OPTIMISATION OF ANISOTROPIC PLATES THAT VARY IN THICKNESSES AND PROPERTIES

Jose E. Herencia, Paul M. Weaver and Michael I. Friswell*
***University of Bristol, Queen's Building, University Walk, Bristol, BS8 1TR, UK 2**

Keywords: *Optimisation, anisotropic, lamination parameters, buckling, Rayleigh-Ritz*

Abstract

A method to optimise anisotropic composite plates that vary in thickness and properties across the width is presented. The optimisation problem is divided into two levels. At the first level, Mathematical Programming (MP) is applied, where the plate is divided into several strips, each of them modelled using lamination parameters accounting for their anisotropy. The laminate of each of the strips is assumed to be symmetric, or mid-plane symmetric with 0, 90, 45 or -45 degree ply angles. The plate is subjected to a combined loading under strength, buckling and practical design constraints. Manufacturing details of the plate are embedded within the design variables. At the second level, the actual lay-ups of the plate's strips are obtained using a Genetic Algorithm (GA), accounting for manufacturability and design practices. The novelty of this work lies in: the inclusion of anisotropy for elastic tailoring, manufacturing and practical design considerations, as well as the application of the Rayleigh-Ritz (RR) method to assess buckling analysis in plates that vary in thickness and properties across the width.

1 Introduction

Composite primary flight structures are commonly designed using stiffened panels. Several effects, such as the stiffener flange over a plate or plates with variable thickness, might be characterized as plates made of several strips with step changes in thickness and other properties across width. In addition, the design of composite materials is directly linked to their manufacture. Common manufacturing practices within the aerospace community limit the ply angles to 0, 90, 45 and -45 degrees.

Early work on plates that change in thickness across the width was performed by Capey [1] and followed by Benthem [2]. They obtained exact solutions by solving the equilibrium equation. The plates were isotropic and idealised as symmetric sections with their edges simply supported. The effect of the thickness variation across the width on the buckling load capability of the plate was assessed.

In contrast to metals, composite laminates might exhibit a certain degree of flexural anisotropy or elastic coupling terms. The presence of flexural anisotropy makes the determination of exact solutions for buckling problems rather complicated. Approximated solutions for buckling of anisotropic flat plates were initially obtained by Ashton and Waddoups [3] using the RR method [4]. The impact of flexural anisotropy on the critical buckling load was highlighted. Levy and Ganz [5] and later Levy [6] also employed the RR method to perform analysis and optimisation of orthotropic plates for buckling accounting for thickness variations. Gutierrez and Laura [7] employed the RR method to estimate the fundamental natural frequencies of a rectangular anisotropic plate, which varied discontinuously in thickness. More recently, Weaver [8]-[9] has provided Closed Form (CF) solutions for flexural anisotropic plates under combined loading with constant thickness across the width.

An alternative to characterise composite materials properties was provided by Tsai and Pagano [10]. Laminated composite properties were obtained by using lamination parameters. Miki and Sugiyama [11] proposed the use of lamination parameters to perform composite optimisation. Haftka and Walsh [12] and Nagendra et al. [13] used integer programming and lamination parameters to carry out lay-up optimisation under buckling and strain constraints on symmetric and balanced laminated plates. Fukunaga et al. [14] used the RR method, MP and lamination parameters to maximise buckling loads of symmetric laminates with flexural anisotropy. However, none of the previous work has investiagated the effect of varying both the thickness and other properties of an anisotropic plate across the width, as well as considering manufacturing and design practices.

The authors' previous work [15], based upon a two level optimisation approach which couples MP with GAs, has shown that material anisotropy can be used to improve the structural performance of composite stiffened panels.

The aim of this paper is to provide an approach to optimise anisotropic composite plates that vary in thickness and properties across the width. The optimisation problem is divided into two levels. At the first level, MP is applied, where the plate is divided into several strips, each of them modelled using lamination parameters accounting for their anisotropy. The laminate of each of the strips is assumed to be symmetric, or mid-plane symmetric with 0, 90, 45 or -45 degree ply angles. The plate is subjected to a combined loading under strength, buckling and practical design constraints. The RR method is developed and applied to asses the buckling behaviour for this particular case. Manufacturing details of the plate are embedded within the design variables. At the second level, the actual lay-ups of the plate's strips are obtained using GAs considering manufacturing and design practices.

2 Plate geometry and loading

Figure 1 defines the plate geometry, material axis and positive sign convention for the loading.

