STOCHASTIC MODELING OF RESIN FLOW IN FIBROUS MEDIA IN LIQUID COMPOSITE MOLDING

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Keywords: *Composite Manufacturing, Robustness, Stochastic Finite Elements Method*

Abstract

An efficient methodology to determine the probabilistic robustness of a Liquid Composite Molding processes on screen by coupling finite elements software with a stochastic method is described. A non-intrusive Stochastic Finite Elements Method, recently proposed by Berveiller [7], is used as the stochastic solver. Interest in the field of liquid composite manufacturing simulation is the possibility to predict influence of random parameters on the injection cycle time with an optimal number of simulations.

1 Introduction

Properties and performance of parts made from fibre reinforced composites depend on material, design and processing. Composite manufacturing technologies play an important role since during process one makes not only the part of the desired shape but also the material itself with specific properties. Variations in material properties will affect the composite part quality and mechanical performance but also the manufacturing process. Because polymer composites are heterogeneous, variations may occur at the constituent level i.e. fibre and matrix. Disturbances may happen also at an upper scale level and can affect variables such as ply orientation and thickness, fibre volume fraction and process-induced defects such as voids. Due to the inherent nature of the manufacturing technology and the raw materials, there will always be uncertainties in the manufactured part and in its mechanical behaviour. Although this paper only focuses on the stochastic modelling of a manufacturing process, methodology used to address this problem is very generic and was basically originated in stochastic structural mechanics.

Liquid Composite Molding (LCM) consists in a variety of composite manufacturing processes that are capable of producing high-quality, complex-shaped fibre reinforced composite parts. They are used mainly in the aerospace, automotive, marine, and civil industries. Among them Vacuum Assisted Resin Transfer Molding (VARTM) became more and more attractive because of the simplicity of mould manufacture and injection equipment required. In VARTM resin is drawn into the fibre preform and mould with vacuum pressure. Generally, fibre preform is put on a one-sided mould and is covered with a flexible top and vacuum sealed. A very porous fabric called distribution medium is placed on top of perform. The low viscosity catalysed resin is then introduced. The resin moves quickly through this layer, and then flows downward through the thickness of the fibre preform. This technology eliminates the need for expensive matched metal tooling and allows for the fabrication of large parts. It is often used with resins that cure at relatively low temperature. The main drawback of this process is the low pressure drop available for driving the resin into the mould. Then mould-filling time will be a relevant parameter that will be affected by the perform permeability and resin viscosity, the latter being controlled by the temperature and the polymerisation state. Since these parameters are not fully controlled in workshops, they may significantly fluctuate from batch to batch and zone to zone in the part. The fill time will be uncertain making the process difficult to qualify and optimise. One way to address that is to incorporate sensors in the mould and develop strategic controllers that will take appropriate control action to drive the process towards success. Strategic location, action mode and amount of sensors and controllers should result from a rigorous analysis of the uncertainties that can affect the

process. As a part and parcel of this, the following sections present a methodology to determine with an optimal number of simulations the probabilistic robustness of a process on screen by coupling finite elements software with a stochastic approach. Without loss of generality, the paper focuses on the influence of random parameters on the injection cycle time.

2 Problem statements

The part modeled in this paper is a 0.15 x 0.5m² rectangular plate, with a 0.1 m thickness. A non-deformable insert (0.15 x 0.3 x 0.02 m^3) is placed in the center of the part as presented in fig. 1.

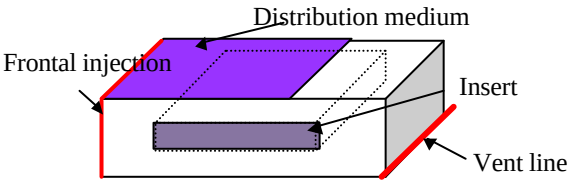


Fig. 1. Studied part

The fiber preform is made of glass woven fabrics with a nominal permeability of 2.9×10^{-10} m² and a 40% fiber volume fraction [10]. The injected resin is an unsaturated polyester, with a 0.22 Pa.s nominal viscosity. The resin viscosity is supposed to be constant during the whole injection. Distribution medium is placed on top of the preform, expected to cover the 0.39 m in the length direction. A 80% porosity value and a 1.0×10^{-8}m² permeability are assigned to this material. A frontal injection is preformed from one side of the mold with a 0.1 MPa injection pressure; a vent line is drawn on the opposite downside of the mold.
In the following sections, influence of variability of the preform permeability, resin viscosity and distribution medium length is studied, to account for operator change and temperature variations from one part manufacturing to another.

