IMPROVED GLOBALIZED NELDER-MEAD METHOD FOR OPTIMIZATION OF A COMPOSITE BRACKET

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Abstract

The optimized design of composite structures requires solving simultaneously both structural and manufacturing problems. It is a difficult task since the objective functions do not have closed form solutions and have multiple local optima that calls for a global search. This paper improves a global search based on several restarts of the Nelder-Mead method, called "Globalized Bounded Nelder-Mead" [GBNM] method [1]. Two issues are addressed here: first, the restart procedure is improved by using a one-dimensional adaptive probability function, and second, nonlinear constraints are included by projecting the infeasible points onto the nonlinear constraints. The improved procedure is more efficient in terms of computational time and probability of finding the global minimum. The improved GBNM is applied to the simultaneous structural and manufacturing design of a Z-shaped composite bracket. Result confirms the proposed approach is more efficient than an evolutionary algorithm in the simultaneous optimization of composite design.

1 Introduction

Composite materials open a new window in engineering by providing excellent mechanical properties. However, this feature is accompanied by the complexity of the design problem, especially when the manufacturing aspects are involved in the structural design. Among difficulties involved in this field is multiplicity of the local solutions that calls for a global optimization.

High computational cost is the main drawback to a global optimization. Several research papers are devoted to speeding up a global search by embedding an efficient local algorithm into a global one. One of these attempts by Luersen et al. [1] is devoted to combine the Nelder-Mead [N-M] method [2] to a random search.

The local-global search proposed by Luersen is called Globalized Bounded Nelder-Mead [GBNM] method. GBNM repeatedly restarts a local search from new points using a probability function. The probability function keeps a memory of past local searches and forces the algorithm to restart the search from regions far from already-known solutions.

It is shown that the restart procedure used by Luersen is computationally time consuming and is not always successful in finding the optimum solution. In this paper, a new restart procedure is introduced to improve the probability of finding the global minimum and to reduce the computational time. In Addition, a backtracking procedure is proposed to incorporate nonlinear constraints into the design problem.

This paper is divided into two main parts: a) improvement and test of the optimization procedure; b) application of the improved method to simultaneous structural and manufacturing optimization of a Z-shaped composite bracket.

2 Optimization Procedure

A global search can be performed by repeatedly restarting a local optimization from different initial points. To avoid finding the same local optima, new initial points should not be close to the previous ones. The restart procedure used for this purpose gives points far from previous local optima and previous initial points more probability to be selected as initial points for the next local search. The restart procedure is shown in Fig. 1.