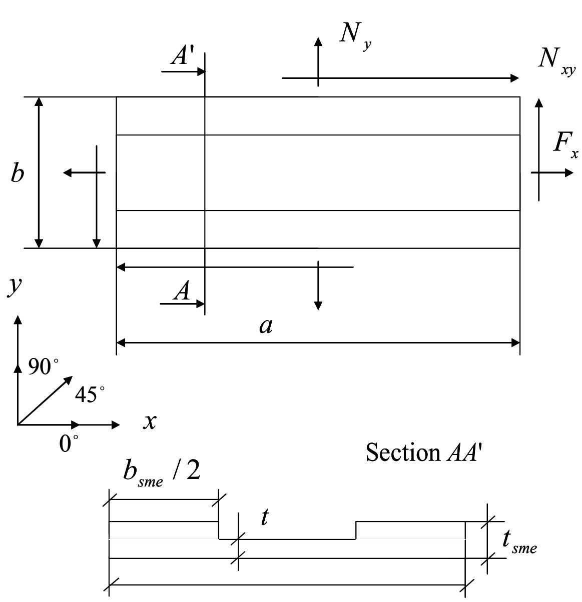


Fig. 1. Plate geometry and loading.

The plate consists of three strips in which the central strip has different thickness and properties compared to the outer strips. Three different types of plate configurations are considered. Those are detailed in the design variables section.

3 Laminate constitutive equations

Laminate constitutive equations for each of the plate's strips are obtained by applying the Classical Lamination Theory (CLT) [16] and lamination parameters (ξ) (e.g. Ref. [10]). Laminates are assumed to be symmetric or mid-plane symmetric laminates with 0, 90, 45 or -45 degree ply angles. Thus,

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} \tag{1}$$

where $[A]$ is the membrane stiffness matrix, $[D]$ is the bending stiffness matrix, $\{N\}$ is the vector of the in-plane running loads, $\{M\}$ is a vector of the running moments, $\{\varepsilon^0\}$ is the vector of in-plane strains and $\{\kappa\}$ is the vector of the middle surface curvatures.

The membrane and bending stiffness matrices can be expressed in terms of material stiffness invariants (U) and eight lamination parameters (ξ). Furthermore, individual plies are considered orthotropic and laminated with fibre angles restricted to 0, 90, 45, and -45 degrees. Consequently, the lamination parameters are reduced to six. Hence,

$$\begin{bmatrix} A_{11} \\ A_{12} \\ A_{22} \\ A_{66} \\ A_{16} \\ A_{26} \end{bmatrix} = h \begin{bmatrix} 1 & \xi_1^A & \xi_2^A & 0 & 0 \\ 0 & 0 & -\xi_2^A & 1 & 0 \\ 1 & -\xi_1^A & \xi_2^A & 0 & 0 \\ 0 & 0 & -\xi_2^A & 0 & 1 \\ 0 & \dfrac{\xi_3^A}{2} & 0 & 0 & 0 \\ 0 & \dfrac{\xi_3^A}{2} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} D_{11} \\ D_{12} \\ D_{22} \\ D_{66} \\ D_{16} \\ D_{26} \end{bmatrix} = \frac{h^3}{12} \begin{bmatrix} 1 & \xi_1^D & \xi_2^D & 0 & 0 \\ 0 & 0 & -\xi_2^D & 1 & 0 \\ 1 & -\xi_1^D & \xi_2^D & 0 & 0 \\ 0 & 0 & -\xi_2^D & 0 & 1 \\ 0 & \frac{\xi_3^D}{2} & 0 & 0 & 0 \\ 0 & \frac{\xi_3^D}{2} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix} \quad (3)$$

The material stiffness invariants are given as follows,

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 3 & 2 & 3 & 4 \\ 4 & 0 & -4 & 0 \\ 1 & -2 & 1 & -4 \\ 1 & -6 & 1 & -4 \\ 1 & -2 & 1 & 4 \end{bmatrix} \begin{bmatrix} Q_{11} \\ Q_{12} \\ Q_{22} \\ Q_{66} \end{bmatrix} \quad (4)$$

The ply stiffness properties (Q) are given by,

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}} \quad (5)$$

$$Q_{12} = \frac{\nu_{12} E_{22}}{1 - \nu_{12}\nu_{21}} \quad (6)$$

$$Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}} \quad (7)$$

$$Q_{21} = Q_{12} \quad (8)$$

$$Q_{66} = G_{12} \quad (9)$$

$$\nu_{21} = \nu_{12} \frac{E_{22}}{E_{11}} \quad (10)$$

The membrane and bending lamination parameters are calculated by the following integrals,

$$\xi^A_{[1\ 2\ 3]} = \frac{1}{h} \int_{-h/2}^{h/2} [\cos 2\varphi, \cos 4\varphi, \sin 2\varphi]\ dz \quad (11)$$

$$\xi^D_{[1\ 2\ 3]} = \frac{12}{h^3} \int_{-h/2}^{h/2} [\cos 2\varphi, \cos 4\varphi, \sin 2\varphi]\ z^2 dz \quad (12)$$

where φ represents the fibre orientation angle at position z and h is the laminate thickness.

4 Optimisation strategy

The optimisation strategy is divided into two levels [15] and is shown in Fig. 2. At the first level, the plate is optimised using gradient based techniques. The optimum dimensions and values of the lamination parameters of the plate design are obtained. At the second level, a GA is used to target the optimum lamination parameters to obtain the actual lay-ups for each of the plate's strips.