3 Stochastic modeling

3.1 General overview

Introduction of randomness into the modeling of a physical phenomenon has received a large attention from the scientific community. Among the few available techniques, Monte-Carlo simulation is widely used because of its simplicity, in both theory and numerical implementation. However, such simulations only provide quantitative results and become very time-consuming when dealing with complex problems.

In general, the Stochastic Finite Elements Method consists in the representation of the overall probabilistic response onto a basis of the vectorial space of real random variables with finite second moment (corresponding to physical systems with finite statistical fluctuations), namely the Polynomial Chaos (see [1] [8]).
The method basically relies on two discretization procedures:

- The first one deals with the geometrical definition of the problem and is similar to the discretization performed within a deterministic framework;
- The second one is related to the discretization along the probabilistic dimension.

A review of a few techniques for representing stochastic processes (also called random fields) can be found in [4] [5] for instance. Among these techniques, the series expansion methods (such as the Karhunen-Loève expansion [1], the Expansion Optimal Linear Estimation [2] or the Orthogonal Series Expansion [3]) seem to provide the most accurate results. An extended presentation of the spectral approach can be found in the book from Ghanem and Spanos [1]. However, the main drawback of such methods is that they basically require a tricky intrusive implementation within commercial Finite Elements codes and thus, they may become inappropriate when dealing with large and complex problems.
Recently, two non-intrusive methods relying on deterministic computations have been proposed. In both cases, the overall stochastic response is expanded onto the Polynomial Chaos and is written as:

$$\tilde{A}(\omega) = \sum_{i=0}^{\infty} a_i Y_i\left(\{\xi_k(\omega)\}_{k=1}^{M}\right) \qquad (1)$$

where $\{\xi_k\}_{k=1}^{M}$ is the set of standard normal random variables used in the expansion of the input random variables, $\{Y_i\}_{i=0}^{\infty}$ are the multidimensional Hermite polynomials and $\{a_i\}_{i=0}^{\infty}$ is a set of coefficients to be determined. For

computational purposes, a truncated expansion is considered (for the sake of clarity, the order of truncation is not reported):

$$\tilde{A}(\omega) \approx \sum_{i=0}^{P-1} a_i Y_i\left(\{\xi_k(\omega)\}_{k=1}^{M}\right) \qquad (2)$$

The number of terms P involved in the representation is related to both the order of expansion p and the dimension M by the relation:

$$P = \frac{(M+p)!}{M!\,p!} \qquad (3)$$

The two methods basically differ in the way the coordinates are computed:

- In the method proposed by Puig et al. [6], the coefficients are determined by using the orthogonality of the Hermite polynomials with respect to the Gaussian probability measure. Such a method then yields a multidimensional integration that can be performed by Monte-Carlo numerical simulations.
- In the methodology proposed by Berveiller [7], the coordinates are computed by considering a set of collocation points and by using a least square minimization method, resulting in a probabilistic regression method.

The second method was used in this study and is further detailed in the next section.

3.2 The probabilistic regression method

The method can be divided into two main steps:

- The first one consists in defining a mapping between the space of physical input data and the normed space. For each random variable, the mapping can be performed by using either an isoprobabilistic transformation or a polynomial expansion.
- The computation of the coefficients involved in the Polynomial Chaos expansion.

3.2.1 Expansion of the input random variables

In this study, each input random variable is represented by a polynomial series expansion in a standard normal variable.

