.0 (0%)
f_3	2777.6	2768.2 (-0.338%)	2773.1 (-0.162%)

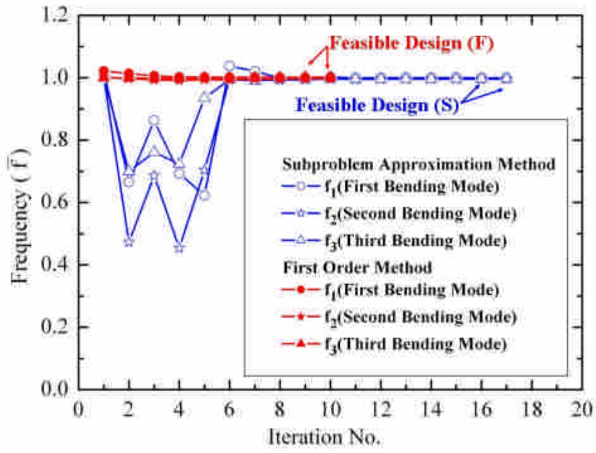
* Subproblem Approximation Method
** First Order Method

4.3 Comparison of Results Between Two Optimization Methods

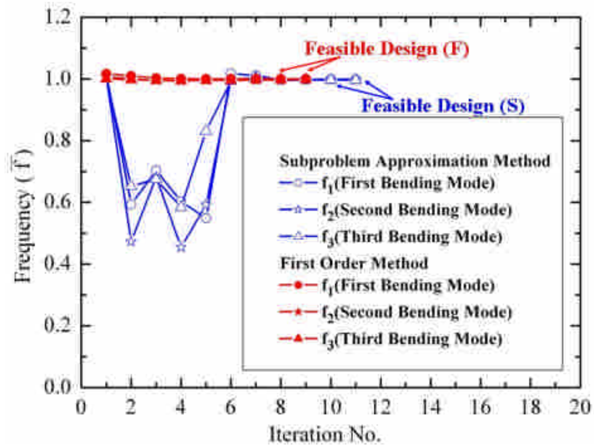
Fig. 5 shows the comparisons between two optimization methods in Type A and Type B xylophone bars. The results of the first order method vary very little from a few iterations. The results of the subproblem approximation method converge after 6 iterations. Although the first order method reach the feasible design in less iterations than those of the subproblem approximation method, the computational time of the first order method is 5.14 and 6.42 times that of the subproblem approximation method in the finite element analysis as described in the previous sections. The subproblem approximation method is more efficient than the first order method.

Fig. 6 shows the optimal shapes of Type A and Type B xylophone bars by the subproblem approximation method. Type A xylophone bar has a undercut of single curve and Type B xylophone bar has a undercut of double curves. The optimal shapes of Type A and Type B xylophone bars obtained from the first order method are very similar to those obtained by the subproblem approximation method.

Fig. 7 and Fig. 8 show the first three bending modes of Type A and Type B xylophone bars. The blue color in the figure indicates the smallest displacement in the mode shape. Fig. 9 shows that there are two nodes in the fundamental mode located symmetrically at 0.205L from the end. The nodes can be used in design to make holes to connect the xylophone bar with xylophone frame by strings.

