VIBRATION OPTIMIZATION OF LAMINATED SHALLOW SHELLS WITH NON-UNIFORM CURVAUTRE

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Abstract

This study proposes a combined approach of the analysis for laminated shallow shells with non-uniform curvature and the layerwise optimization (LO) method to achieve the optimum free vibration behaviors. Shell structures with non-uniform curvature are recently found in automobile and other design-oriented structural applications. In the analysis part, an interpolating function is introduced in polynomial form and the corresponding curvature is derived as a function of the position. The obtained curvature is substituted into the total potential energy of the shell and the analytical procedure is shown to derive a frequency equation by the Ritz method. After examining the solution accuracy, the optimization process is applied in numerical examples to accomplish the maximum fundamental frequencies of the shallow shells.

1 Introduction

Significant progress has been made over the last two decades in the development of increasingly more efficient composite structures, and many industries continue to investigate strategies for fully exploiting the potential of composites for a variety of structural forms including the laminated shallow shells. In addition to their potential for high specific stiffness and strength, a key advantage of composite laminates is the ability to tailor the properties of the laminate through lay-up design. It is therefore appropriate clearly that efforts be applied to fully optimize this tailoring process.

The vibration analysis of shallow shells has a long history of academic and practical interest, as summarized in a monograph [1] by Leissa and in two review papers by Qatu [2] and Liew, Lim and Kitipornchai [3]. A basic theory was fully developed in reference [4], where a complete set of equations for elastic deformation of laminated composite shallow shells are presented for static and vibration behaviors. Since the publication of this reference, a number of relevant technical papers have appeared on vibration of laminated composite shallow shells.

As for tailoring, Raouf [5] considered the effect of tailoring on the dynamic characteristics of composite panels using fiber orientation. Narita, Ito and Zhao [6] applied a genetic algorithm to determine the maximum fundamental frequency of laminated shallow shells that are supported by shear diaphragms and for the same problem they used Kuhn-Tucker condition to derive the maximum fundamental frequency of laminated shallow shells [7]. The effect of using various solutions upon optimizing vibration characteristics of laminated shallow shells are also studied [8]. These papers [5-8] are however limited to a simple case with the edges fully supported by shear diaphragms, where a simplified frequency formula is derivable by neglecting the cross-elasticity terms.

The present paper proposes a new strategy for optimizing the vibration characteristics of the laminated composite shallow shells with *non-uniform* curvature subjected to any sets of typical boundary conditions. First, a semi-analytical method is introduced by extending a method used for uniform laminated shallow shell [9] to deal with the problem. For surface modeling, an interpolating function with unknown coefficients is introduced to represent the required surface shape and is determined by matching at some representative points on the surface. The variable curvature determined from the surface is substituted into the total potential energy of the shell. A frequency equation is derived by the Ritz method to yield the frequency parameter which is used as an objective function in the present optimization.

Secondly, the analytical method is combined with the layerwise optimization (LO) scheme, recently developed for laminated shallow shells with constant cylindrical curvature [10]. Numerical examples demonstrate the accuracy of the present Ritz solution to determine natural frequencies of cylindrically curved panels with various edge conditions, and also the extension of the LO approach to the shallow shells with non-uniform curvature is shown to be quite effective in obtaining the optimum fiber orientation angles which maximize the fundamental frequencies of the laminated shallow shells.

2 Analysis and Optimization Procedure

2.1 Vibration Analysis

Consider a shallow shell with rectangular planform of $a \times b$, and the rise of the shell is expressed by a polynomial in terms of x and y

$$\phi(x,y) = c_0 + c_{10}x + c_{01}y + c_{20}x^2 + c_{11}xy + ... \quad (1)$$

where $c_0, c_{10}, \ldots$ are unknown coefficients, and are determined by interpolating the surface on some representative points. For purpose of illustrating the procedure, the curved surface modeled for automobile bonnet and roof shapes is introduced in Fig.1 that satisfies

$$\phi(-\frac{a}{2}, y) = \phi(\frac{a}{2}, y) = \phi(x, -\frac{b}{2}) = 0, \quad \phi(0, \frac{b}{2}) = H \quad (2)$$

