CRACK PROBLEM IN A FUNCTIONALLY GRADED MAGNETO-ELECTRO-ELASTIC COATING-HOMOGENEOUS ELASTIC SUBSTRATE STRUCTURE

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Abstract

A functionally graded magneto-electro-elastic material layer bonded to an elastic substrate is investigated. The functionally graded magneto-electro-elastic layer contains an edge crack that is perpendicular to the surface of the medium. Integral transform and dislocation density functions are employed to reduce the problem to the solution of a system of singular integral equations. Both impermeable crack and permeable crack assumptions are considered. Numerical results show the effect of crack configuration and loading combination parameter on the field intensity factors and energy release rate of the functionally graded magneto-electro-elastic strip bonded to an elastic substrate structure.

1 Introduction

Composite materials consisting of a piezoelectric phase and a piezomagnetic phase simultaneously process piezoelectric, piezomagnetic and magneto-electric effects, and thus they have wide applications in microwave electronics, optoelectronics and electronic instrumentation [1, 2]. Due to multi-field coupled effects, a magnetic field may induce an electric field and an elastic field in a magneto-electro-elastic solid, and vise versa. The coupled properties of piezoelectric/piezomagnetic composites offer great opportunities for engineers to create intelligent structures and devices that are capable of responding to internal and/or environment changes. In practical applications, a magneto-electro-elastic layer is usually bonded to an elastic substrate. Due to the difference of the thermo-physical properties between the magneto-electro-elastic layer and the substrate, the surface of the magneto-electro-elastic layer may suffer cracking during processing or under influence of the environment. Therefore, study of the surface cracking of laminated magneto-electro-elastic medium is of great importance. The problem of an edge crack has been study in references [3, 4] for homogeneous elastic layers under mechanical load.

On the other hand, the development of functionally graded materials (FGMs) has demonstrated that they have the potential to reduce the stress concentration and increase the fracture toughness. Consequently, the concept of FGMs can be extended to the magneto-electro-elastic materials to improve the reliability of magneto-electro-elastic materials and structures. These new kinds of materials with continuously varying properties may be called functionally graded magneto-electro-elastic materials.

The problem of an edge or an embedded crack perpendicular to the surface of the functionally graded magneto-electro-elastic layer has been considered by Ma et al. [5] for coupled magneto-electro-mechanical load. The dynamic fracture behavior of an edge crack or an embedded crack in functionally graded magneto-electro-elastic strip has been considered by Feng and Su [6, 7].

For many practical devices, functionally graded magneto-electro-elastic structures are surface-mounted. An important fracture mode is that the functionally graded magneto-electro-elastic layer contains a crack perpendicular to its surface. It is the purpose of this paper to investigate the fracture behavior of a functionally graded magneto-electro-elastic layer bonded to an elastic substrate and containing an edge crack perpendicular to its surface.

2. Formulation of the problem

The problem under consideration is described in Fig.1. A functionally graded magneto-electro-elastic strip of width h contain a crack of length c ($0<c<h$) perpendicular to the interface bonded to a homogeneous elastic substrate of width h_e. The system of rectangular Cartesian coordinates (x, y, z) is introduced in the layered material in such a way that the crack is located along the x-axis, and the y-axis is parallel to the interface. In the following, the subscripts x, y, z will be used to refer to the direction of coordinates, respectively. The functionally graded magneto-electro-elastic layer exhibits transversely isotropic behavior and is poled in z-direction.

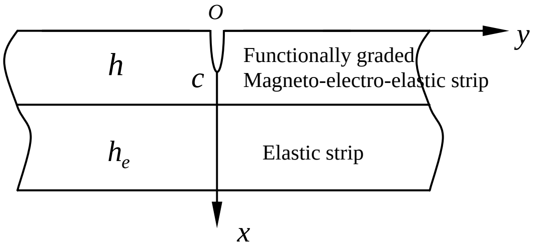

Fig.1 Geometry of the crack problem in the functionally graded coating-homogeneous substrate structure