The following sections propose two improvements to the Luersen's GBNM. The first

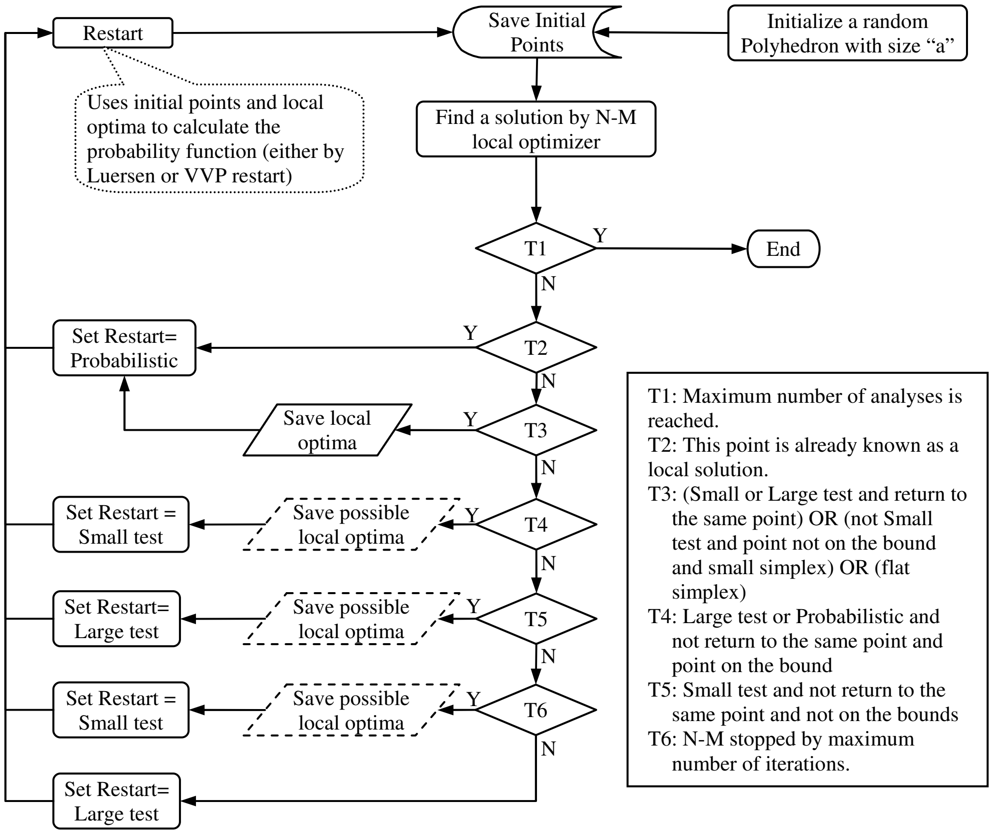


Fig. 1. Restart and convergence tests linking in GBNM

deals with the restart procedure in section 2.1. The second, described in 2.2, addresses the issue of how to include nonlinear constraints in the optimization algorithm.

2.1 Improved Restart Procedure

Luersen used a Multi-Dimensional Probability [MDP] density to assign the sampling probability of a point. It randomly selects N_r points and calculates the probability function by MDP. The point with highest probability value is used as an initial point for the next local search. The probability distribution achieved by this procedure is not equivalent to a normal distribution. It is strongly dependent on the number of points (N_r) and is independent of the probability density function. Luersen restart is computationally expensive because of the computational time needed for MDP.

Here, we propose, an adaptive probability density called the Variable Variance Probability [VVP]. The new probability function is based on the minimum distance to the points already sampled and represented as:

$$\phi(x) = \frac{1}{\sqrt{2\pi}\sigma}(1 - e^{-(d_{\min}{}^2/2\sigma^2)}) \qquad (1)$$

$$d_{\min} = \min_{i=1,\dots,m} d_i = \sqrt{\sum\nolimits_{k=1}^{n} \left(\frac{x_{k,i} - x_k}{x_{ku} - x_{kl}}\right)^2} \qquad (2)$$

where, $\phi(x)$ is the sampling probability of point x, n is the number of design variables, x_i, $_{i=1,\dots,m}$ are already-known solutions and previous initial points. d_i is the non-dimensional distance between point x and point x_i. The variance of the normal probability density is updated in each restart by $\sigma = (3\sqrt[n]{m})^{-1}$. It gradually decreases when the number of sampled points is increased.

The VVP restart also uses a selection procedure different than that of Luersen restart. In new selection procedure, N_r points are randomly selected to create a selection pool, which is a set of points whereby each has a number of copies proportional to its probability value. A new point is randomly selected from this pool.

2.2 Non-linear Constraints

N-M method is originally introduced for unconstrained optimization, but the variables in an engineering problem are usually constrained not only by upper and lower bounds on variables (i.e. box constraints) but also by nonlinear constraints.

GBNM uses a projection procedure on the box constrained variables. Projection of variables is mathematically specified by:

$$x_i = \begin{cases} x_i^{lower\,bount} & if \;\; x_i < x_i^{lower\,Bound} \\ x_i^{upper\,bound} & if \;\; x_i > x_i^{upper\,bound} \end{cases} \qquad (3)$$

GBNM, as used by Luersen, can not deal with nonlinear constraints; however, the variables in a composite optimization problem are often constrained by nonlinear functions, e.g. strength constraints. Identical to the projection of box-constrained variables, a projection procedure is used for nonlinear constraints. The projection of nonlinear constraint includes a backtracking procedure illustrated in pseudo code in Fig 2. This procedure shifts the infeasible point toward the original feasible one, until it reaches the boundary of the feasible region. Thus, it guarantees the feasibility of the final solution.

```
Input α, n, x_new, x_f
If x_new is feasible
        Return x_new;
Set i=0
Repeat
        Set x_new=x_f+α(x_new-x_f)
        Set i=i+1;
Until ( i>n) or (x_new is feasible)
If i>n
        Set x_new=x_f
Return x_new
```