Substitution of Eq.1 into Eq.2 yields an expression for the non-uniformly curved surface

$$\phi(x,y) = \frac{H}{2}\left[1 - \left(\frac{2x}{a}\right)^2\right]\left[1 + \left(\frac{2y}{b}\right)\right] \quad (3)$$

and the curvatures (curvature radiuses) are obtained by differentiating twice as

$$\frac{1}{R_x} = \left(\frac{4H}{a^2}\right)\left(1 + \frac{2y}{b}\right), \quad \frac{1}{R_y} = 0, \quad \frac{1}{R_{xy}} = \frac{8Hx}{a^2 b} \quad (4)$$

under the assumption $(\partial\varphi/\partial x)^2 = (\partial\varphi/\partial y)^2 = 0$. Equations (4) indicate that the curvature changes in linear fashion and unknown coefficients c_0, c_{10} and c_{01} in Eq.1 are not included in the curvatures.

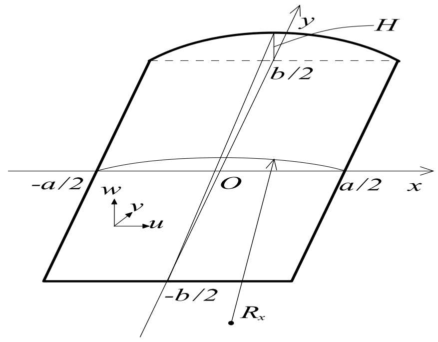


Fig. 1. Example of Laminated composite shallow shells with non-uniform curvature

In the Donnell type thin shell theory [4], displacements $u^*(x,y,z)$, $v^*(x,y,z)$ and $w^*(x,y,z)$ at an arbitrary point within the shell are given in terms of the displacements $u(x,y)$, $v(x,y)$ and $w(x,y)$ on the middle plane by

$$u^* = u - z\frac{\partial w}{\partial x} \;,\quad v^* = v - z\frac{\partial w}{\partial y} \;,\quad w^* = w \quad (5)$$

where z is a coordinate measured from the middle plane. The strains are then given by

$$\varepsilon_x^* = \varepsilon_x + z\kappa_x \;,\quad \varepsilon_y^* = \varepsilon_y + z\kappa_y \;, \quad (6)$$
$$\gamma_{xy}^* = \gamma_{xy} + z\kappa_{xy}$$

where κ_x and κ_y are the normal curvatures defined at the middle plane of the shell, and κ_{xy} is a twisting curvature as

$$\kappa_x = -\frac{\partial^2 w}{\partial x^2} \;,\quad \kappa_y = -\frac{\partial^2 w}{\partial y^2} \;,\quad \kappa_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} \quad (7)$$

The relations between the displacements (u, v and w) and the strains (ε_x, ε_y and γ_{xy}) are given in the shallow shell theory by

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{w}{R_x}, \quad \varepsilon_y = \frac{\partial v}{\partial y} + \frac{w}{R_y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_{xy}} \quad (8)$$

The difference between the present and conventional shell analyses is easily recognized, since the curvatures in Eqs.(8) are not constant but functions of the coordinate (x,y).

In using the Ritz method, one has to evaluate the total potential energy and the strain energy for the shallow shell is

$$V = V_s + V_{bs} + V_b \quad (9)$$

where V_s is the energy caused by the in-plane motion, V_{bs} is the energy by coupling between in-plane and out-of-plane motions and V_b is the energy by out-of-plane motions:

$$V_s = \frac{1}{2}\iint \{\varepsilon\}^T [A]\{\varepsilon\} dxdy \tag{10}$$

$$V_{bs} = \frac{1}{2}\iint (\{\kappa\}^T [B]\{\varepsilon\} + \{\varepsilon\}^T [B]\{\kappa\}) dxdy$$

$$V_b = \frac{1}{2}\iint \{\kappa\}^T [D]\{\kappa\} dxdy$$

where $\{\varepsilon\}$ and $\{\kappa\}$ are the strain and curvature vectors, and for the laminated composite material, $[A]$, $[B]$ and $[D]$ become the stiffness matrices of the composites for in-plane motion, coupling motion of in-plane and out-of-plane motion, and out-of-plane motion, respectively. The kinetic energy for the shell is defined by

$$T = \frac{\rho h}{2}\int_{-b/2}^{b/2}\int_{-a/2}^{a/2}\left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right] dx\iota \tag{11}$$