Case of the distribution medium

The length L of the draining material is modeled as a random variable uniformly distributed around its mean theoretical value $\underline{L} = 0.39$. Its support is defined as $[0.38, 0.40]$. The length is written as:

$$L(\omega) \approx \sum_{i=0}^{\infty} L_i H_i(\xi) \qquad (4)$$

where the coefficients $\{L_i\}_{i=0}^{\infty}$ are to be computed. By using the orthogonality of the polynomial basis, these coordinates are found to be [7] (see also [9]):

$$\begin{cases} L_0 = (a+b)/2 \\ L_{2i} = 0 \\ L_{2i+1} = \dfrac{(-1)^i (b-a)}{2^{2i+1}\sqrt{\pi}\, i!\,(2i+1)} \end{cases} \qquad (5)$$

where $a = 0.38$ and $b = 0.40$. Due to the singularities of the distribution, it is necessary to include many terms in the expansion [9], as shown on Fig. 2.

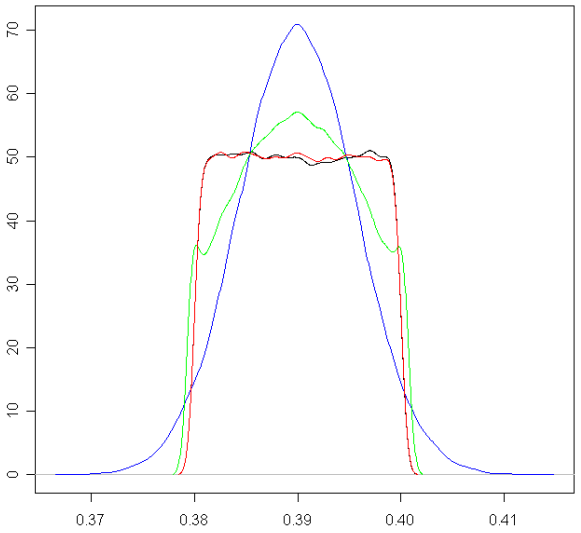


Fig. 2. Comparison of the PDFs of the random length of draining material for different orders of expansion
(Black: *exact*, blue: *p=1*, green: *p=3*, red: *p=13*).

Case of the bulk preform

The permeability of the preform K is written as $K = K^* \cdot 10^{-10}$ and K^* is modeled as a Log-normal distributed random variable with parameters $l = 1.0509$ and $\zeta = 0.1684$ (see [10] for the description of the experimental analysis).
Then, one has:

$$K^*(\omega) \approx \sum_{i=0}^{\infty} K^*_i \, H_i(\xi) \qquad (6)$$

In the case of a Log-normal random variable with parameters (λ, ζ), the coefficients can be determined analytically and are given by (see [7]):

$$K^*_i = \frac{\zeta^i \, exp\left(\lambda + 0.5 \cdot \zeta^2\right)}{i!} \qquad (7)$$

Fig. 3 shows the PDFs, assessed by Monte-Carlo numerical simulations (100 000 realizations), for different orders of expansion (note that for the sake of clarity, the multiplicative factor is not reported on the plot).

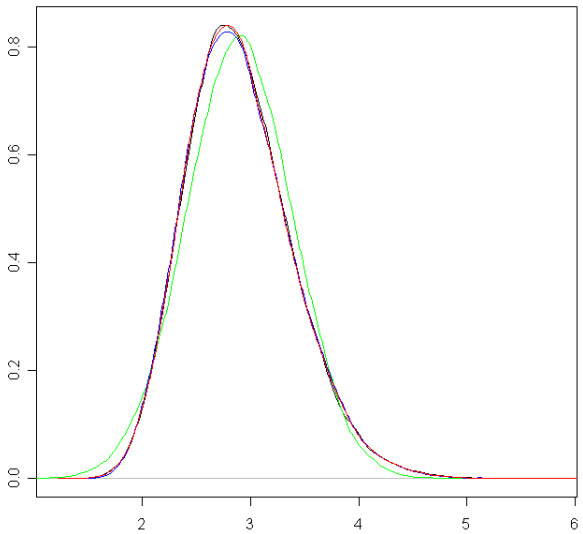


Fig. 3. Comparison of the PDFs of the random permeability of the bulk preform for different orders of expansion
(Black: *exact*, green: *p=1*, blue: *p=2*, red: *p=3*).

Case of the viscosity

The viscosity is modeled as a Log-Normal distributed random variable with parameters $l = -1.5182$ and $\zeta = 0.0907$. The coordinates of the projection are computed by using Eq. (7). Fig. 4 shows the PDFs for different order of expansion.