Fig. 2. Backtracking procedure projects an infeasible point onto the nonlinear constraints

2.3 Mathematical Test Functions

Three mathematical test functions are used in this section to assess the improvement suggested in previous sections. The first is Griewank's test function [1], with the global minimum of -1 and several local minima.

$$A_1(x) = \sum_{i=1}^{n} \frac{x_i^2}{400n} - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right) \qquad (4)$$
$$x_i \in [-1000, 1000]; \quad i = 1, \ldots, n$$

Next is Ackley's test function [8], with the global minimum of -21.80 and several local minima.

$$A_2(x) = -20e^{-\frac{1}{5}\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}} - e^{-\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)} \qquad (5)$$
$$x_i \in [-15, 30], \quad i = 1, \ldots, n$$

The last is a combination of six-hump camelback [1] test function, with the global minimum of -6.18 and 6^6 local minima.

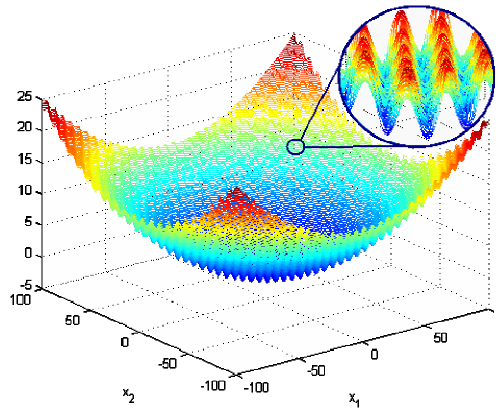


Fig. 3. Griewank's test function (A_1) with two design variables

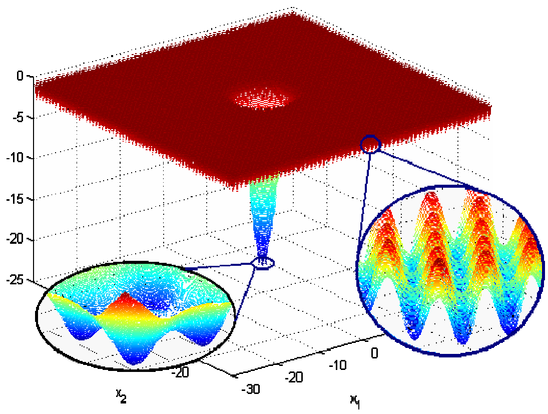


Fig. 4. Ackley's test function (A_2) with two design variables

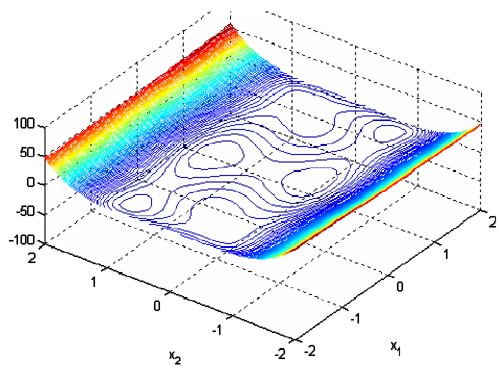


Fig. 5. Six-hump camelback test function (*cb(x)*) with two design variables

$$A_3(x) = \sum_{i=1}^{n} cb(x_{2i-1}, x_{2i}),\ x_i \in [-2,2]\ \ i = 1,\ldots,n; \qquad (6)$$

$$cb(x) = 4x_1^2 - 2.1x_1^4 + \tfrac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4 \qquad (7)$$

Figs. 3 to 5 show the test functions in a two dimensional design space. However, to test the optimization procedure, these functions are optimized with 12 design variables.