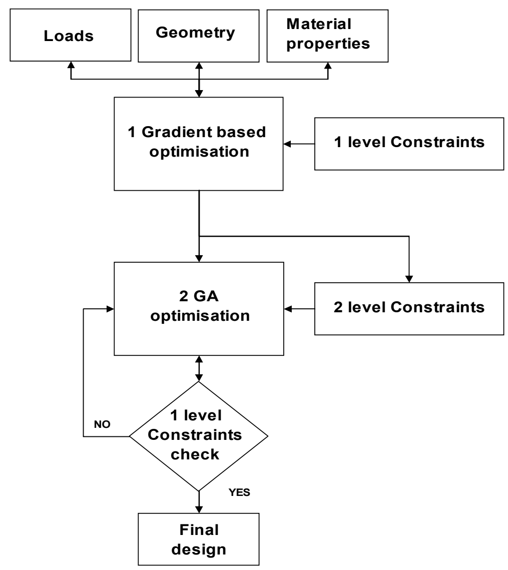


Fig. 2. Optimisation strategy.

4.1 First level - gradient based optimisation

The gradient based optimisation is carried out in MATLAB [17]. The basic mathematical optimisation problem is stated as follows,

$$\begin{array}{lll} \text{Minimise} & M(\vec{x}) & \\ \text{Subject to} & G_j(\vec{x}) \le 0 & j = 1, \ldots, n_G \\ & x_i^l \le x_i \le x_i^u & i = 1, \ldots, n \end{array} \quad (13)$$

where M is the objective function or mass of the plate, G are the design constraints such as strength, buckling or practical design rules, and $\vec{x}$ is the vector of the design variables.

4.1.1 Objective function

The objective function is the mass of the plate. Therefore,

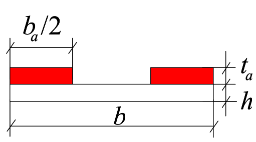
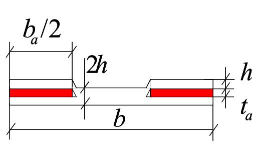
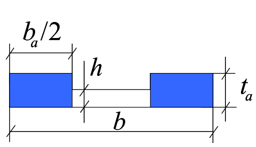
$$M = a\left(\rho_a b_a t_a + \rho\, b^* t\right) \tag{14}$$

where a is the length of the plate, t and t_a are the plate and reinforcement thicknesses, ρ and ρ_a are the plate and reinforcement densities, and b^* and b_a are the plate and reinforcement widths. Table 1 relates the above properties to the different plate designs considered.

4.1.2 Design variables

The design variables, depending on the plate design, are shown in Table 1. Note that in plate designs **a** and **b** the outer strip is made of part of the plate and the reinforcement. In contrast, in plate design **c** the outer strip is the reinforcement.

Table 1. Table of design variables

Plate design	Plate configuration	Design variables $\vec{x}$	
		Central strip	Outer strip
a		h $\xi^{A,D}_{[1\ 2\ 3]}$	t_a b_a $\xi^{A,D}_{[1\ 2\ 3]}$
		$t = h$ $t_{sme} = h + t_a$ $b_{sme} = b_a$ $b^* = b$	
b		h $\xi^{A,D}_{[1\ 2\ 3]}$	t_a b_a $\xi^{A,D}_{[1\ 2\ 3]}$
		$t = 2h$ $t_{sme} = 2h + t_a$ $b_{sme} = b_a$ $b^* = b$	
c		h $\xi^{A,D}_{[1\ 2\ 3]}$	t_a b_a $\xi^{A,D}_{[1\ 2\ 3]}$
		$t = h$ $t_{sme} = t_a$ $b_{sme} = b_a$ $b^* = b - b_a$	

4.1.3 Design constraints

The design constraints considered for the optimisation of the plate are described in the following sections.

1) Lamination parameter feasible region

The lamination parameters feasible region is extracted from Ref. [15]. For instance, the membrane and bending lamination parameter feasible regions are given by,

$$2\left|\xi_1^{A,D}\right| - \xi_2^{A,D} - 1 \le 0 \tag{15}$$

$$2\left|\xi_3^{A,D}\right| + \xi_2^{A,D} - 1 \le 0 \tag{16}$$

Further details on these constraints can be found in Ref [18]. These constraints are imposed on the central and outer strips of the plate.

2) Strength constraints

Failure strength constraints are considered by restricting the laminate in-plane strains longitudinally, transversally and in shear, for both the tension and compression cases. Laminate strains under in-plane loading are determined by CLT. Hence,

$$\left\{\varepsilon^0\right\} = [A]^{-1}\{N\} \tag{17}$$

The strength load factor is given by the ratio between the allowable and applied strain. Therefore,

$$\lambda_i^j = \frac{\varepsilon_{ai}^j}{\varepsilon_i^{0j}} \qquad i = T, C; \quad j = x, y, xy \tag{18}$$

where ε_a is the allowable strain, ε^0 is the applied strain, x, y, and xy represent the longitudinal, transversal and shear directions, respectively. Note that T and C denote tension and compression.