When the structure is subjected to anti-plane mechanical and in-plane electric and magnetic loads, the crack problem involves the anti-plane elastic field coupled with the in-plane electric and magnetic field. The constitutive equations for the magneto-electro-elastic materials are as follows:

$$\tau_{kz} = c_{44} w_{,k} + e_{15}\phi_{,k} + f_{15}\psi_{,k} \quad (1a)$$

$$D_k = e_{15} w_{,k} - \varepsilon_{11}\phi_{,k} - g_{11}\psi_{,k} \quad (1b)$$

$$B_k = f_{15} w_{,k} - g_{11}\phi_{,k} - \mu_{11}\psi_{,k} \quad (1c)$$

where τ_{kz}, D_k, B_k $(k = x, y)$ are the anti-plane shear stress, in-plane electric displacement and magnetic induction, respectively; c_{44}, e_{15}, f_{15}, ε_{11}, g_{11} and μ_{11} are the elastic, the piezoelectric, the piezomagnetic, dielectric, electromagnetic and the magnetic constants, respectively; w, ϕ and ψ are the mechanical displacement, electric potential and magnetic potential of the functionally graded magneto-electro-elastic layer, respectively.

The material properties of the functionally graded magneto-electro-elastic are assumed to be one-dimensionally dependent as:

$$\begin{aligned}&(c_{44}, \varepsilon_{11}, e_{15}, f_{15}, g_{11}, \mu_{11}) \\ &= (c_{440}, \varepsilon_{110}, e_{150}, f_{150}, g_{110}, \mu_{110})\exp(\beta x)\end{aligned} \quad (2)$$

where c_{440}, ε_{110}, e_{150}, f_{150}, g_{110} and μ_{110} are the material constants at $x = 0$; β is a parameter to describe the material gradient distribution.

Substituting Eqs. (1) into the basic equations of functionally graded magneto-electro-elastic boundary value problem, i.e.,

$$\tau_{xz,x} + \tau_{yz,y} = 0,\ D_{x,x} + D_{y,y} = 0,\ B_{x,x} + B_{y,y} = 0 \quad (3)$$

We can obtained the equilibrium equation for functionally graded magneto-electro-elastic materials as follows,

$$c_{440}\left(\nabla^2 w + \beta w_{,x}\right) + e_{150}\left(\nabla^2\phi + \beta\phi_{,x}\right) + f_{150}\left(\nabla^2\psi + \beta\psi_{,x}\right) = 0 \quad (4a)$$

$$e_{150}\left(\nabla^2 w + \beta w_{,x}\right) - \varepsilon_{110}\left(\nabla^2\phi + \beta\phi_{,x}\right) - g_{110}\left(\nabla^2\psi + \beta\psi_{,x}\right) = 0 \quad (4b)$$

$$f_{150}\left(\nabla^2 w + \beta w_{,x}\right) - g_{110}\left(\nabla^2\phi + \beta\phi_{,x}\right) - \mu_{150}\left(\nabla^2\psi + \beta\psi_{,x}\right) = 0 \quad (4c)$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator in the variables x and y.

For convenience of mathematics and similar to the method mentioned by Feng and Su [6], we assume

$$\phi = d_1 w + e_1\chi + f_1\zeta,\ \psi = d_2 w + e_2\chi + f_2\zeta \quad (5)$$

where

$$d_1 = \frac{e_{150}\mu_{110} - f_{150}g_{110}}{\mu_{110}\varepsilon_{110} - g_{110}^2},\ d_2 = \frac{\varepsilon_{110}f_{150} - e_{150}g_{110}}{\mu_{110}\varepsilon_{110} - g_{110}^2} \quad (6a)$$

$$e_1 = \frac{-\mu_{110}}{\mu_{110}\varepsilon_{110} - g_{110}^2},\ e_2 = \frac{g_{110}}{\mu_{110}\varepsilon_{110} - g_{110}^2} \quad (6b)$$

$$f_1 = \frac{g_{110}}{\mu_{110}\varepsilon_{110} - g_{110}^2},\ f_2 = \frac{-\varepsilon_{110}}{\mu_{110}\varepsilon_{110} - g_{110}^2} \quad (6c)$$

are the known constants. The governing Eqs.(4) can be expressed as

$$\nabla^2 w + \beta w_{,x} = 0 \quad (7a)$$

$$\nabla^2\phi + \beta\phi_{,x} = 0 \quad (7b)$$

$$\nabla^2\psi + \beta\psi_{,x} = 0 \quad (7c)$$

The constitutive relations Eqs.(1) can be rewritten as

$$\tau_{xz} = \exp(\beta x)\left(m_{10} w_{1,x} + m_{20}\chi_{,x} + m_{30}\zeta_{,x}\right) \quad (8a)$$

