CROSS SECTIONAL PROPERTIES OF THIN-WALLED COMPOSITE BEAMS

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Abstract

In the design of composite sections beam theories are used, which require the knowledge of the cross sectional properties, i.e. the bending-, the shear-, the torsional-, the axial stiffnesses and the coupling terms. In the classical analysis the properties are calculated by assuming kinematical relationships, e.g. cross sections remain plane after the deformation of the beam. These assumptions may lead to inaccuracy or to contradictory results. In the paper a new theory is presented, in which no kinematical assumption is applied, rather the properties are derived from the accurate (three dimensional) equations of beams using limit transition. The theory includes the shear displacements both in the in-plane and in the torsional deformations, and it is applied both for open and for closed cross sections.

1 Introduction

Fiber reinforced plastic (composite), thin-walled beams are widely used in the aerospace industry and are increasingly applied in the infrastructure.

In beam theories the stresses and strains of an arbitrary point of the cross section is calculated from the displacements of the beam's axis. To reach this relationship the displacements of the axis are defined, and kinematical assumptions are made.

For example, when a *beam deforms only in a plane* (e.g. in the *x-z* plane), in the classical beam theory [1], (when the shear deformation is neglected), only the displacement of the axis in the *z* direction (w) is needed to calculate the strains and deformations of any point of the cross section. When the shear deformation is taken into account, according to Timoshenko's beam theory [2], two displacement functions of the axis are required: the displacement perpendicular to the axis (w) and the rotation of the cross section (χ_z).

The plane cross section assumption leads to an overestimation of the shear stiffness and contradicts the three dimensional equations of the beam: the plane cross section results in a uniform shear strain and a uniform shear stress, however, according to the equilibrium equations the shear stress and strain distribution is parabolic (Fig.1).

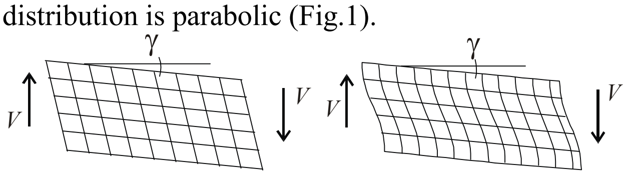

Fig. 1. Shear deformation assuming uniform and parabolic shear strain

This contradiction was recognized already by Timoshenko, and the shear stiffness was calculated as follows [1]: the axial stresses are calculated on the basis of the kinematical assumption (i.e. cross sections remain plane), while the shear stresses are calculated from the equilibrium equation, and the shear stiffness is evaluated with the use of the strain energy. This leads to the usage of the shear correction factor which, in many cases, gives satisfactory results. However, as will be shown in the next section, for composite beams it may be inaccurate.

When a *beam is subjected to torsion*, in the classical (Vlasov) theory only the rotation of the cross section (ψ) about the beam's axis is needed to calculate the displacements of any point of the cross section. (See [1] for isotropic and [2, 3] for composite beams.) When the axial warping is constrained, an open section beam carries the torque load mainly by the bending and shear of the flanges. Note, however that according to Vlasov's theory the shear deformation of the walls – in restrained warping – is neglected. To overcome this shortcoming, analogously to Timoshenko's beam theory, a new displacement function was introduced [2, 4, 6], (in addition to the rotation of the cross section, ψ): the rate of twist due to warping (ϑ^{B}). The rotational stiffnesses were calculated analogously to the in plane stiffnesses [2].

When we consider the spatial (in plane and torsional) deformations of the beam, seven displacements are taken into account [2]:

$$u\,,\quad v\,,\quad w\,,\quad \psi\,,\ \chi_y,\ \chi_z,\ \vartheta^{\mathrm{B}}$$

u is the axial displacement, while v and w are the displacements perpendicular to the axis, ψ is the rotation of the cross section about the beam's axis; χ_y, χ_z are the rotation of the cross sections about the z and y axes, and ϑ^{B} is the rate of twist due to warping. For doubly symmetrical orthotropic cross sections the material law can be written in the following form:

$$\begin{Bmatrix} \hat{N}_x \\ \hat{M}_z \\ \hat{M}_y \\ \hat{M}_\omega \\ \hat{V}_y \\ \hat{V}_z \\ \hat{T}_\omega \\ \hat{T}_{\mathrm{SV}} \end{Bmatrix} = \left\langle \begin{matrix} \overline{EA} \\ \overline{EI}_{zz} \\ \overline{EI}_{yy} \\ \overline{EI}_{\omega\omega} \\ S_{yy} \\ S_{zz} \\ S_{\omega\omega} \\ \overline{GI}_{\mathrm{t}} \end{matrix} \right\rangle \begin{Bmatrix} \varepsilon_x \\ 1/\rho_z \\ 1/\rho_y \\ \Gamma \\ \gamma_y \\ \gamma_z \\ \vartheta^{\mathrm{s}} \\ \vartheta \end{Bmatrix} \tag{1}$$

where <> denotes a diagonal matrix, $\hat{N}_x$ is the axial force (Fig.2), $\hat{M}_z$ and $\hat{M}_y$ are the moments about the z and y axis, $\hat{M}_\omega$ is the bimoment, $\hat{V}_y$ and $\hat{V}_z$ are the shear forces, $\hat{T}_\omega$ is the restrained warping induced torque, while $\hat{T}_{\mathrm{SV}}$ is the Saint Venant torque, the sum of which gives the torque:

$$\hat{T} = \hat{T}_\omega + \hat{T}_{SV} \tag{2}$$

The generalized strains, on the right side of Eq.1, are calculated from the displacements [2].

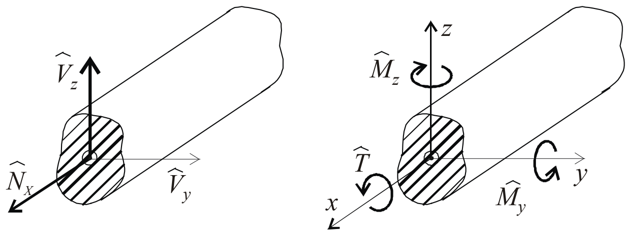

Fig. 2. Internal forces in a beam

2 Problem statement

To show the weakness of the above theory we consider the example of a thin walled beam, which consists of three walls connected at their axes:

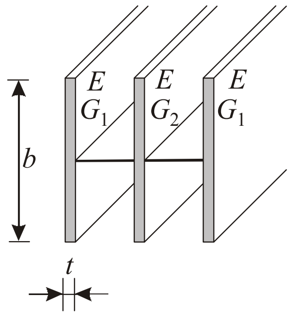

Fig. 3. Beam, which consists of three parallel wall segments

The Young modulus of the material are the same in each wall, and it is denoted by E, while the shear stiffness of the middle wall is much lower than those of the other two: $G_2 << G_1$. According to the classical shear deformation (Timoshenko) theory the axial stresses in the walls are the same, and hence, from the equilibrium the shear stresses are identical too. These stresses lead to the following bending and shear stiffnesses:

$$\overline{EI} = 3E\frac{b^3 t}{12} \qquad S = G_2\frac{bt}{1.2} \tag{3}$$

These results are incorrect: the middle wall, because of the low shear stiffness, does not play a role, and hence the stiffnesses should be

$$\overline{EI} = 2E\frac{b^3 t}{12} \qquad S = 2G_1\frac{bt}{1.2} \tag{4}$$

which are failed to be predicted by the classical shear deformation theory. Note that the same problem may arise in the case of torsion.

In this paper we consider thin walled beams which consist of flat wall segments as shown in the figure below.