where ρ is defined as the averaged mass per unit volume of the shell. Next, the displacement functions are assumed by the double series form as

$$u(\xi,\eta,t) = \sum_{i=0}^{M-1}\sum_{j=0}^{N-1} P_{ij} X_i(\xi) Y_j(\eta) \sin\omega t, \tag{12}$$

$$v(\xi,\eta,t) = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} Q_{kl} X_k(\xi) Y_l(\eta) \sin\omega t$$

$$w(\xi,\eta,t) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} R_{mn} X_m(\xi) Y_n(\eta) \sin\omega t$$

where $\xi=2x/a$ and $\eta=2y/b$ are non-dimensional coordinates, and P_{ij}, Q_{kl} and R_{mn} are unknown coefficients. Functions $X_i(\xi)$, $Y_j(\eta)$,…, and $Y_n(\eta)$ are

$$X_i(\xi) = \xi^i (\xi+1)^{BC11} (\xi-1)^{BC31} \tag{13}$$

$$X_k(\xi) = \xi^k (\xi+1)^{BC12} (\xi-1)^{BC32}$$

$$X_m(\xi) = \xi^m (\xi+1)^{BC13} (\xi-1)^{BC33}$$

$$Y_j(\eta) = \eta^j (\eta+1)^{BC21} (\eta-1)^{BC41}$$

$$Y_l(\eta) = \eta^l (\eta+1)^{BC22} (\eta-1)^{BC42}$$

$$Y_n(\eta) = \eta^n (\eta+1)^{BC23} (\eta-1)^{BC43}$$

that satisfy the geometrical boundary conditions along the shell edges. In Eqs.(13), the $BCpq$ are the boundary indexes where p denotes the shell edge (p=1～4: the left-hand-side edge, the lower edge, the right-hand-side edge and the upper edge, respectively, in Fig.1) and q denotes each of the displacements (q=1～3: u,v,w). More specifically, $BCpq$=0 and $BCpq$=1 indicate that the in-plane displacements u, v (q=1,2) are free and fix, respectively. For the out-of-displacement w (q=3), $BCpq$=0, $BCpq$=1 and $BCpq$=2 indicate that the out-of-displacement w (q=3) is free, simply supported and clamped, respectively. The use of the indices $BCpq$ makes it possible to satisfy the kinematical boundary conditions [10].

After the equations are rewritten by using the non-dimensional quantities, the displacements (12) are substituted into the functional in term of the strain and kinetic energies

$$L = T_{\max} - V_{\max} \tag{14}$$

Then an eigenvalue equation is derived by the minimizing process

$$\frac{\partial L}{\partial P_{ij}} = \frac{\partial L}{\partial Q_{kl}} = \frac{\partial L}{\partial R_{mn}} = 0 \tag{15}$$

where the subscripts are i (k,m)=0,1,2,..., M-1 , j (l,n)=0,1,2,...,N-1. In a matrix form,

$$\left(\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{12} & k_{22} & k_{23} \\ k_{13} & k_{23} & k_{33} \end{bmatrix} - \Omega^2 \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix}\right)\begin{Bmatrix} P_{ij} \\ Q_{kl} \\ R_{mn} \end{Bmatrix} = \tag{16}$$

where Ω is a frequency parameter defined by

$$\Omega = \omega a^2 \left(\frac{\rho h}{D_0}\right)^{1/2}, D_0 = \frac{E_T h^3}{12(1-\nu_{LT}\nu_{TL})} \tag{17}$$

and k_{11},.., k_{33} and m_{11},.., m_{33} are the elements of the stiffness and mass matrices (not presented due to the limited space), where the integrals included in the elements can be integrated exactly.

2.2 Layerwise Optimization

An optimum structural design is generally composed of two parts: the structural analysis and the optimization scheme. The first part has been developed in Sec.2.1 and the second part is formulated here. The object function for optimization is taken to be a frequency parameter Ω defined in Eq.(17) and is denoted by Ω_1 for the fundamental mode. The term "fundamental frequency" indicates the lowest eigenvalue for given conditions.