Failure strength constraints are implemented as follows,

$$\frac{1}{\lambda_i^j} - 1 \le 0 \qquad i = T, C; \quad j = x, y, xy \tag{19}$$

These constraints are applied to each of the plate's strips.

3) Buckling constraints

The plate is assumed to be flat and with its four edges simply supported. The actual cross section of the plate is idealised as shown in Fig. 3, following Ref. [1].

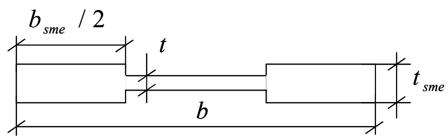


Fig. 3. Idealised plate cross-section.

It is assumed that the neutral axis passes through the centre of each of the strips of the plate. Furthermore, for plate design **a**, as the total laminate at the strip edge might present a certain degree of unsymmetry, smeared properties are assumed and the reduced bending stiffness approach [4] is taken. Thus,

$$t_{sme} = t + t_a \tag{20}$$

$$b_{sme} = b_a \tag{21}$$

$$[D]_{sme} = [D^*] - [B^*]\,[A^*]^{-1}[B^*] \tag{22}$$

with

$$[A^*] = [A] + [A]_a \tag{23}$$

$$[B^*] = \frac{t_a}{2}[A] - \frac{t}{2}[A]_a \tag{24}$$

$$[D^*] = [D] + [D]_a + \frac{t_a^{\,2}}{4}[A]_a + \frac{t^2}{4}[A]_a \tag{25}$$

where $[A]$ and $[A]_a$ are the membrane stiffness matrices of the plate and reinforcement, as well as $[D]$ and $[D]_a$ are the bending stiffness matrices of the plate and reinforcement, respectively.

The RR method is used to perform the buckling analysis. The RR method is based on the principle of minimum potential energy. The potential energy of a system has at equilibrium an extremal value [19]. For the neutral equilibrium the potential energy due to bending (V_T) is balanced by a factor (λ) of the work done by the external loads (W_T). Hence,

$$V_T - \lambda\, W_T = 0 \tag{26}$$

The potential energy due to bending is given by,

$$V_T = \frac{1}{2}\iint_{Area} \{\kappa\}^T [D]\,\{\kappa\}\; dy\; dx \tag{27}$$

where $[D]$ is fully populated.

The work done by the external loads is given by,

$$W_T = \frac{-1}{2}\iint_{Area} \left[\begin{array}{c} N_x\left(\dfrac{\partial w}{\partial x}\right)^2 + N_y\left(\dfrac{\partial w}{\partial y}\right)^2 \\ + 2N_{xy}\dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{array} \right] dy\; dx \tag{28}$$

For the solution procedure, the out-of-plane displacement shape is represented by a double sine Fourier series, since it satisfies the simply supported boundary conditions at the plate edges. Thus,

$$w = \sum_i^m \sum_j^n A_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \tag{29}$$

where A_{mn} are undetermined coefficients.

The critical buckling load is given by the lowest value or critical factor (λ_b), which is obtained by minimizing Eq. (26) with respect to the A_{mn} coefficients. Hence,

$$\frac{\partial}{\partial A_{mn}}(V_T - \lambda W_T) = 0 \tag{30}$$

This provides an eigenvalue problem in λ, where the smallest non-zero solution is the critical factor. Therefore, the critical buckling load is given by,

$$\{N^{cr}\} = \lambda_b\{N\} \tag{31}$$

As the plate consists of three strips, the expression for the potential energy and external work is given as a sum of the potential energy and external work of each of the plate's strips. Hence,

$$V_T = \sum_{i=1}^{3} V_i \tag{32}$$

$$W_T = \sum_{i=1}^{3} W_i \tag{33}$$

The initial and final widths of the plate's strips used for the integration over the plate width, are given by,

$$b_i = \begin{cases} 0 \\ b_{sme}/2 \\ b - b_{sme}/2 \end{cases}$$

and

$$b_f = \begin{cases} b_{sme}/2 \\ b - b_{sme}/2 \\ b \end{cases} \tag{34}$$

with $i, f = 1,2,3$.

Once the critical buckling factor is identified, the buckling constraint is expressed as,

$$1 - \lambda_b \leq 0 \tag{35}$$

4) Practical design constraints

Practical design rules are taken from Ref. [15]. Those constraints are described in the following sections.

Percentages of ply angles

Niu [20] suggested that in composite design at least 10% of each ply angle should be provided. The maximum and minimum percentages of the ply angles for the reinforcement and plate are limited. The percentages of the 0, 90, 45, and -45 degree ply angles for each of those elements are expressed as,

$$p_i = \frac{2t_i}{h}100 \; i = 0,90,45,-45 \,; h = t, t_a \tag{36}$$

The design constraint imposed for the maximum and minimum percentages of the 0, 90, 45, and -45 degree ply angles, is as follows,

$$1 - \frac{p_i^{\max}}{p_i} \leq 0 \tag{37}$$

$$\frac{p_i^{\min}}{p_i} - 1 \leq 0 \tag{38}$$

Plate-reinforcement Poisson's ratio mismatch

The reduction of the Poisson's ratio mismatch is critical in composite bonded structures [20]. The difference between the plate and the reinforcement Poisson's ratio is limited by a tolerance (ζ) to reduce the mismatch. An acceptable value of ζ is assumed to be 0.05.