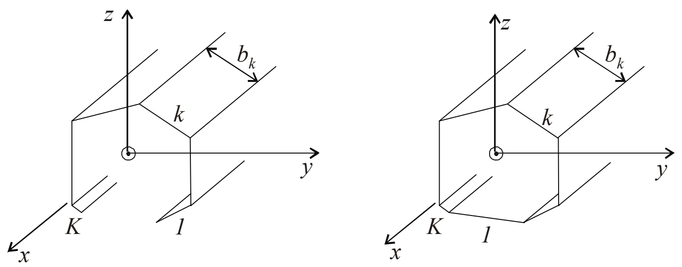

Fig. 4. Open and closed section beams

3 Solution

To overcome the problem presented in the previous section the following solution is presented: thin walled beams can be solved accurately using the three dimensional equations if the displacements, strains and stresses (and as a consequence the loads) vary trigonometrically along the axial coordinate of the beam:

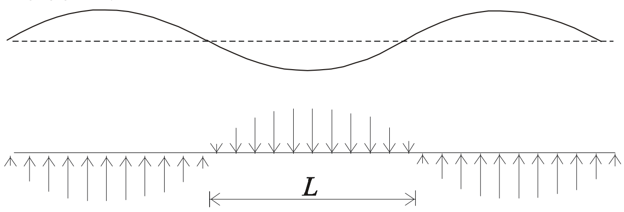

Fig. 5. Variation of the load along the beam

These equations are formulated and then the cross sectional properties are developed by limit transition, assuming that L (i.e. the half wavelength of the trigonometrical functions) is large compared to the seize of the section.

It is important to note that we keep the "classical" definition of the beam forces and strains, and the seize of the stiffness matrix is not increased either: only the stifnesses are calculated in a consistent manner.

It may be stated that for the previously presented theories the "best" stiffnesses are determined.

4 Solution for one wall segmant

To obtain an accurate solution for a composite beam first a single, flat wall element (Fig.6.) is considered. (For simplicity only symmetrical layups are considered.)

4.1 Basic equations of one anisotropic plate

The axial strain, transversal strain and the shear strain are related to the displacements of the wall as

$$\varepsilon_x = \frac{\partial u}{\partial x}$$
$$\varepsilon_s = \frac{\partial w}{\partial s} \tag{5}$$
$$\gamma_{xs} = \frac{\partial u}{\partial s} + \frac{\partial w}{\partial x}$$

while the constitutive equations are

$$\begin{Bmatrix} N_x \\ N_s \\ q \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_s \\ \gamma_{xs} \end{Bmatrix} \tag{6}$$

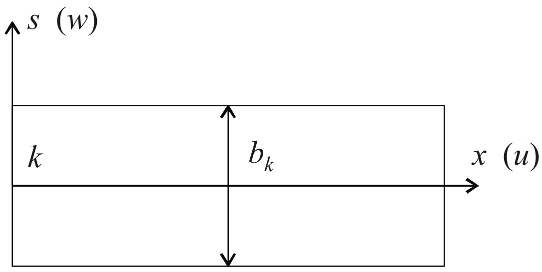

Fig. 6. Coordinate system attached to a wall segment and the displacements

where A_{ij} are the elements of the tensile stiffness matrix of a layered composite plate [2]. It is assumed that N_s , i.e. the resultant forces per unit length perpendicular to the beam axis, is small compared to N_x and q, and hence ε_s can be eliminated form Eq.6. We obtain

$$\begin{Bmatrix} N_x \\ q \end{Bmatrix} = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{16} \\ \overline{A}_{16} & \overline{A}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \gamma_{xs} \end{Bmatrix} \tag{7}$$

Where $\overline{A}_{ij}$ can be calculated from the values of A_{ij} . The distributed loads acting on a plate element are related to the internal forces by the equilibrium equations:

$$\frac{\partial N_x}{\partial x} + \frac{\partial q}{\partial s} + p_s = 0 \tag{8}$$

$$\frac{\partial q}{\partial x} + p_s = 0 \tag{9}$$

4.2 Solution for the trigonometric loads

As we stated above the solution of one plate is assumed in the form of trigonometric functions. The displacements of the axis of the wall is assumed in the following form:

$$w_o(x) = \tilde{w}_o \sin \alpha x - \tilde{\tilde{w}}_o \cos \alpha x \tag{10}$$

$$u_o(x) = -\tilde{u}_o \cos \alpha x + \tilde{\tilde{u}}_o \sin \alpha x \tag{11}$$

where $\tilde{u}_o$, $\tilde{w}_o$, $\tilde{\tilde{u}}_o$, $\tilde{\tilde{w}}_o$, are yet unknown constants and

$$\alpha = \frac{\pi}{L} \tag{12}$$

Here length, L is shown in Fig. 5.