The design variables are taken to be a set of fiber orientation angles in the K layers of the upper (or lower) half of the plate cross-section:

$$[\theta_1/\theta_2/\ldots/\theta_k/,\ldots/\theta_K]_S \qquad (18)$$

where θ_k is the fiber orientation angle in the k th layer (k=1:outermost, $k=K$: innermost) and the subscript "s" denotes symmetric lamination. Therefore, the optimization problem may be written in standard form as

$$\begin{aligned}&\text{Find : } \vec{\theta} = [\theta_1/\theta_2/\ldots/\theta_K]_{S,opt}\\ &\text{which maximizes : } \Omega_1\end{aligned} \qquad (19)$$

subject to the constraints:

$$-90° \le \theta_k \le 90° (k=1,2,\ldots,K)$$

It is known that the approach of using each fiber orientation angle directly as a design variable is straightforward but the number of design variables increases in proportion to the number of layers, resulting in a multi-dimensional search optimization problem. The layerwise optimization (LO) approach makes use of a simple physical observation that in the bending of shallow shells, the outer layer has a greater stiffening effect than an inner layer and therefore has a greater influence. This physical fact suggests that the outer layer plays a more influential role in determining the natural frequency of laminated shallow shells. It is intended here to extend the LO approach, already successfully used for plate optimization, to the shallow shell problems with slight non-uniform curvature.

Therefore, the following approach to the solution of the optimization problem is advocated.

The optimum stacking sequence $[\theta_1/\theta_2/\ldots/\theta_K]_{S,opt}$ *for the maximum fundamental frequency of a laminated shallow shell can be obtained by determining the optimum fiber angle for each layer sequentially working from the outermost to the innermost layer.*

The difference between the flat plate problems and the present shallow shell problem is that the in-plane motion is coupled with the out-of-plane motion. Due to this difference the applicability of the LO procedure to the present problem is questionable, but it will be demonstrated that the LO procedure works quite effectively for shallow shells because the bending is still dominant in the vibration of these shallow shells.

If $\Omega_1^{(k)}{}_{opt}$ is assumed to be the maximum value of the frequency parameter obtained in the k th step (Note that the same k indicating the layer number is used because it deals with the k th layer), the following procedure, based on the foregoing assumption, may be used to determine $\Omega_{1,opt}$:

Step 0: Assume a laminated shell made of K hypothetical layers in the upper (lower) half of the cross-section with mass but no rigidity.

Step 1: Find $\theta_{1,opt}$, using a one-dimensional search with a certain increment $\Delta\theta$, which maximizes the fundamental frequency $\Omega_1^{(1)}{}_{opt}$ of the laminated shallow shell with an anisotropic lamina (i.e., with E_L, E_T, G_{LT} and ν_{LT}) in the first outermost layer. The (K-1) inner layers remain hypothetical with no rigidity.

Step 2: Find $\theta_{2,opt}$, using a one-dimensional search, which maximizes $\Omega_1^{(2)}{}_{opt}$ of the laminated shell with an anisotropic lamina in the second layer and an anisotropic first layer with $\theta_1=\theta_{1,opt}$. The inner (K-2) layers remain hypothetical with no rigidity.

Step 3 to K-1: The foregoing process is repeated to yield $\theta_{3,opt},\ldots\theta_{(K-1),opt}$.

Step K: Find $\theta_{K,opt}$ which maximizes the $\Omega_{K,opt}$ of the laminated panel with an anisotropic lamina in the K-th innermost layer. This last step determines the optimum lay-up $[\theta_1/\theta_2/\ldots/\theta_K]_{S,opt}$ which maximizes the fundamental frequency $\Omega_{1,opt}=\Omega_1^{(K)}{}_{opt}$ of the panel.

The above set of Steps from 1 to K is considered as one cycle of the LO iterative solution procedure. In the first cycle, the inner layers are assumed to have zero stiffness, and the fiber orientation angles determined at Step K in the first cycle, i.e. $[\theta_1/\theta_2/\ldots/\theta_K]_{S,opt}$, must be a better initial approximation for the second cycle of Steps 1 to K. The iterative cycles continue until a converged solution is obtained.

3 Numerical Results and Discussions

3.1 Numerical Examples

Numerical results are given for symmetrically laminated, 8-layered shallow shells and the elastic constants used are for carbon/ epoxy composite: CFRP material:

E_L = 138GPa, E_T = 8.96GPa,

G_{LT} = 7.1GPa, ν_{LT} =0.30.