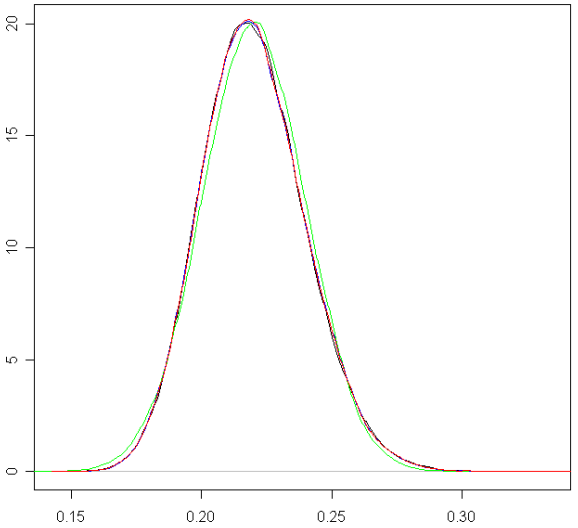


Fig. 4. Comparison of the PDFs of the random viscosity for different orders of expansion
(Black: *exact*, green: *p=1*, blue: *p=2*, red: *p=3*).

3.2.2. Probabilistic regression

The probabilistic regression method was introduced by Berveiller et al. [7] [11]. We recall that the aim is to compute the coefficients involved in the representation of the overall response of the system (see Eq. (1)).
For that purpose, let us consider n outcomes of the random vector $\underline{\xi}$ gathering the standard normal variables involved in the projection of the input random variables. By using the expansions detailed in the previous section, one then determines the n realizations of the input random variables. For each outcome, the overall response can be computed by using any commercial software. The method basically consists in determining the set of coefficients which minimize the quantity:

$$DA = \sum_{k=1}^{n} \left[A^{(k)} - \tilde{A}\left(\underline{\xi}^{(k)}\right) \right]^2 \qquad (8)$$

where $\{A^{(k)}\}_{k=1}^{n}$ is the set of deterministic results and the $\{\tilde{A}(\xi^{(k)})\}_{k=1}^{n}$ are the projections of the probabilistic response (see Eq. (1)).
The collocation points are chosen as follows [7] [12] [13]:

- for an expansion at the *p*-th order, we first compute the *(p+1)* roots of the *(p+1)*-th order Hermite polynomial;
- all the vectors of length *M* are then computed by using all possible combinations of the roots;
- the *n* combinations that maximize the Gaussian probability measure are finally selected.

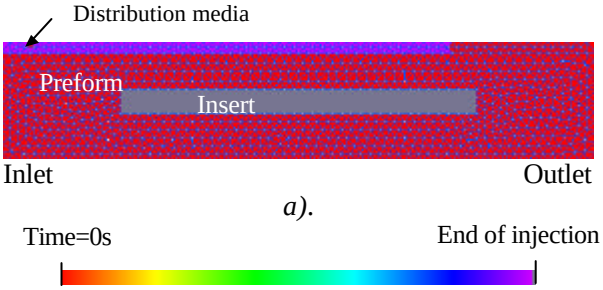
Note that a few parametric studies carried out by Berveiller (see [7]) tend to prove that $n = (M-1)P$ provides accurate results.

3.3 Results

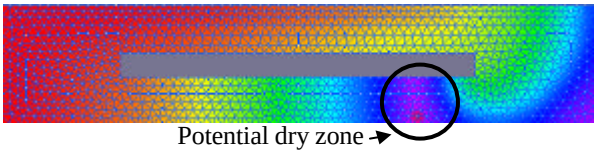
Once the coordinates of the stochastic parameters are determined, Monte-Carlo numerical simulations are performed in order to assess some statistical properties of the parameters. Note that since we have a complete representation of the probabilistic quantities in terms of standard normal random variables, these simulations are very cost-effective compared to direct Monte-Carlo simulations.

3.3.1. Statistical properties

The deterministic computations are realized with LIMS, a finite element solver associated with a control volume approach to model injection of resin in a porous medium [14]. The 2D geometry implemented represents the middle plane of the part, cut along the flow direction as presented in fig. 5a.



a).



b).