The Poisson's ratio mismatch design constraint is implemented as,

$$\left|\nu_{xy} - \nu_{xy}^a\right| - \zeta \leq 0 \tag{39}$$

This constraint is applied to the outer strips of the plate for plate design **a**.

4.2 Second level - GA based optimisation

A standard GA [21]-[22] is used to solve the discrete lay-up problem. The GA used has the following operators: initial population, crossover, reproduction, mutation and elitism. Note that the GA is applied separately to each of the strips (plate and reinforcement).

4.2.1 Fitness function

The fitness function used is given by the sum of the square difference between the optimum and actual lamination parameters [18] plus extra penalty terms to account for ply contiguity constraints. Therefore,

$$f(\vec{y}) = \sum_{i=1}^{3} wf_i^A \left(\xi_i^A - \xi_{iopt}^A\right)^2 + \sum_{i=1}^{3} wf_i^D \left(\xi_i^D - \xi_{iopt}^D\right)^2 + \sum_{k=1}^{4} \Theta_k \tag{40}$$

where $\vec{y}$ is the design variable vector or gene representing the lay-up, $wf_i^{A,D}$ are the weighting factors for the lamination parameters and Θ_k are penalty functions terms to limit the number of plies of the same orientation stacked together. The value of Θ_k is 1, when more than 4 plies of the same orientation are stacked together [12] otherwise it is 0.

4.2.2 Design variables - Genes

The design variables are the thickness and the 0, 90, 45 and -45 degree ply angles that constitute the actual lay-ups for the central and outer strips of the plate (plate and reinforcement). Those variables are characterised as chromosomes in genes within the GA. The corresponding encoded chromosomes to ply angles are: 1, 2, 3, 4, 5, 6 and 7 for ± 45, 90_2, 0_2, 45, -45, 90 and 0 degrees, respectively.

The total laminate thickness is given by h, the encoded ply angle is θ and n corresponds to half or half plus one plies depending on whether the skin laminate is symmetric or mid-plane symmetric.

h	θ_1	θ_2	..	..	θ_{n-1}	θ_n

$\theta_i = 1,2,3,4,5,6,7$

Fig. 3. Gene with chromosomes for the laminate.

5 Numerical examples

First of all, the RR method developed in this paper was compared against Finite Element (FE) analysis using MD NASTRAN [23]. Figs. 4 and 5 show the buckling coefficient for a simply supported isotropic and angle ply (45 degrees) plate of design **a** with an aspect ratio (*a*/*b*) of 10, under longitudinal loading. A typical aluminium alloy was used for the isotropic plate with the following properties: $E = 72000$ N/mm^2, $G = 26900$ N/mm^2 and $\nu = 0.3$. The angle ply plate used AS4/3502, which has the following properties: $E_{11} = 127553.8$ N/mm^2, $E_{22} = 11307.47$ N/mm^2, $\nu_{12} = 0.3$, $G_{12} = 5998.48$ N/mm^2. The RR method was used with 400 terms in the double sine series ($m = n = 20$). An FE model of the plate design type **a**, was set up and a linear buckling analysis was performed in MD NASTRAN using SOL 105 [24]. The plate was modelled using quadrilateral elements with four nodes (CQUAD4) and maintaining a minimum of five nodes per half wave length [24]. Membrane and bending properties for the plate and reinforcement respectively were introduced as PSHELL and MAT2 cards. Note that the FE analysis was only performed for reinforcement and plate width ratios (b_a / b) of 0.2, 0.4, 0.6 and 0.8.

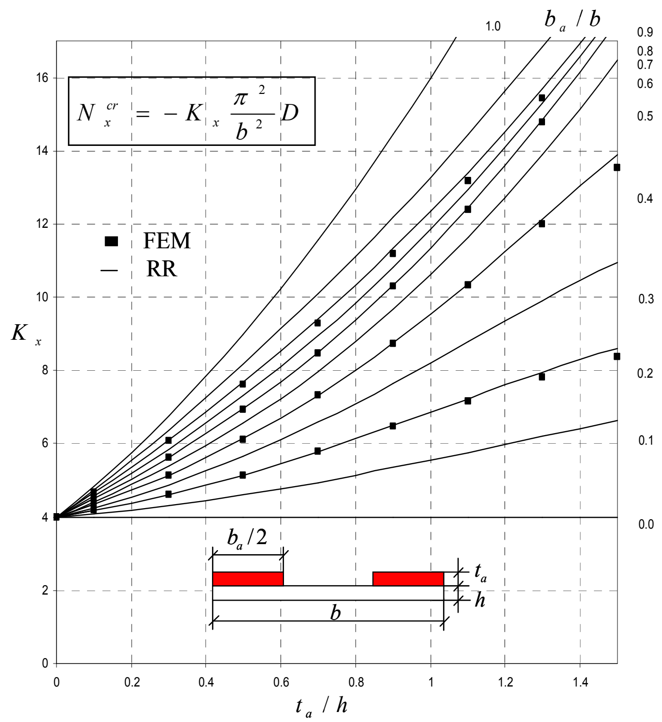


Fig. 4. Buckling coefficient for an isotropic plate design **a** with 4 edges simply supported (*a*/*b* = 10).