The planform has a square (*a/b*=1) and the thickness is moderate value (*h/a*=0.01). The boundary conditions are denoted by letters F, S and C, where F stands for free, S for simply supported and C for clamped edges, respectively. The kinematic boundary conditions for S and C are defined, e.g., along $x=-a/2$ edge, $v=w=0$ and $u=v=w=\partial w/\partial x=0$, respectively.

The value of the rise *H* is taken as $\beta=H/a$=0.0251 and 0.0635, which have the same height with *a/R*=0.2 and 0.5, respectively, for cylindrically curved shells. The stacking sequence is taken as general lay-up $[\theta_1/\theta_2/\theta_3/\theta_4]_s$ (s: symmetric) and the degree 45(°) is written as $[(45/-45)_2]_S$ for simplicity.

Three types of curvature models are considered for comparison:

PLT model: PLaTe model (no curvature)
SGC model: shell with SinGle Curvature
(constant curvature *a/R*=0.2 or 0.5)
NUC model: shell with Non-Uniform Curvature
(as shown in Fig. 1)

The solution accuracy for natural frequencies of these plates and shells were already clarified in a previous report [9].

Table 1 Optimum stacking sequence and corresponding frequency parameter of shallow shells with various curvature.

		$[\theta_1/\theta_2/\theta_3/\theta_4]_{s,opt}$	Ω_{opt}
SSSS	PLT	[45/−45/−45/−45]s	56.32
	NUC(β=0.0251)	[45/−45/−45/45]s	72.77
	SGC(a/Rx=0.2)	[45/−45/45/−45]s	113.4
	NUC(β=0.0635)	[55/−50/−40/35]s	104.4
	SGC(a/Rx=0.5)	[5/0/−45/75]s	172.3
CCCC	PLT	[0/90/90/0]s	93.67
	NUC(β=0.0251)	[90/0/0/0]s	136.1
	SGC(a/Rx=0.2)	[0/0/0/0]s	239.4
	NUC(β=0.0635)	[0/90/0/0]s	215.4
	SGC(a/Rx=0.5)	[5/−20/40/−45]s	296.8

3.2 Maximizing the Fundamental Frequencies

The application of the present design method is to maximize the fundamental frequencies of the three types of plates and shallow shells. Table 3 presents the maximum frequency parameters $\Omega_{1,opt}$

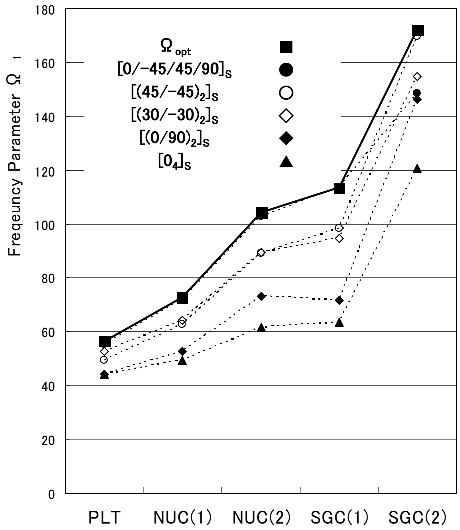


Fig. 2. Comparison of the optimum frequency and frequency parameters of 8-layer shallow square shells for typical lay-ups (SSSS shell, PLT:plate, NUC(1):β=0.0251, NUC(2):β=0.0635, SGC(1): *a/R*=0.2, SGC(2):*a/R*=0.5)