The two dimensional displacement of the wall is:

$$u(x,s) = -(\tilde{u}_s + \tilde{u}_o)\cos \alpha x + \left(\tilde{\tilde{u}}_s + \tilde{\tilde{u}}_o\right)\sin \alpha x \tag{13}$$

In Eq.13 $\tilde{u}_s$ and $\tilde{\tilde{u}}_s$ are functions of s. The loads on the wall are also assumed in the form of trigonometric functions:

$$p_s(x,z) = \tilde{p}_z \sin \alpha x + \tilde{\tilde{p}}_z \cos \alpha x \tag{14}$$

$$p_s(x,z) = \tilde{p}_x \cos \alpha x + \tilde{\tilde{p}}_x \sin \alpha x \tag{15}$$

After algebraic manipulation from Eqs. 5, 7, 8 and 11 we obtain:

$$\begin{bmatrix} 1 & \\ & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\tilde{u}}_s \\ \ddot{\tilde{\tilde{u}}}_s \end{Bmatrix} + 2\alpha \frac{\bar{A}_{16}}{\bar{A}_{66}} \begin{bmatrix} & -1 \\ 1 & \end{bmatrix} \begin{Bmatrix} \dot{\tilde{u}}_s \\ \dot{\tilde{\tilde{u}}}_s \end{Bmatrix} \cdot$$
$$\alpha^2 \frac{\bar{A}_{16}}{\bar{A}_{66}} \begin{Bmatrix} \tilde{u}_s \\ \tilde{\tilde{u}}_s \end{Bmatrix} = \alpha^2 \frac{\bar{A}_{16}}{\bar{A}_{66}} \begin{Bmatrix} \tilde{\tilde{w}}_o \\ \tilde{w}_o \end{Bmatrix} + \tag{16}$$
$$\frac{\alpha^2}{\bar{A}_{66}} \begin{Bmatrix} \tilde{p}_x \\ \tilde{\tilde{p}}_x \end{Bmatrix} + \alpha^2 \frac{\bar{A}_{16}}{\bar{A}_{66}} \begin{Bmatrix} \tilde{u}_o \\ \tilde{\tilde{u}}_o \end{Bmatrix}$$

where dot denotes the derivation with respect to s.

Eq.16 is a second order differential equation system for the unknown functions $\tilde{u}_s$ and $\tilde{\tilde{u}}_s$. When the axial loads $\tilde{p}_x$ and $\tilde{\tilde{p}}_x$ are *constant* (uniform load), it can be shown that the general solution is:

$$\begin{aligned} \tilde{u}_s &= -\frac{\bar{A}_{16}}{\bar{A}_{66}} \tilde{\tilde{w}}_o + \\ &\left(C_1 \frac{1}{\alpha} \cosh \Omega \alpha s + C_3 \frac{1}{\alpha} \sinh \Omega \alpha s \right) \cos \beta \alpha s + \\ &\left(C_2 \frac{1}{\alpha} \cosh \Omega \alpha s + C_4 \frac{1}{\alpha} \sinh \Omega \alpha s \right) \sin \beta \alpha s \end{aligned} \tag{17}$$

$$\begin{aligned} \tilde{\tilde{u}}_s &= -\frac{\bar{A}_{16}}{\bar{A}_{66}} \tilde{w}_o + \\ &\left(C_1 \frac{1}{\alpha} \cosh \Omega \alpha s + C_3 \frac{1}{\alpha} \sinh \Omega \alpha s \right) \sin \beta \alpha s + \\ &\left(C_2 \frac{1}{\alpha} \cosh \Omega \alpha s + C_4 \frac{1}{\alpha} \sinh \Omega \alpha s \right) \cos \beta \alpha s \end{aligned} \tag{18}$$

where

$$\beta = -\frac{\bar{A}_{16}}{\bar{A}_{66}} \tag{19}$$

$$\Omega = \sqrt{\frac{\bar{A}_{11}}{\bar{A}_{66}} - \frac{\bar{A}_{16}^2}{\bar{A}_{66}^2}} \tag{20}$$