Fig. 5. a). Simulation input b) Example of a Filling profile

For each determinist simulation, preform permeability, resin viscosity and length of the distribution medium are implemented. Two different end conditions are then applied. For the first one, injection is completed when resin reaches the vent. Note that full injection is not guaranteed and that dry zones can be observed, as shown in Fig. 5b. Injection time t_1 is then recorded. For the second one, injection is continued until the complete filling of the part. This simulation is characterized by the total injection time t_2.
The first set of calculations (60 computations) considers the Berveiller method [7]. For comparison, another set of calculations is conducted considering the Monte Carlo method (10 000 computations).
Table 1 presents the statistical parameters.

Table 1. Statistical parameters.

Property	Mean	CV
Time 1, t_1	175.9 [s]	13%
Time 2, t_2	200.3 [s]	15%

Figs. (6) to (9) show the PDFs and the empirical CDFs of the random parameters assessed by 10 000 simulations.

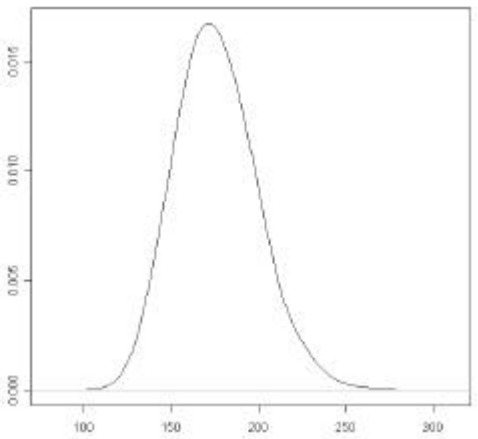


Fig. 6. Probability density function of t_1.

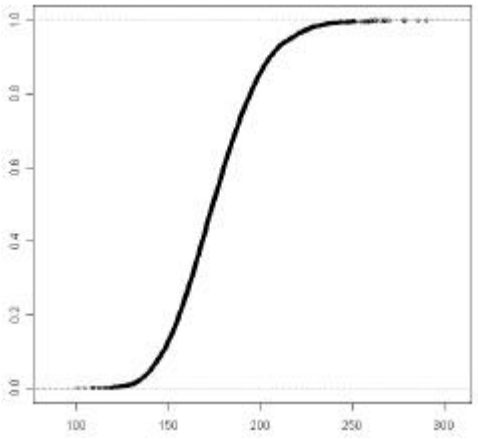


Fig. 7. Cumulative distribution function of t_1.

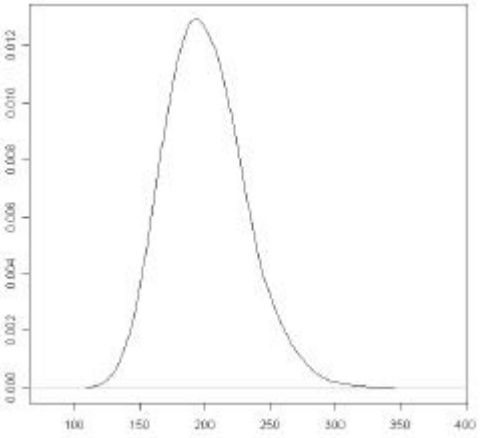


Fig. 8. Probability density function of t_2.

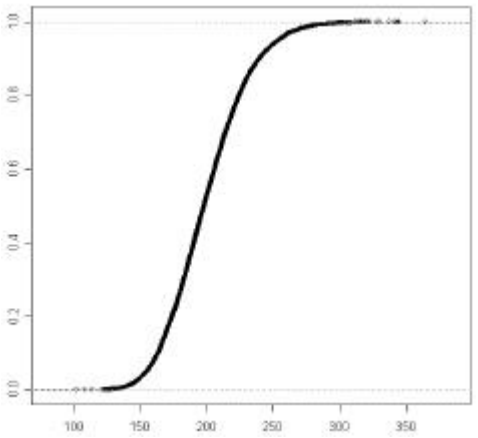


Fig. 9. Cumulative distribution function of t_2.