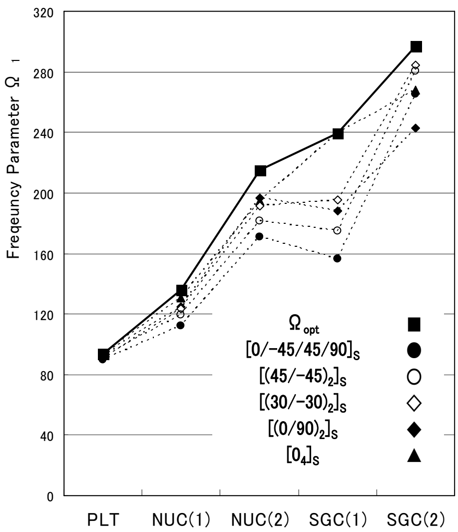
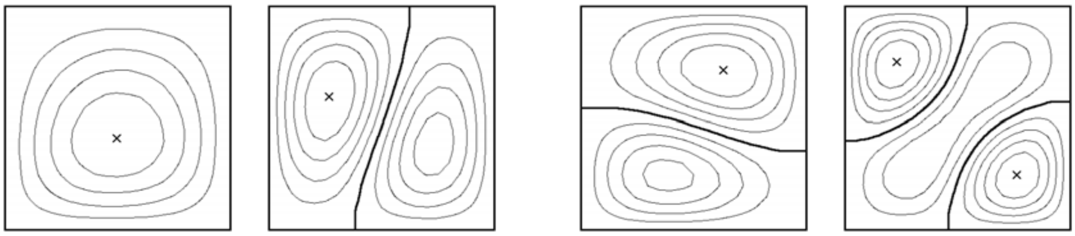
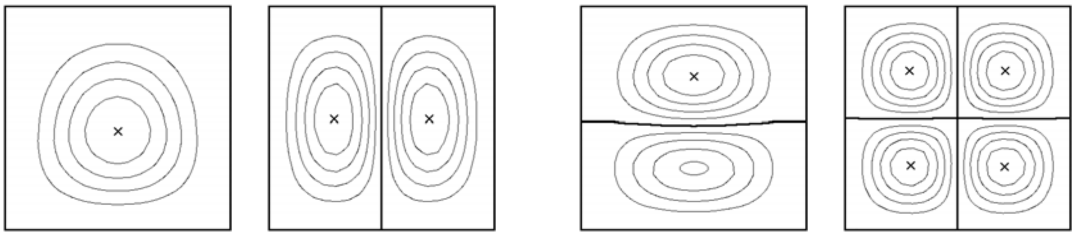


Fig. 3. Comparison of the optimum frequency and frequency parameters of 8-layer shallow square shells for typical lay-ups (CCCC shell, PLT:plate, NUC(1):β=0.0251, NUC(2):β=0.0635, SGC(1): *a/R*=0.2, SGC(2):*a/R*=0.5)



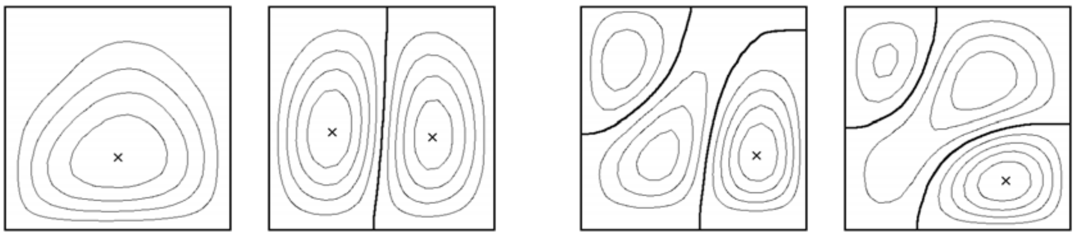
Ω_1=72.77 Ω_2=142.0 Ω_3=153.6 Ω_4=227.2

(a) NUC (β=0.0251, SSSS, [45/-45/-45/45]s)



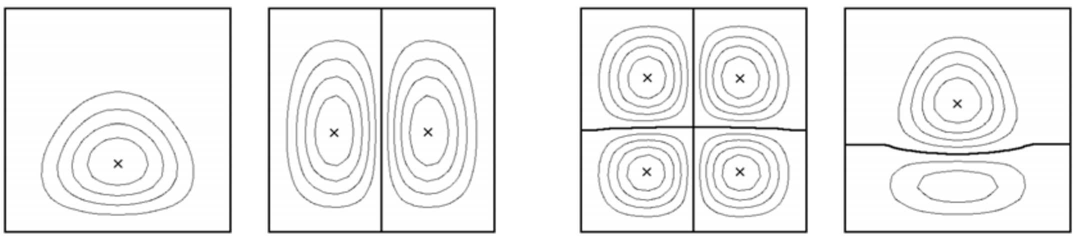
Ω_1=136.1 Ω_2=187.6 Ω_3=231.9 Ω_4=270.5

(b) NUC (β=0.0251, CCCC, [90/0/0/0]s)



Ω_1=104.4 Ω_2=184.8 Ω_3=214.7 Ω_4=233.5

(c) NUC (β=0.0635, SSSS,[55/-50/-40/35]s)