2.4 Optimization Set up

The control parameters in N-M optimization procedure are set to be 1, 2, 0.5, and 0.9, respectively for reflection, expansion, contraction, and shrink coefficients [2]. The N-M method is terminated when the simplex is small or flat, or when the maximum number of iterations is reached. A simplex is small when:

$$\underset{i=1,\ldots,n}{Max}\left(\left| \frac{x_i^{k+1} - x_i^k}{x_i^u - x_i^l} \right| \right) \le \varepsilon_1 \qquad (8)$$

where, k is the number of iterations, subscripts "*u*", and "*l*" represent the upper and lower bound on variables x_i. ε_1 is a predetermined small number. Similarly, the simplex is called flat when:

$$| f_H - f_L | \le \varepsilon_2 \qquad (9)$$

where, f_H and f_L are the highest and lowest function values at the current simplex and ε_2 is a given small number.

The N-M's initial simplex is a polyhedron with the edge size of 20% of the design space. The small-test and large-test in Fig. 1 restart the N-M method from the best point of the current simplex with a polyhedron of size 2% and 10% of the domain size. N_r in selection procedure is set to 10, and α and *n* in the backtracking procedure are set to be 0.9 and 10, respectively. A genetic algorithm with the population size of 20 and crossover and mutation fraction of 0.8 and 0.01 is used for comparison.

2.5 Numerical Results

Three test functions described in section 2.3 with 12 variables are minimized using GBNM with random restart, Luersen restart, and VVP restart. The optimization is conducted for *1000*, *5000*, and *10000* iterations. Results are compared to those obtained by the genetic algorithm.

Fig. 6 compares the probability of finding the global minimum by each algorithm. The figure shows the average value of 100 runs for three test functions. Within *1000* iterations, all restart procedures perform almost similarly. But, when the number of iterations is increased, the difference in performance is noticeable. VVP restart shows more probability to find the global minimum with the same number of iterations.

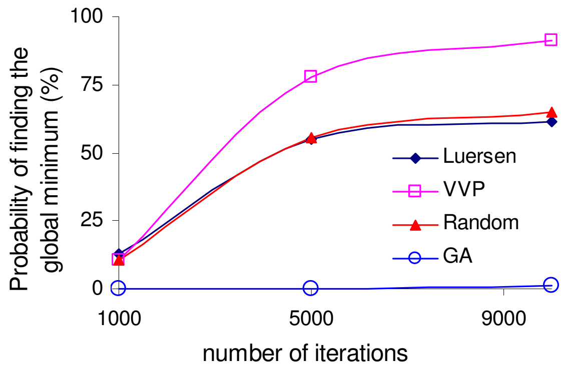


Fig. 6. Average probability of finding the global solution in 100 runs for three test functions

Fig. 7 plots the average runtime for each algorithm on the three test functions. It shows that Luersen restart is about 5 to 10 times slower than the random and the VVP restarts. Furthermore, computational time of the VVP and random restarts increases linearly with the number of iterations, although time needed for Luersen restart is increased exponentially.

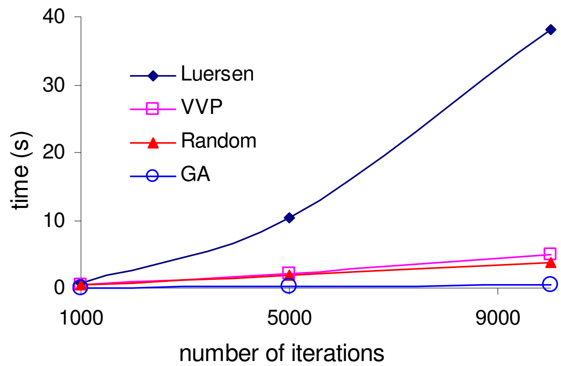


Fig. 7. Average computational time of GBNM with different restart procedures and GA

A GA is applied to all the test functions with the same number of function analyses. In all cases, GBNM finds a better solution compare to the GA, see Fig. 6. The computational time is not valid for comparison, because the two optimization methods have different operators. In practical problems, the computational time is strongly related to the number of function analyses.

To test the nonlinear constraint handling procedure, two hyper-spheres are added as constraints to the third test function (A_3). Points within the two hyper-spheres are feasible. The GBNM with VVP restart and backtracking procedure is applied to this test function with two and 12 variables. With two variables, all the local minima found within 1000 iterations. With 12 design variables, the constrained problem is more challenging since the number of local optima is increased from 6^6 to 9^6. But the solution found is feasible and sufficiently close to the global one (i.e the minimum found is -4.83 compare to the global minimum of -5.45). The results are not compared with GA or GBNM with Luersen restart because neither of these algorithms is able to directly work with nonlinear constraints.