$$\tau_{yz} = \exp(\beta x)\left(m_{10} w_{1,y} + m_{20}\chi_{,y} + m_{30}\zeta_{,y}\right) \quad (8b)$$

$$D_x = \exp(\beta x)\chi_{,x},\ D_y = \exp(\beta x)\chi_{,y} \quad (8c)$$

$$B_x = \exp(\beta x)\zeta_{,x},\ B_y = \exp(\beta x)\zeta_{,y} \quad (8d)$$

where,

$$m_{10} = e_{150} + \frac{\varepsilon_{110} f_{150}^2 - 2e_{150} f_{150} g_{110} + \mu_{110} e_{150}^2}{\mu_{110}\varepsilon_{110} - g_{110}^2} \quad (9a)$$

$$m_{20} = \frac{f_{150} g_{110} - e_{150}\mu_{110}}{\mu_{110}\varepsilon_{110} - g_{110}^2} \quad (9b)$$

$$m_{30} = \frac{e_{150} g_{110} - f_{150}\varepsilon_{110}}{\mu_{110}\varepsilon_{110} - g_{110}^2} \quad (9c)$$

The constitutive equations of the elastic layer are

$$\tau_{xz}^e = c_{44}^e w_{,x}^e , \ \tau_{yz}^e = c_{44}^e w_{,y}^e \tag{10}$$

where the subscript e represents the elastic layer, c_{44}^e is elastic constant of the elastic layer, w^e is the z-component of the displacement vector for the elastic layer. The equilibrium equation for the elastic layer is as follows

$$w_{,x}^e + w_{,y}^e = 0 \tag{11}$$

The mixed boundary value problem shown in Fig.1 must be solved under the following conditions. For the magneto-electrically impermeable crack, the boundary conditions are

$$\tau_{xz}(0,y) = D_x(0,y) = B_x(0,y) = 0 , \ -\infty < y < \infty \tag{12a}$$

$$D_x(h,y) = B_x(h,y) = 0 , \ -\infty < y < \infty \tag{12b}$$

$$\tau_{yz}(x,0) = -\tau_0 , \ D_y(x,0) = -D_0 , \ B_y(x,0) = -B_0 , \ 0 < x < c \tag{12c}$$

$$w(x,0) = \phi(x,0) = \psi(x,0) = 0 , \ c \le x \le h \tag{12d}$$

$$\tau_{xz}(h,y) = \tau_{xz}^e(h,y) , \ w(h,y) = w^e(h,y) , \ -\infty < y < \infty \tag{12e}$$

For the magneto-electrically permeable crack, the boundary conditions are

$$\tau_{xz}(0,y) = D_x(0,y) = B_x(0,y) = 0 , \ -\infty < y < \infty \tag{13a}$$

$$D_x(h,y) = B_x(h,y) = 0 , \ -\infty < y < \infty \tag{13b}$$

$$\tau_{yz}(x,0) = -\tau_0 , \ 0 < x < c \tag{13c}$$

$$w(x,0) = 0 , \ c \le x \le h \tag{13d}$$

$$\phi(x,0) = \psi(x,0) = 0 , \ 0 \le x \le h \tag{13e}$$

$$\tau_{xz}(h,y) = \tau_{xz}^e(h,y) , \ w(h,y) = w^e(h,y) , \ -\infty < y < \infty \tag{13f}$$

The electric displacement $D_y(x,0)$ and magnetic induction $B_y(x,0)$ on the crack surfaces consist of two components. The first is the imposed electric displacement $-D_0$ and magnetic induction $-B_0$ for $D_y(x,0)$ and $B_y(x,0)$. The second is the unknown caused by $-\tau_0$ for both of $D_y(x,0)$ and $B_y(x,0)$ [6].