The results show an excellent agreement between the RR method and FE analysis, having a maximum difference of approximately 2.8% and 5.6% for the isotropic and anisotropic (angle ply of 45 degrees) plate. It is observed that the difference in results between the RR method and FE increases with anisotropy.

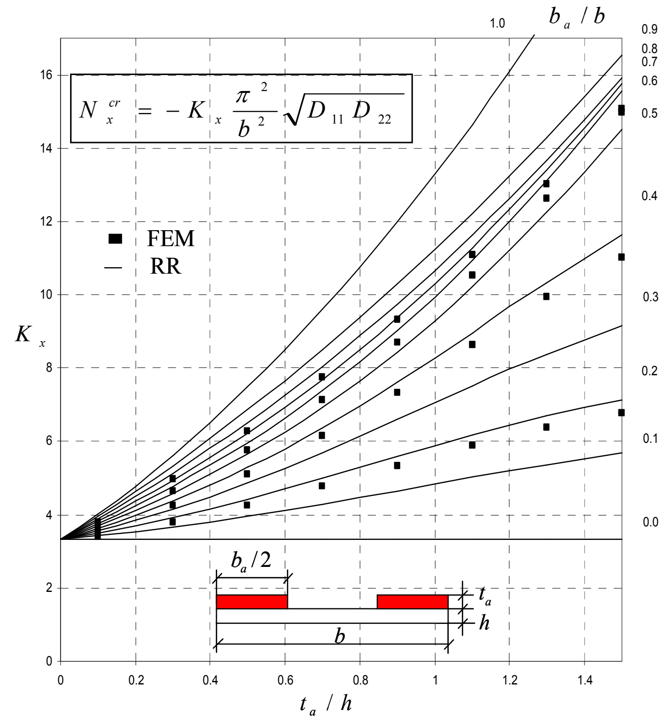


Fig. 5. Buckling coefficient for an anisotropic (angle ply of 45 degrees) plate design **a** with 4 edges simply supported (*a*/*b* = 10).

Ref. [15] was also used to evaluate the RR method. The material was also AS4/3502. Plate design **a** was used as the data was extracted from a stiffened panel. The skin and stiffener flange were modelled as the plate and reinforcement respectively. The length and width of the plate were 762 mm and 203.2 mm respectively. The width of the reinforcement was 60.96 mm. Two cases were considered. The normal forces for cases A and B were $F_x = -381953.21$ N and $F_x = -390266.33$ N respectively. In all cases the shear load was $N_{xy} = -875.63$ N/mm^2. Table 2 shows the plate and reinforcement lay-ups for those cases. Note that the laminate in case B exhibits membrane and flexural anisotropy. Table 3 lists the buckling load factors using the RR method with 400 terms in the double sine series (m = n = 20) and the FE analysis as previously stated.

Examining the buckling load factors from Table 3, it is clearly seen that good agreement is found between the RR method and FE for case A (approx. 3.8% difference). In contrast for case B, the RR method and FE show a difference of approximately 13%.

Table 2. Plate designs **a** properties ($_S$ –symmetric, $_{MS}$ – mid-plane symmetric).

Case	Lay-up-[0%/45%/-45%/90%]
A	Plate (32 plies)-[0/37.5/37.5/25] $[\pm45/90_4/(\pm45)_5]_S$ Reinforcement (68 plies)-[47/26.5/26.5/0] $[(\pm45)_4/0_2/(\pm45)_2/(0_4/\pm45)_3/0_2]_S$
B	Plate (33 plies)-[0/42/21/36] $[\pm45/90_4/45/90_2/45_2/(\pm45)_2/45/-45]_{MS}$ Reinforcement (67 plies)-[58/21/21/0] $[(\pm45)_3/0_2/45/(0_2/\pm45)_2/0_4/45/(0_4/-45)_2/0/0]_{MS}$

Table 3. Buckling load factors comparison

Case	FE	RR
A	1.04	1.08
B	1.08	1.22

Differences between analyses are expected especially when laminates contain anisotropy as the convergence of the RR method toward the actual solution will be slow. This is because the shape chosen for the out-of-plane displacement (Eq. (29) only satisfies the geometric boundary conditions but not the natural boundary conditions of the problem. Note that the FE estimation of the actual solution is mesh and element type related. Nevertheless, the RR method is easy to encode and faster than FE. So for initial sizing optimisation where design evaluation is more important than analysis accuracy the RR method is a good option.