It is shown that GBNM can generally find a better solution than an evolutionary optimizer like a GA. Among three restart procedures, VVP restart is faster and it can find a better minimum for a general multimodal function. With the backtracking procedure, GBNM can also deal with nonlinear constrained problems.

3. Composite Design Problem

Finding the optimum structural design of composite parts is a difficult task due to the high degree of freedom in tailoring material properties and shape design. A variety of optimization methods, from simple mathematical methods, such as a linear programming, to combinations of computationally expensive methods, such as GA and topology optimization, has been used for this purpose. The GA is among the most popular ones in this field [6] because of capability of global optimization and independency to gradient information.

The optimum design of a composite part is a trade-off between structural and manufacturing aspects. Composite designers usually simplify the problem by separating the two parts [7] and performing the process tuning after the structural design. But, there are several researches [7-9] confirming the approach to the design and optimization of composite materials must be multidisciplinary. Such an approach, called simultaneous optimization, is studied in this paper.

Simultaneous optimization is more complex than the separate design, because it requires taking into account a large number of variables and local minima. Figs. 8 to 10 show the structural and manufacturing objectives of a two-layer rectangular plate made of laminated composite materials. Maximum structural objective (Fig. 8) corresponds to the maximum strength. Maximum manufacturing

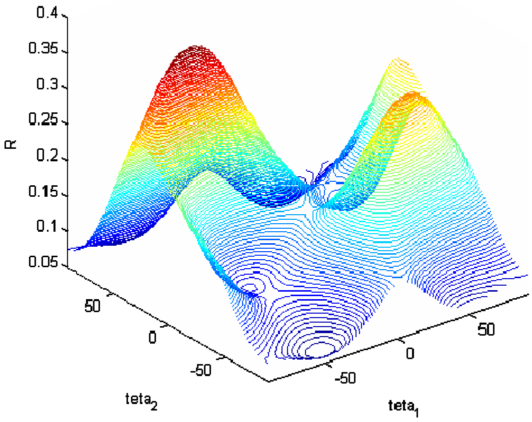


Fig. 8. Structural objective of two-layer laminated composite plate

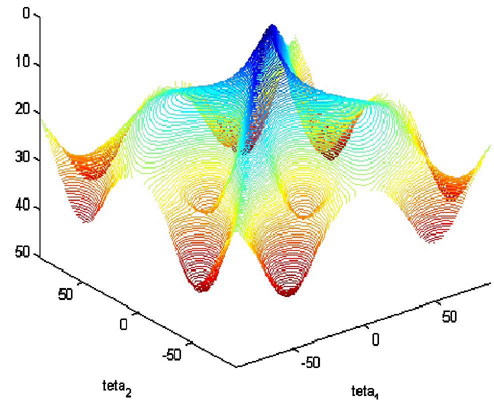


Fig. 9. Manufacturing objective of two-layer laminated composite plate

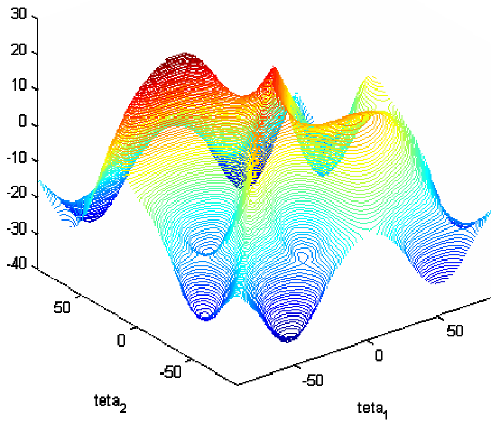


Fig. 10. Simultaneous structural and manufacturing objective of two-layer laminated composite plate

Table 1. Structural, geometrical, and manufacturing design variables, objectives, and constraints