3. Methods of solutions

Employing the Fourier transform on the variable x and the Fourier sine transform on the variable y and considering at infinity the quantity in the left side of Eqs.(4) must limited, it can be shown that

$$w(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_1(s) e^{-m_1 y} e^{-isx} ds + \frac{2}{\pi}\int_{-\infty}^{\infty} \left[A_2(s) e^{m_2 x} + A_3(s) e^{m_3 x}\right] \sin(sy) ds \tag{14a}$$

$$\chi(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} B_1(s) e^{-m_1 y} e^{-isx} ds + \frac{2}{\pi}\int_{-\infty}^{\infty} \left[B_2(s) e^{m_2 x} + B_3(s) e^{m_3 x}\right] \sin(sy) ds \tag{14b}$$

$$\zeta(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} C_1(s) e^{-m_1 y} e^{-isx} ds + \frac{2}{\pi}\int_{-\infty}^{\infty} \left[C_2(s) e^{m_2 x} + C_3(s) e^{m_3 x}\right] \sin(sy) ds \tag{14c}$$

where $A_j(s)$, $B_j(s)$, $C_j(s)$, $(j = 1,2,3)$ are unknown functions to be determined and

$$m_1(s) = \sqrt{s(s+i\beta)} \tag{15a}$$

$$m_2(s) = -\beta/2 - \sqrt{\beta^2/4 + s^2} \tag{15b}$$

$$m_3(s) = -\beta/2 + \sqrt{\beta^2/4 + s^2} \tag{15c}$$

The solution of the governing equations (11) for the elastic layer can be expressed in the following form:

$$w^e(x,y) = \frac{2}{\pi}\int_0^{\infty} \left[D_1(s) e^{sx} + D_2(s) e^{-sx}\right] \sin(sy) ds \tag{16}$$

To make the solution satisfy the boundary conditions (12), we introduce the following dislocation density function

$$g_w(x) = w_{,x}(x,0) \tag{17a}$$

$$g_\phi(x) = \phi_{,x}(x,0) \tag{17b}$$

$$g_\psi(x) = \psi_{,x}(x,0) \tag{17c}$$

and applying Eq.(14) and (5), we obtain

$$A_1(s) = \frac{i}{s}\int_0^c g_w(t) e^{ist} dt \tag{18a}$$

$$B_1(s) = \frac{i}{s}\int_0^c g_{w\phi\psi}^{e\varepsilon g}(t) e^{ist} dt \tag{18b}$$

$$C_1(s) = \frac{i}{s}\int_0^c g_{w\phi\psi}^{fg\mu}(t) e^{ist} dt \tag{18c}$$

where

$$g_{w\phi\psi}^{e\varepsilon g}(t) = e_{150} g_w(t) - \varepsilon_{110} g_\phi(t) - g_{110} g_\psi(t) \tag{19a}$$

$$g_{w\phi\psi}^{fg\mu}(t) = f_{150} g_w(t) - g_{110} g_\phi(t) - \mu_{110} g_\psi(t) \tag{19b}$$

Using Eqs.(8), (14), (18), together with the boundary conditions (12), it follows that

$$A_2 m_2 + A_3 m_3 = \int_0^c g_w(t) F_1(s,t) dt \tag{20a}$$

$$B_2 m_2 + B_3 m_3 = \int_0^c g_{w\phi\psi}^{e\varepsilon g}(t) F_1(s,t) dt \tag{20b}$$

$$C_2 m_2 + C_3 m_3 = \int_0^c g_{w\phi\psi}^{fg\mu}(t) F_1(s,t) dt \tag{20c}$$

$$B_2 m_2 e^{m_2 h} + B_3 m_3 e^{m_3 h} = \int_0^c g_{w\phi\psi}^{e\varepsilon g}(t) F_2(s,t) dt \tag{20d}$$

$$C_2 m_2 e^{m_2 h} + C_3 m_3 e^{m_3 h} = \int_0^c g_{w\phi\psi}^{fg\mu}(t) F_2(s,t) dt \tag{20e}$$

$$A_2 m_2 e^{m_2 h} + A_3 m_3 e^{m_3 h} - \frac{c_{44}^e}{m_{10}\exp(\beta h)} s\left[D_1 e^{sh} - D_2 e^{-sh}\right] = \int_0^c g_w(t) F_2(s,t) dt \tag{20f}$$

$$A_2 e^{m_2 h} + A_3 e^{m_3 h} - D_1 e^{sh} - D_2 e^{-sh} = \int_0^c g_w(t) F_3(s,t) dt \tag{20g}$$