Next, the two level approach was applied to optimise a plate under strength, buckling and practical design constraints. The plate length and width were 800 mm and 200 mm, respectively. The plate and reinforcement were made of AS4/3502 with a ply thickness of 0.132 mm and a density of 1.578 10^{-6} kg/mm^3. The plate has to withstand a normal load in compression of 350 kN, a transverse running load in compression of 100 N/mm and a shear load of -250 N/mm. The plate strain cannot exceed ±3600με in the longitudinal and transverse directions, and ±7200με in shear. For design practice the minimum width of the outer strip cannot be less than 50 mm and laminates must contain at least 10% of each ply angle. At the first level, gradient based optimisation was carried out under strength, buckling and practical design constraints. At the second level, a GA code was used with a population of 40, 200 generations, a 0.7 probability of crossover, a 0.05 probability of mutation, with all weighting factors for the lamination parameters equal to 1, ply contiguity constraints and locating at least one set of ±45 degree plies at the outer surface of the plate and reinforcement laminates. The weights, load factors and dimensions for the different plate designs are shown in Table 4. Table 5 lists the lay-ups for the three plate designs.

Table 4. Continuous (W_c) and discrete (W_d) weights, load factors and reinforcement widths for the optimum plate designs. FE buckling load factors are shown in brackets.

Plate design	W_c/W_d [kg]	λ_b	λ_s	b_{sme} [mm]
a	1.57/ 1.62	1.07 (0.98)	0.99	50.01
b	1.56/ 1.69	1.22 (1.18)	0.97	62.95
c	1.51/ 1.58	0.98 (0.95)	1.03	50.21

Table 5. Lay-ups for the optimum plate designs ($_S$ – symmetric, $_{MS}$ – mid-plane symmetric).

Plate design	Lay-up-[0%/45%/-45%/90%]
a	Plate (32 plies)-[31/31/19/19] $[\pm45/45_2/90/(\pm45)_2/0_3/90_2/0_2]_S$ Reinforcement (67 plies)-[64/12/12/12] $[\pm45/0_2/90/(\pm45/0_2)_2/0_290_2/0_4/-45/0_4/45/0_4/90/0/0]_{MS}$
b	Plate (14 plies)-[14/58/14/14] $[\pm45/90/45_3/0]_S$ Reinforcement (72 plies)-[64/11/11/14] $[\pm45/90/\pm45/0_4/90_2/0_4/45/0_2/\pm45/0_3/-45/(0_4/90)_2/0_2]_S$
c	Plate (32 plies)-[31/19/12/38] $[\pm45/45/0/45/(90_2/0_2)_2/90_2/-45]_S$ Reinforcement (93 plies)-[65/11/13/11] $[(\pm45/0_3)_2/0/-45/0_4/\pm45/0_2/45/0_4/90/0_4/\pm45/0_4/-45/90_2/0_4/90_2/0/0]_{MS}$

From the results, it is clearly seen that the lightest and the heaviest designs correspond to plate design **c** and **b**, respectively. However, the plate design **c** does not maintain continuity in plies between the outer and central plate's strips. A weight penalty for maintaining continuity in plies across the width are approximately 2.5% and 7% for plate designs **a** and **b**, respectively.

It is observed, as one might expect, that the reinforcement tends to have high percentages of 0 degree plies to transfer axial loading whereas the plate consists of more ±45 (unbalanced) and 90 degree plies to improve buckling behaviour under combined loading. Note that the plate laminates contain membrane and flexural anisotropy. In this case and as it is shown in Ref. [15] material anisotropy and hence elastic tailoring is used to an advantage.

Furthermore, it is observed that the RR and FE buckling load factors do not show significant differences (max. approx. 9%).

Conclusion

A method to optimise anisotropic composite plates that vary in thickness and properties across the width has been presented. The optimisation problem is divided into two levels. At the first level, MP is applied, where the plate is divided into several strips, each of them modelled using lamination parameters accounting for their anisotropy. The laminate of each of the strips is assumed to be symmetric, or mid-plane symmetric with 0, 90, 45 or -45 degree ply angles. The plate is subjected to a combined loading under strength, buckling and practical design constraints. Three types of plate configuration are considered depending on manufacturing or design requirements. The buckling behaviour is assessed by the RR method taking into account variation in thickness and properties across the width. At the second level, the actual lay-ups of the plate's strips are obtained using a GA bearing in mind design practices.

The two level approach has been applied to optimise a plate with a reinforcement subject o combined loading and under strength, buckling and practical design rules. It has been shown that the lightest plate design corresponds to a plate that does not maintain continuity in plies between adjacent strips. The least weight penalty for keeping this continuity is approximately 2.5% and corresponded to plate design **a**.

The RR method herein developed captures the buckling behaviour of a plate that varies in thickness and properties across the width or has a reinforcement. It can be applied to produce design charts as shown in Fig. 4-5. Additionally, it can be applied to stiffened panels to evaluate the local buckling behaviour of the skin when the stiffener flanges act as a reinforcement.