	Structural objectives/constraints				Manufacturing objectives/constraints	
Design variable	min: W	Max: R	$\delta_y \leq 1mm$	D ≥2	$\lvert\Delta\theta\rvert \leq 0.5^{\circ}$	min : $\lvert\Delta\theta\rvert$
θ_i $_{i=1,\ldots,4}$	○[a]	●[b]	●	●	●	●
E	●	●	●	●	●	●
S	●	●	●	○	●	●
R	●	●	●	●		

a: ○= Indirect or weak interconnection; b: ●=Direct and strong coupling;

objective (Fig. 9) corresponds to the minimum warpage and maximum permeability. The simultaneous objective function illustrated in Fig. 10 shows that the best simultaneous design (i.e. maximum objective value) is neither the best manufacturing nor the best structural design, but it is a trade-off that lies between them.

There are only a few papers working on simultaneous optimization of composite materials, and most of them use genetic algorithm [8-9], despite its low convergence rate and inherently high computational cost. Section 2 has shown that the GBNM is generally faster than an evolutionary algorithm for small number of function analyses. In this section the improved GBNM, described in preceding sections, is used for simultaneous optimization of a composite bracket.

3.1 Composite Bracket Design Problem

The Z-shaped composite bracket shown in Fig. 11 is made of 16 plies balanced symmetric laminate of graphite/epoxy (AS4/8552) with fiber orientation of $[\pm\theta_1/\pm\theta_2/\pm\theta_3/\pm\theta_4]_s$. The optimization problem is to find the optimum value of the objective functions shown in the first row of the Table 1. This table also shows the strong/weak effect of the design variables on the objectives by a solid/non-solid circle in the corresponding cell.

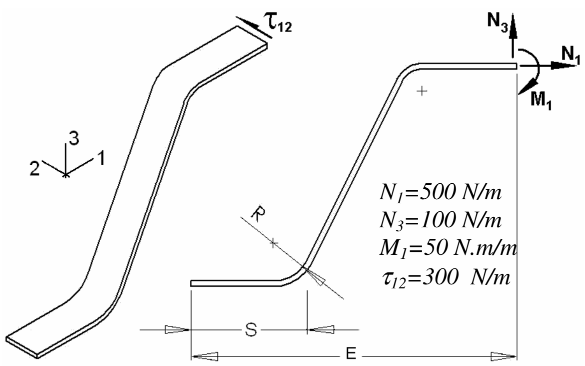


Fig. 11. Geometrical variables and applied loads on the composite bracket

The problem has nonlinear constraints because it should not fail or delaminate anywhere within a safety factor of 1.5 and 2, respectively. Delamination is calculated in the curved regions where the angle shape causes high interlaminar normal stresses. The vertical deflection of less than $1mm$ and the spring-in of less than $0.5°$ are strictly required for an acceptable design.

3.2 Problem Simulation

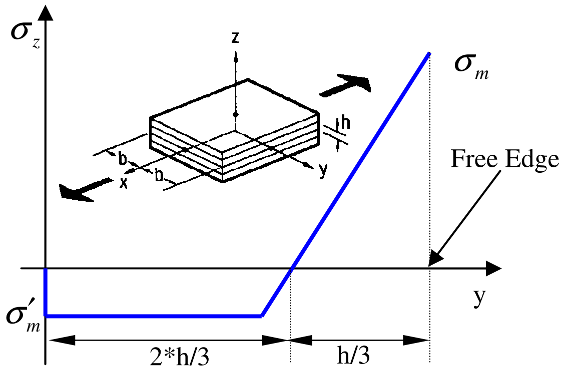
To evaluate the objective function, an appropriate processing and structural simulation is required. A semi-analytical model is developed in MATLAB© for quick evaluation of the objectives and constraints.

Objectives of the coupled problem include: failure index, vertical deflection, and spring-in. Failure index (R) is calculated by first ply failure in classical lamination theory and Hashin stress criterion [10]. Vertical deflection is calculated by a numerical integration and energy method [11]. Finally, spring-in, which is the angular deformation of a part after demoulding, is a function of cure shrinkage and thermal expansion, and is given by:

$$\Delta\theta = \theta\left[\left(\frac{(\alpha_l - \alpha_t)\Delta T}{1+\alpha_t \Delta T}\right) + \left(\frac{\phi_l - \phi_t}{1+\phi_t}\right)\right] \qquad (10)$$

where, θ is angle of the bracket, $\Delta\theta$ is spring-in, ΔT is temperature change which is the difference between the cure temperature and room temperature. ϕ and α are coefficients of shrinkage and thermal expansion, subscripts "l" and "t" respectively stand for longitudinal and through thickness direction.