$$D_1(s) e^{s(h+h_e)} - D_2(s) e^{-s(h+h_e)} = 0 \tag{20h}$$

where the expressions of functions $F_j(s,t)$ $(j=1,2)$ are

$$F_1(s,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{-s}{m_1^2(\rho)+s^2} e^{i\rho t} d\rho \tag{21a}$$

$$F_2(s,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{-s}{m_1^2(\rho)+s^2} e^{i\rho(t-h)} d\rho \tag{21b}$$

$$F_3(s,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{i}{\rho}\frac{-s}{m_1^2(\rho)+s^2} e^{i\rho(t-h)} d\rho \quad 21c)$$

By using the theory of residues, the integrals in Eqs.(21) may be evaluated as follows:

$$F_1(s,t) = \frac{s}{m_2(s) - m_3(s)} e^{-tm_3(s)} \tag{22a}$$

$$F_2(s,t) = \frac{s}{m_2(s) - m_3(s)} e^{(h-t)m_2(s)} \tag{22b}$$

$$F_3(s,t) = -\frac{1}{2s} + \frac{m_3(s)}{\left[m_3(s) - m_2(s)\right]s} e^{(h-t)m_2(s)} \tag{22c}$$

From Eqs.(22), unknowns $A_2(s)$, $A_3(s)$, $B_2(s)$, $B_3(s)$, $C_2(s)$, $C_3(s)$, $D_1(s)$ and $D_2(s)$ can be obtained and omitted here.

Substituting the expressions of $A_2(s)$, $A_3(s)$, $B_2(s)$, $B_3(s)$, $C_2(s)$, $C_3(s)$, $D_1(s)$ and $D_2(s)$ into Eqs.(8), and by using the these expressions and the boundary condition (12), we obtain the following integral equations:

$$\int_0^c [H(x,t)]\begin{Bmatrix} g_w(t) \\ g_\phi(t) \\ g_\psi(t) \end{Bmatrix} dt + \frac{2}{\pi}\int_0^c [G(x,t)]\begin{Bmatrix} g_w(t) \\ g_\phi(t) \\ g_\psi(t) \end{Bmatrix} dt = e^{-\beta x}\begin{Bmatrix} \tau_y(x) \\ D_y(x) \\ B_y(x) \end{Bmatrix} \tag{23}$$

where the kernels $H(x,t)$, $G(x,t)$ are given by

$$\left[H(x,t)\right] = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \left[H_0\right] e^{is(t-x)} ds \tag{24}$$

$$\left[G(x,t)\right] = \int_0^{\infty} \left[G_0\right] s ds \tag{25}$$

where $\left[H_0\right]$ is a constant matrix which depend only on the material properties. It can be shown from Eq. (24) that

$$\left[H\right] = \frac{1}{\pi}\left[H_0\right]\frac{1}{t-x} \tag{26}$$

By applying Eq. (23) to crack surface boundary conditions (12), and using (26), we obtain the following singular integral equation:

$$\frac{1}{\pi}[H_0]\int_0^c \frac{1}{t-x}\begin{Bmatrix} g_w(t) \\ g_\phi(t) \\ g_\psi(t) \end{Bmatrix} dt + \frac{2}{\pi}\int_0^c [G(x,t)]\begin{Bmatrix} g_w(t) \\ g_\phi(t) \\ g_\psi(t) \end{Bmatrix} dt = \begin{Bmatrix} -\tau_0 \\ -D_0 \\ -B_0 \end{Bmatrix} \tag{27}$$

4. Solution of integral equation

This integral equation can be used to determine the as yet unknown functions g_w, g_ϕ, and g_ψ. In order to simplify the analysis, the interval [0, c] is normalized by defining

$$x = \frac{1}{2}(1+r),\ t = \frac{1}{2}(1+u) \tag{28a}$$

$$g_w(t) = G_w(u),\ g_\phi(t) = G_\phi(u),\ g_\psi(t) = G_\psi(u) \tag{28b}$$

$$-\tau_\infty = f_w(r),\ -D_0 = f_\phi(r),\ -B_0 = f_\psi(r) \tag{28c}$$

and then the integral equation (27) would become

$$\frac{1}{\pi}[H_0]\int_0^c \frac{1}{u-r}\begin{Bmatrix} G_w(u) \\ G_\phi(u) \\ G_\psi(u) \end{Bmatrix} dt + \frac{2}{\pi}\int_0^c [G'(r,u)]\begin{Bmatrix} G_w(u) \\ G_\phi(u) \\ G_\psi(u) \end{Bmatrix} dt = \begin{Bmatrix} f_w(r) \\ f_\phi(r) \\ f_\psi(r) \end{Bmatrix} \tag{29}$$