When the axial loads are *linearly varying* through the thickness, and the distribution is given by the following equations:

$$\begin{aligned} \tilde{p}_x &= \tilde{p}_{x1} s \\ \tilde{\tilde{p}}_x &= \tilde{\tilde{p}}_{x1} s \end{aligned} \tag{21}$$

The particular solution of Eq.16 is:

$$\tilde{u}_s = 2 \frac{\bar{A}_{16}}{\bar{A}_{11}^2} \tilde{\tilde{p}}_{x1} + \frac{\alpha}{\bar{A}_{11}} \tilde{p}_{x1} s \tag{22}$$

$$\tilde{\tilde{u}}_s = 2 \frac{\bar{A}_{16}}{\bar{A}_{11}^2} \tilde{p}_{x1} - \frac{\alpha}{\bar{A}_{11}} \tilde{\tilde{p}}_{x1} s \tag{23}$$

If both uniform and linearly varying axial loads ($\tilde{p}_x$ and $\tilde{\tilde{p}}_x$) are considered, the solution is obtained as the sum of Eqs.17, 18 and 22, 23:

$$\begin{aligned}\tilde{u}_s = & -\frac{\overline{A}_{16}}{\overline{A}_{66}}\tilde{\tilde{w}}_o + 2\frac{\overline{A}_{16}}{\overline{A}_{11}^2}\tilde{\tilde{p}}_{x1} + \frac{\alpha}{\overline{A}_{11}}\tilde{p}_{x1}s \\ & \left(C_1\frac{1}{\alpha}\cosh\Omega\alpha s + C_3\frac{1}{\alpha}\sinh\Omega\alpha s\right)\cos\beta\alpha s + \\ & \left(C_2\frac{1}{\alpha}\cosh\Omega\alpha s + C_4\frac{1}{\alpha}\sinh\Omega\alpha s\right)\sin\beta\alpha s\end{aligned} \tag{24}$$

$$\begin{aligned}\tilde{\tilde{u}}_s = & -\frac{\overline{A}_{16}}{\overline{A}_{66}}\tilde{w}_o + 2\frac{\overline{A}_{16}}{\overline{A}_{11}^2}\tilde{p}_{x1} - \frac{\alpha}{\overline{A}_{11}}\tilde{\tilde{p}}_{x1}s \\ & \left(C_1\frac{1}{\alpha}\cosh\Omega\alpha s + C_3\frac{1}{\alpha}\sinh\Omega\alpha s\right)\sin\beta\alpha s + \\ & \left(C_2\frac{1}{\alpha}\cosh\Omega\alpha s + C_4\frac{1}{\alpha}\sinh\Omega\alpha s\right)\cos\beta\alpha s\end{aligned} \tag{25}$$

These functions contain 2 × 5 constants:

$$\tilde{u}_o,\ \tilde{w}_o,\ \tilde{p}_{x1},\quad C_1,\quad C_3 \tag{26}$$

$$\tilde{\tilde{u}}_o,\ \tilde{\tilde{w}}_o,\ \tilde{\tilde{p}}_{x1},\quad C_2,\quad C_4 \tag{27}$$

We may observe that for orthotropic walls, when $A_{16} = \overline{A}_{16} = 0$, the two displacement functions (Eqs.24 and 25) are uncoupled:

$$\begin{aligned}\tilde{u}_s = & \frac{\alpha}{\overline{A}_{11}}\tilde{p}_{x1}s \\ & + C_1\frac{1}{\alpha}\cosh\Omega\alpha s + C_3\frac{1}{\alpha}\sinh\Omega\alpha s\end{aligned} \tag{28}$$

$$\begin{aligned}\tilde{\tilde{u}}_s = & -\frac{\alpha}{\overline{A}_{11}}\tilde{\tilde{p}}_{x1}s \\ & + C_2\frac{1}{\alpha}\cosh\Omega\alpha s + C_4\frac{1}{\alpha}\sinh\Omega\alpha s\end{aligned} \tag{29}$$