Delamination is a critical mode of failure in composite materials, and it is due to the interlaminar stresses. In a flat plate, interlaminar stresses are created only by the free-edge effect [12], but in a curved part, the 3D stress field also creates significant interlaminar stresses. Here, delamination due to free-edge effect is calculated based on



$$\sigma'_m = \tfrac{1}{5}\sigma_m,$$
$$\sigma_m = \sigma_m(z) = 45M(z)/14h^2,$$
$$M(z) = \int_z^h \sigma_y(\xi)(\xi - z)d\xi,$$
$$\sigma_y(z) = \text{Transverse stress}$$

Fig. 12. Approximate distribution of interlaminar normal stress due to free-edge effect [12]

Pagano's model [12]. In this model the interlaminar normal stress is estimated as shown in Fig. 12.

Interlaminar normal stresses created by the angle-shape effect are shown in Fig. 13. Sequentially solving the equilibrium equations for all layers, starting form the innermost layer results in interlaminar normal stress between layers n and $n+1$:

$$\sigma_{z,n-(n+1)} = \frac{\sum_{k=1}^{n} \sigma_{x,k} t_k}{R + \sum_{k=1}^{n} t_k} \tag{11}$$

where, t_i shows the thickness of i^{th} layer, and R is the inner radius of the curved part. If off-axis stresses change along the curve, Eq. (11) would be valid only for a differential angle $d\theta$.

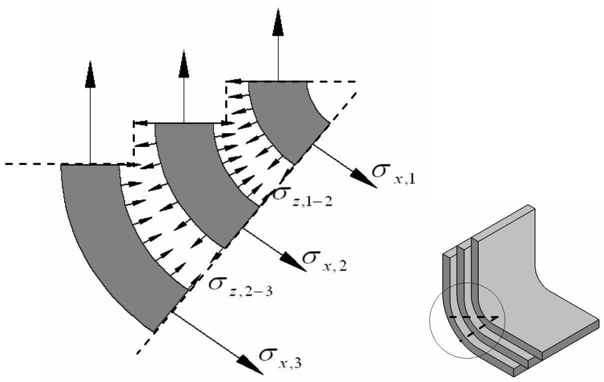


Fig. 13. Normal stresses in an angle shaped part

Interlaminar shear stresses are of minor importance with respect to interlaminar normal stresses, thus an approximation of shear stress in a prismatic member under a transverse load is used.

The semi-analytical models of the first ply failure, delamination, deflection, and spring-in will be used during the optimization process in the next section.

3.3 Numerical Results

The improved GBNM is applied to the simultaneous structural and manufacturing design of the bracket. The following weighted summation of the objectives is used as a cost function to be minimized:

$$\min\ F(x) = \alpha\left(\frac{Weight}{0.005} - \frac{R}{1.5}\right) + \beta\left(\frac{|\Delta\theta|}{0.25}\right); \quad \alpha, \beta \in \{0,1\}$$
$$S.T. \begin{cases} R \geq 1.5 \\ D \geq 2.0 \\ S_r \geq 0.01m \\ \alpha(|\delta| - 0.001m) \leq 0 \\ \beta(|\Delta\theta| - 0.5^\circ) \leq 0 \end{cases} \tag{12}$$

where R and D are delamination and load factor. S_r is the shoulder length after applying the fillet. δ is the vertical deflection in meters. $\Delta\theta$ is the spring-in expressed in degrees. α an β are two dimensionless factors defining the relative importance of structural and processing objectives. The optimum found by $\alpha = 1$ and $\beta = 0$ only considers the structural objectives and constraints, thus called the structure-only design. In contrast the case with $\alpha = 0$ and $\beta = 1$ is the manufacturing-only design. Optimization with $\alpha = \beta = 1$ is called a simultaneous design. Here, α and β are restricted to 0 and 1, but in general, a designer can set them to any real value by considering the relative importance of structural and manufacturing design.