Convergence rate

A study on the convergence rates of some statistical properties was carried out. Figs. (10) and (11) show the convergence of some properties as a function of the number of collocation points used in the probabilistic regression. Note that the values are normed by the value obtained by converged results from the direct Monte-Carlo simulations (approximately 10^5 realizations)

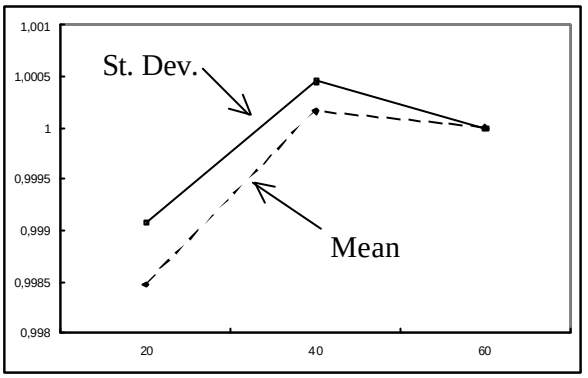


Fig. 10. Convergence rates of the mean and standard deviation of t_2.

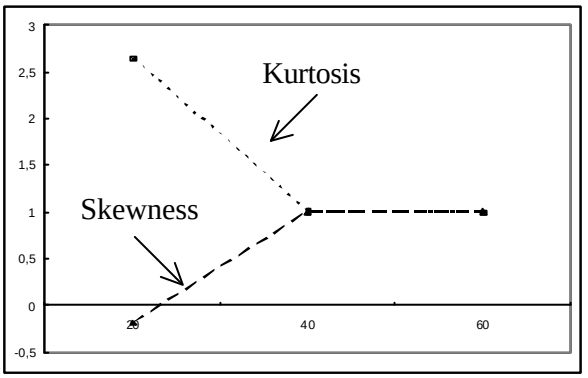


Fig. 11. Convergence rates of the skewness and kurtosis coefficients of t_2.

3.3.2. Robustness

For a given property (for instance, the total injected time t_2) G, we define the associated measure of robustness as:

$$R_G = 1 - Prob\left(G - G^{th} > 0\right) \qquad (9)$$

where G^{th} denotes the theoretical (deterministic) value of the property. Since we have a representation of the probabilistic parameters, such measures can readily be obtained from direct Monte-Carlo simulations (approximately 10^5 realizations). Results for times t_1 and t_2 are presented in Table 2.

Robustness values associated to both processing times are much lower than 1, which indicates that injection times are scattered and the process considered in this study has to be improved.

Table 2. Measures of Robustness.

Property	Measure of Robustness
Time 1, t_1	0.49
Time 2, t_2	0.50

4 Conclusion

Sophisticated models for flow impregnation, heat transfer and resin viscosity and curing have been proposed and implemented in softwares. The ability of the digital simulation to mimic real situation depends on how well properties of interest are characterized at every location. Unfortunately, experience gained so far provides the evidence that material characteristics at a location sometimes are not repeatable from part to part and from zone to zone.

A rigorous and efficient methodology has been presented in this paper to assess the probabilistic nature of a manufacturing process. Without loss of generality, it has been applied to the VARTM of a part where the mould filling time was the property of interest. Influence of random parameters on the injection cycle time has been predicted with an optimal number of simulations. This method can be used to optimize a manufacturing process and determine the optimum cycle time for instance. It can be also used to undertake sensitivity analysis to define random parameters that have the strongest influence on the process. Then one can implement a specific on-line and off-line control of these parameters to improve the stability of the manufacturing process.

In this study random parameters (resin viscosity, preform permeability and length of the distribution medium) were modeled assuming that they are homogeneous over the corresponding part and thus, can be modeled by random variables. This problem can be avoided by representing the material properties as random fields. Further work is needed to construct relevant parametric or non-parametric probabilistic models associated to random properties of interest regarding the manufacturing process: preform permeability, resin viscosity, heat transfer, clearance between a fiber preform and the mould walls or inserts that will lead to disturbing race-tracking effects during the filling stage. This work will provide a more realistic probabilistic modeling method for assessing robustness of the manufacturing process.

5 References

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