Furthermore, elastic tailoring is used to an advantage.

Acknowledgements

The authors thank the EC for the Marie-Curie Excellence Grant MEXT-CT-2003-002690.

References

[1] Capey, E. C. "The buckling under longitudinal compression of a simply supported panel that changes in thickness across the width". Ministry of Supply, Aeronautical Research Council, C. P. No. 235, London, 1956.

[2] Benthem, J. P. "The buckling load of flat panels that change in thickness across the bay". Nationaal Luchtvaartlaboratorium, National Aeronautical Research Institute, Amsterdam, 1958. NLL-TR S. 527.

[3] Ashton, J. E. and Waddoups, M. E. "Analysis of anisotropic plates". *Journal of Composite Materials*, Vol. 3, pp. 148-165, 1969.

[4] Ashton, J. E. and Whitney, J. M. "*Theory of laminated plates*". Technomic Publishing Co., Inc., 750 Summer St., Stamford, Conn. 06901, 1970.

[5] Levy, R. and Ganz, A. "Analysis of optimized plates for buckling". *Computers & Structures*, Vol. 41, No. 6, pp. 1379-1385, 1991.

[6] Levy R. "Rayleigh-Ritz optimal design of orthotropic plates for buckling". *Structural Engineering & Mechanics*, Vol. 4, No. 5, pp. 541-552, 1996.

[7] Gutiérrez, R. H, and Laura, P. A. A. "Fundamental frequency of transverse vibration of a rectangular, anisotropic plate of discontinuously varying thickness". *Journal of Sound and Vibration*, 248(3), pp. 573-577, 2001.

[8] Weaver, P. M. "On optimisation of long anisotropic flat plates subject to shear buckling loads". *45th AIAA/ASME/ASCE/AHS/ACS Structures, Structural Dynamics & Materials Conference*, Palm Springs, California, 19-22 April 2004.

[9] Weaver, P. M. "Approximate analysis for buckling of compression loaded long rectangular plates with flexural/twist anisotropy". *Proceedings of the Royal Society of A: Mathematical, Physical and Engineering Sciences*. Vol. 462, No. 2065, 2006.

[10] Tsai, S. W., Halpin, J. C. and Pagano, N. J. "*Composite materials workshop*". Technomic publishing Co., Inc., 750 Summer St., Stamford, Conn. 06902. pp. 233-253. 1968.

[11] Miki, M. and Sugiyama, Y. "Optimum design of laminated composite plates using lamination parameters". AIAA-91-0971-CP, 1991.

[12] Haftka, R. T. and Walsh, J. L. "Stacking sequence optimisation for buckling of laminated plates by integer programming". *AIAA Journal*, Vol. 30, No. 3, pp. 814-819, 1992.

[13] Nagendra, S., Haftka, R. T. and Gürdal, Z. "Stacking sequence optimisation of simple supported laminates with stability and strain constraints". *AIAA Journal*, Vol. 30, No. 8, pp. 2132-2137, 1992.

[14] Fukunaga, H., Sekine, H., Sato, M. and Iino, A. "Buckling design of symmetrically laminated plates using lamination parameters". *Computers and Structures*, Vol. 57, No. 4, pp. 643-649, 1995

[15] Herencia, J. E, Weaver, P. M and Friswell, M. I, "Local optimisation of long anisotropic laminated fibre composite panels with T shape stiffeners". *47th AIAA/ASME/ASCE/AHS/ ACS Structures, Structural Dynamics & Materials Conference*, Newport, RI, 1-4 May 2006.

[16] Jones, R. M. "*Mechanics of composite materials*". 2nd edition. Taylor and Francis Inc., 325 Chestnut street, Philadelphia, PA 19106. pp. 26-31. 1999.

[17] MATLAB software V.7.1. The MathWorks, Inc., 1994-2006.

[18] Diaconu, C. G. and Sekine, H. "Layup optimisation for buckling of laminated composite shells with restricted layer angles". *AIAA Journal*, Vol. 42, No. 10, pp. 2153-2163, 2004.

[19] Lekhnitskii, S. G. "*Anisotropic plates*". Gordon and Breach, Science Publishers, Inc., 150 Fifth Avenue, New York, N. Y. 10011. USA, 1968.

[20] Niu, C. Y. M. "*Composite airframe structures-Practical design information and data*". Hong Kong Conmilit Press Limited. 1992.

[21] Coley, D. A. "*An introduction to genetic algorithms for scientist and engineers*". World Scientific Publishing Co. Pte. Ltd., P O. Box 128, Farrer Rd, Singapore 912805, 1999.

[22] Gürdal, Z., Haftka, R. T. and Hajela, P. "*Design optimization of laminated composite materials*". John Wiley and Sons, Inc. 1999.

[23] MD NASTRAN 2006r1. MSC Software. 2006.

[24] MSC/NASTRAN Linear static analysis, User's guide, Chapter 13, Linear buckling, 2003.