Ω_1=215.4 Ω_2=236.8 Ω_3=296.5 Ω_4=310.1

(d) NUC (β=0.0635, CCCC, [0/90/0/0]s)

Fig.4 Mode shapes and frequency parameters of the lowest four modes for the shallow shells with optimum lay-ups and non-uniform curvature and the corresponding optimum lay-up $[\theta_1/\theta_2/\ldots/\theta_K]_{S,opt}$ obtained by the present design method for the shells with two typical boundary conditions of fully simply supported (SSSS) and clamped (CCCC) edges. For the SSSS plate (PLT) and relatively shallow NUC and SGC shells (β=0.0251 and a/R_x=0.2), the optimum lay-ups are [45/-45/-45/-45]s or similar lay-ups, but for relatively deep NUC and SGC shell (β=0.0635 and a/R_x=0.5) different lay-ups are found. For the CCCC shells, where the boundaries are strongly constrained, the effects of different lay-ups are diminished and the optimum lay-ups are mostly combinations of 0 and 90 degree.

Comparisons are made in Figs.2 and 3 to demonstrate that the shells with the optimum lay-ups $[\theta_1/\theta_2/\theta_3/\theta_4]_{s,opt}$ actually give higher fundamental frequencies than shells with other stacking sequences. Typical stacking sequences of symmetric 8-layer shells are chosen for comparison purposes, namely $[0_4]$s, $[(0/90)_2]$s, $[(30/-30)_2]$s, $[(45/-45)_2]$s and [0/-45/45/90]s. The first two are specially orthotropic shells denoted by ▲ and ◆ in the figures. The third and fourth cases are the shells with alternating angle-ply sequences $[(30/-30)_2]$s and $[(45/-45)_2]$s denoted by ◇ and ○, respectively. The last one ([0/-45/45/90]s) is a quasi-isotropic lay-up, denoted by ●. It is observed that all of the present optimum solutions (denoted by ■) have higher fundamental frequencies than the shells with the five typical lay-ups without exception.

3.3 Vibration Mode Shapes of the Shallow Shells with the Optimum Lay-ups

The vibration mode shapes of the shallow shells with the optimum lay-up configurations are presented for the NUC model in Fig.4(a)-(d) to study effects of varying the degree of curvature and boundary conditions. Corresponding frequency parameters Ω_1, Ω_2, Ω_3 and Ω_4 are given below of the mode shapes. There are four combinations of two degrees of shallowness ($\beta=H/a$=0.0251 and 0.0635) and two boundary conditions (SSSS and CCCC). In the figures, the mark "×" represents the maximum displacement point and thin lines denote the displacement contour lines when displacements are normalized by the maximum displacement and divided into five equal increments. The thick lines represent the nodal lines (i.e., lines of zero displacement).

In case (a) of relatively shallow shell (β=0.0251) with weak boundary constraints (SSSS), the effect of diagonally located fiber directions (45 or -45 degree) can be seen clearly on the distorted nodal and contour lines. In contrast, case (b) has strong boundary constraint (CCCC) and parallel fiber orientation along the edges (0 or 90 degree), and these suppress the effect of non-uniform curvature giving almost straight nodal lines. For relatively deep shells (β=0.0635), the mode shapes in (c) and (d) are similar to (a) and (b), respectively, but the effect of larger curvature than (a) and (b) causes more distorted in mode shapes except for the second mode.,

4 Conclusions

A new combination of the analysis method and LO approach is proposed to determine the optimum lay-up design of the shallow shells with non-uniform curvature. The curvature variation is expressed in polynomials and is differentiated twice to obtain three curvatures of $1/R_x$, $1/R_y$ and $1/R_{xy}$. The Ritz method is used to derive a frequency equation. In numerical examples, the shell geometry (NUC model) is proposed to model bonnet and roof structures of automobiles as a standard model. By comparing the frequency parameters and mode shapes of the shells with optimum lay-ups among three models, the effects of varying the shallowness, boundary conditions are discussed on the optimum lay-up designs.

Thus the validity and effectiveness are demonstrated in numerical examples, and it is hoped that the present design method will be used to design the lay-up configurations for shell-type laminated composite structures with non-uniformly curved surfaces.

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