4.3 Resultants

Using the above derived displacements we can determine the strains and the internal forces from Eqs. 5 and 7. They can be separated similarly as the displacements and the loads:

$$\varepsilon_x(x,s) = \tilde{\varepsilon}_x \sin\alpha x + \tilde{\tilde{\varepsilon}}_x \cos\alpha x \tag{30}$$

$$\gamma_{xs}(x,s) = \tilde{\gamma}_{xs} \cos\alpha x + \tilde{\tilde{\gamma}}_{xs} \sin\alpha x \tag{31}$$

$$N_x(x,z) = \tilde{N}_x \sin\alpha x + \tilde{\tilde{N}}_x \cos\alpha x \tag{32}$$

$$q(x,z) = \tilde{q} \cos\alpha x + \tilde{\tilde{q}} \sin\alpha x \tag{33}$$

Then the resultant forces on a wall segment are defined, and calculated as

$$\begin{aligned}N &= \int_{(b)} N_x ds \\ M &= \int_{(b)} N_x z ds \\ V &= \int_{(b)} q ds\end{aligned} \tag{34}$$

The resultant loads (which depend on the axial coordinate, x only) are the moment: m, the axial load: p_{xo} , and the transverse load: p_{so} , which can be calculated from the equilibrium equations, which result in:

$$\begin{aligned}m &= \frac{\partial M}{\partial x} - V \\ p_{xo} &= -\frac{\partial N}{\partial x} \\ p_{so} &= -\frac{\partial V}{\partial x}\end{aligned} \tag{35}$$

These loads can be written in the following form:

$$\begin{aligned}m &= \tilde{m}\cos\alpha x + \tilde{\tilde{m}}\sin\alpha x \\ p_{xo} &= \tilde{p}_{xo}\cos\alpha x + \tilde{\tilde{p}}_{xo}\sin\alpha x \\ p_{so} &= \tilde{p}_{so}\sin\alpha x + \tilde{\tilde{p}}_{so}\cos\alpha x\end{aligned} \tag{36}$$

Note again that these functions depend on the constants given in Eqs.26 and 27.

5 Exact solution for a beam

Using the above derived displacements for one wall segment we can determine the exact solution of a beam consisting of several wall segments. The displacements of each wall are given by Eqs.24 and 25 which depend on the constants given in Eqs.26 and 27.

The constants C_1 , C_3 , C_2 , C_4 can be determined from the continuity conditions. At each intersection the displacement and the shear force must be continuous (Fig.4.):

$$
\begin{aligned}
&\tilde{u}_{s,k-1}\left(+\frac{b_{k-1}}{2}\right)=\tilde{u}_{s,k}\left(-\frac{b_k}{2}\right)\\
&\tilde{\tilde{u}}_{s,k-1}\left(+\frac{b_{k-1}}{2}\right)=\tilde{\tilde{u}}_{s,k}\left(-\frac{b_k}{2}\right)\\
&\tilde{q}_{k-1}\left(+\frac{b_{k-1}}{2}\right)=\tilde{q}_{k}\left(-\frac{b_k}{2}\right)\\
&\tilde{\tilde{q}}_{k-1}\left(+\frac{b_{k-1}}{2}\right)=\tilde{\tilde{q}}_{k}\left(-\frac{b_k}{2}\right)
\end{aligned}
\tag{37}
$$

while for an open section beam the shear force on the free edges must be zero

$$
\begin{aligned}
&\tilde{q}_1\left(-\frac{b_1}{2}\right)=0\\
&\tilde{q}_K\left(-\frac{b_K}{2}\right)=0\\
&\tilde{\tilde{q}}_1\left(-\frac{b_1}{2}\right)=0\\
&\tilde{\tilde{q}}_K\left(-\frac{b_K}{2}\right)=0
\end{aligned}
\tag{38}
$$

Using the continuity conditions we obtain an exact solution of the beam, which depends only on (Eqs. 26 and 27)

$$\tilde{u}_o,\ \tilde{w}_o,\ \tilde{p}_{x1}, \tag{39}$$

$$\tilde{\tilde{u}}_o,\ \tilde{\tilde{w}}_o,\ \tilde{\tilde{p}}_{x1} \tag{40}$$

We may observe that the displacements of the beam (Eqs. 10 and 13) depends on α (Eq.11), which is a function of the “length”, *L*.