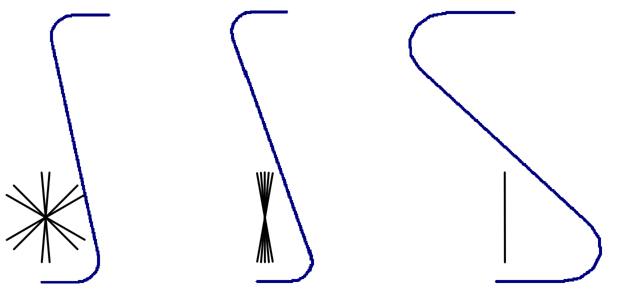


Fig. 14. Optimum geometry of the bracket and fiber orientation of the laminate

Table 2. Optimum design of the composite bracket obtained by improved GBNM after 5,000 iterations

		Structure-Only	Simultaneous	Mfg-Only
Fiber Orientation		$[\pm5/\pm45/\pm60/\pm60]_s$	$[10/0_4/5_8]_s$	$[0_8]_s$
Geometry	E (mm)	21	10	6
	S (mm)	20	20	50
	R (mm)	8	7	12
Structural Objectives/Constraints	Weight (gr)	6.83	6.91	9.34
	Deflection (mm)	-0.98	-0.24	0.141
	Load Factor (R)	2.17	1.74	1.55
	Delamination Factor	12	23	40
Mfg. Obj./Cons.	Spring-in	-0.27	-0.014	-0.005

Since the problem includes variables of different kinds and scales, the variables are normalized to their design domain. The stopping criteria are set to be one percent of the smallest discrete portion of the design domain for the small simplex and 10^{-4} for a flat simplex. The optimization procedure is performed up to *5000* iterations. The optimum point obtained in continuous optimization process is rounded off toward the near discrete value. Table 2 and Fig. 14 show the best solutions to the structural-only, manufacturing-only, and simultaneous design problem.

Table 2 shows that the simultaneous design is a trade-off between the structure-only and the manufacturing-only design. The geometry, fiber orientation, and the performance criteria of the simultaneous design are a trade-off between the two other designs. The simultaneous approach obtains a solution that is 18% better than the structural-only design in manufacturing aspect and it losses only 1.18% of structural performance.

The genetic algorithm needs a penalty function and its performance depends on the appropriate selection of penalty factor. Here, a GA with different penalty functions is used, and the best result obtained is compared with the improved GBNM. After *2000* function analyses, solution by the improved GBNM is 17% better than GA. The difference is reduced to 10% after *5000* function analyses, but still GBNM finds a better design.

Table 2 shows only the best solution, although there are around 50 other local solutions found during the optimization process that a designer can select from. In this respect, this optimization procedure is comparable to an evolutionary procedure that provides a family of optimal solutions instead of just one specific solution. This feature is important, especially for multi-objective optimization.

The composite bracket is optimized for structural, manufacturing, and simultaneous objectives. The result attests to the necessity of the simultaneous approach for composite material design. It also confirms that for small of number of function analyses the improved GBNM is more efficient than a common evolutionary algorithm, both in terms of time and the optimal solution found.

4. Conclusion

A local-global search based on several restarts of the N-M local optimizer is introduced. The VVP restart is presented to improve the performance of this local-global search. Different mathematical functions are used to show that the VVP restart generally performs better than Luersen and random restarts and even an evolutionary algorithm like GA. The computational cost of VVP restart is much less than that of Luersen restart. A backtracking procedure is also presented and tested to incorporate nonlinear constraints into the design problem. The resulting algorithm can work on a problem with nonlinear constraints. It is simple to use and is capable of being terminated at any time. The numerical results show that it is generally faster than an evolutionary algorithm for a small number of function analyses. Finally, it provides a family of solutions (local optima) instead of just one specific solution.

The developed optimization procedure is applied to simultaneous structural and manufacturing optimization of a Z-shaped composite bracket and compared with genetic algorithm. The observed trade-off between structural and manufacturing optimum designs confirms the use of a simultaneous approach in this field. The proposed procedure performed better than an evolutionary algorithm on this type of problem by providing a better solution with the same number of function analyses.

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