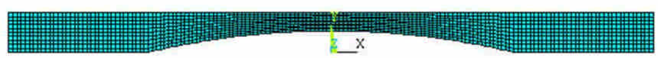


(a) Type A

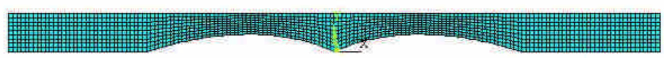


(b) Type B

Fig. 5. Comparisons of results between two kinds of optimization method

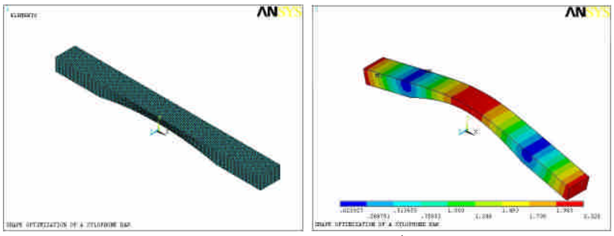


(a) Type A

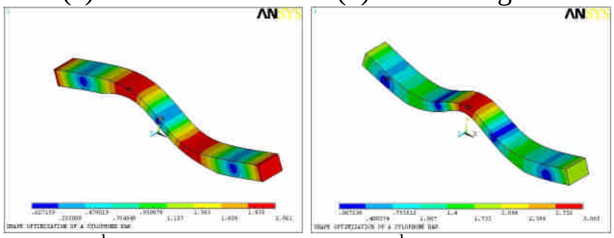


(b) Type B

Fig. 6. The optimal shapes of xylophone bars by Subproblem approximation method

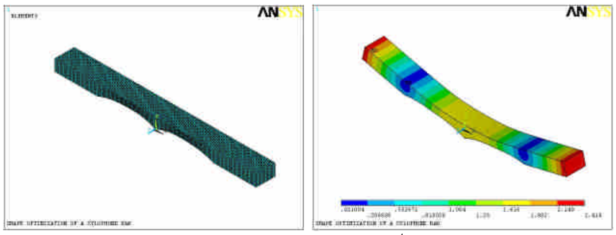


(a) FEA Model (b) 1st Bending Mode

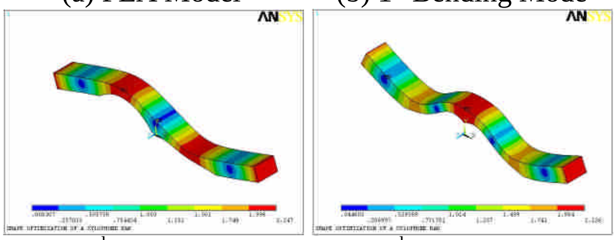


(c) 2nd Bending Mode (d) 3rd Bending Mode

Fig. 7. The mode shapes of Type A xylophone bar



(a) FEA Model (b) 1st Bending Mode



(c) 2nd Bending Mode (d) 3rd Bending Mode

Fig. 8. The mode shapes of Type B xylophone bar

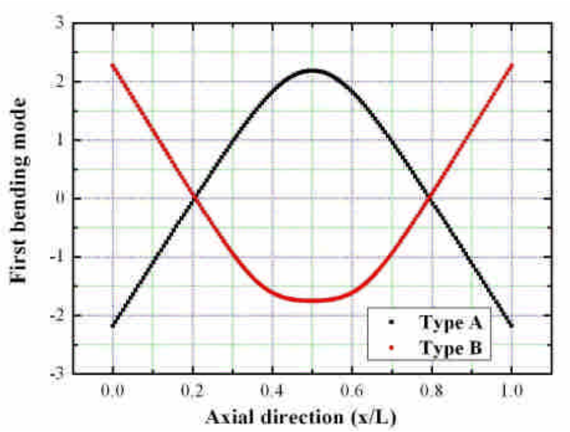


Fig. 9. Two nodes appear in the 1st bending mode of xylophone bar

5 Conclusions

According to the finite element analysis and experimental results, the following conclusions can be made for designing two types of xylophone bars.

(1). The fiberglass/vinylester composite materials can replace the rosewood as a new material for xylophone bars.

(2). In finite element optimization analysis, the iteration number and error of the first order method is less than those of the subproblem approximation method for both Type A or Type B xylophone bars. The first order method is more accurate than the subproblem approximation method in optimization analysis for both Type A and Type B xylophone bars.

(3). The computational time of the first order method is more than that of the subprobelm approximation method. The subproblem approximation method is more efficient than the first order method in the optimization analysis for both Type A and Type B xylophone bars.

(4). The finite element method combined with both optimization methods, the subproblem approximation method and the first order method, can be used to efficiently design the composite xylophone bars.

(5). The mode shapes of xylophone bar can help the designer to locate the position of holes at which the string can be used to connect xylophone bars with the frame.

Acknowledgement

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