The hyperbolic and trigonometrical functions in the displacements (Eqs.24 and 25) can be eliminated, if the Taylor series expansions of the functions with respect to α are determined. We must keep in mind that parameter *L* is related to the length of the beam, which is large compared to *b*, and hence α is small. In such a way we obtain functions where the displacements are polynomials, similarly as in the classical beam theory. However, at least for trigonometrically varying loads, we obtain an “exact” solution of the beam. The accuracy of the solution depends on the number of terms in the Taylor series expansion.

5 Cross sectional properties of a beam

When the displacements of each wall is given the strain energy of the beam can be formulated. For length *L*, for each wall segment, the strain energy is:

$$U=\frac{1}{2}\int_0^L\int_{(b)}\left(N_x\varepsilon_x+q\gamma_{xs}\right)ds\,dx \tag{41}$$

which can be written (see Eq.30-33) in the following form

$$U=\frac{L}{4}\int_{(b)}\left(\tilde{N}_x\tilde{\varepsilon}_x+\tilde{q}\tilde{\gamma}_{xs}+\tilde{\tilde{N}}_x\tilde{\tilde{\varepsilon}}_x+\tilde{\tilde{q}}\tilde{\tilde{\gamma}}_{xs}\right)ds \tag{42}$$

To derive the cross sectional properties of the beam we compare the strain energy (Eq.42) to that of a beam given in [6].

From the condition that for the same displacements of the axis of the beam the strain energy obtained from the exact solution (Eq. 42) and from the beam equations are the same the properties can be determined. In the calculation we keep the same number of terms in the Taylor series expansion for both the exact and the beam solution.

This calculation requires an extensive algebraic manipulations of the equations, which, excepts for very simple cases, can be done numerically only.

For example, in the case treated in the Problem statement, this new theory results in Eq.4.

Another simple example, if we have an orthotropic beam, which consists of one anisotropic wall only. The constitutive equation is:

$$\begin{Bmatrix}\hat{N}_x\\ \hat{M}_z\\ \hat{V}_y\end{Bmatrix}=\mathbf{C}\begin{Bmatrix}\varepsilon_x\\ 1/\rho_z\\ \gamma_y\end{Bmatrix} \tag{43}$$

where the stiffness matrix, **C** is:

$$\begin{bmatrix} \frac{\overline{A}_{66}}{b \det \overline{A}} & & -\frac{\overline{A}_{16}}{b \det \overline{A}} \\ & \frac{12\overline{A}_{66}}{b^3 \det \overline{A}} & \\ -\frac{\overline{A}_{16}}{b \det \overline{A}} & & \frac{1.2}{b\overline{A}_{66}}\left(1+\frac{5}{6}\frac{\overline{A}_{16}^2}{\det \overline{A}}\right) \end{bmatrix} \tag{44}$$

where $\det \overline{A} = \overline{A}_{11}\overline{A}_{66} - \overline{A}_{16}^2$. For orthotropic wall ($A_{16} = \overline{A}_{16} = 0$) this matrix simplifies to the well known form:

$$\begin{bmatrix} \frac{1}{b\overline{A}_{11}} & & \\ & \frac{12}{b^3\overline{A}_{11}} & \\ & & \frac{1.2}{b\overline{A}_{66}} \end{bmatrix} \tag{45}$$

5 Conclusion

In the classical analysis the properties of a beam are calculated by assuming kinematical relationships, e.g. cross sections remain plane after the deformation of the beam. These assumptions may lead to inaccuracy or to contradictory results. In the paper a new theory is presented, in which no kinematical assumption is applied, rather the properties are derived from the accurate (three dimensional) equations of beams using limit transition.

This calculation requires an extensive manipulations of the equations, which, excepts for very simple cases, can be done numerically only. However, this solution does not contain the shortcomings of the classical derivations.

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