Where

$$G'(r,u) = \frac{c}{2} G(x,t) \tag{30}$$

It can be observed that the weight function of the solution of (29) is $w(u) = (1-u)^{-1/2}$, and hence the solution of the integral equation (29) may be expressed as [8~10]

$$G_w(u) = \frac{1}{\sqrt{1-u}}\sum_{n=0}^{\infty} c_n T_n(u) \tag{31a}$$

$$G_\phi(u) = \frac{1}{\sqrt{1-u}}\sum_{n=0}^{\infty} d_n T_n(u) \tag{31b}$$

$$G_\psi(u) = \frac{1}{\sqrt{1-u}}\sum_{n=0}^{\infty} e_n T_n(u) \tag{31c}$$

where T_n is the Chebyshev polynomial of the first kind and $c_0, c_1, c_2, \cdots$, $d_0, d_1, d_2, \cdots$ and $e_0, e_1, e_2, \cdots$ are unknown constants. Note that the single-values condition (11) becomes

$$\int_{-1}^{1} G_w(t) dt = 0,\ \int_{-1}^{1} G_\phi(t) dt = 0,\ \int_{-1}^{1} G_\psi(t) dt = 0 \tag{32}$$

From Eqs.(31) and (32) and the orthogonality of Chebyshev polynomials it can be shown that $c_0 = 0$, $d_0 = 0$, $e_0 = 0$. The remaining constants are then determined by substituting from (31) into (29).

The singularity of the integral equation (29) may be removed by using the following relations

$$\int_{-1}^{1}\frac{T_n(u)du}{(u-r)\sqrt{1-u}}=T_n(r)\int_{-1}^{1}\frac{du}{(u-r)\sqrt{1-u}}+\int_{-1}^{1}\frac{T_n(u)-T_n(r)}{(u-r)\sqrt{1-u}}du \quad (33)$$

where in the second term on the right-hand side in the integrand is bounded, the first integral is given by

$$\int_{-1}^{1}\frac{du}{(u-\sigma)\sqrt{1-u}}=\frac{\log|B(\sigma)|}{\sqrt{1-\sigma}} \quad (\sigma<1)\,,\quad B(\sigma)=\frac{1+\sqrt{(1-\sigma)/2}}{1-\sqrt{(1-\sigma)/2}} \quad (34)$$

In the problem under consideration from Eqs.(28), (33) and (34), it may be seen that $\sigma=r$, which, with $-1<r<1$, indeed satisfies the condition $\sigma<1$. All other integrals involving the solution are evaluated by using Gaussian quadrature, and the resulting functional equation is solved by using the collocation method [11]. Since it is not possible to investigate the regularity of the corresponding infinite matrix, the convergence of the solution may be examined by varying the number of the term retained in the series. Because of the nature of the solution in this problem, it is preferable to concentrate the collocation point u_k near the ends. They are, thus, chosen as

$$T_N(u_l)=0\,,\ u_l=\cos\left\{(2l-1)\frac{\pi}{2N}\right\} \quad (l=1,2,\cdots,N) \quad (35)$$

where N is the number of terms retained in the series.

5. Field intensity factors and energy release rates

After determining the coefficients $c_1,c_2,\cdots,c_N$, $d_1,d_2,\cdots,d_N$ and $e_1,e_2,\cdots,e_N$ for the edge crack problem, the stress intensity factors (SIFs), the electric displacement intensity factors (EDIFs) and the magnetic induction intensity factors (MIIFs) at the crack tips may be defined and evaluated as

$$K_{III}(c)=\lim_{x\to c}\sqrt{2(x-c)}\tau_{yz}(x,0)=-e^{-\beta c}\sqrt{c}\sum_{n=1}^{\infty}\left[c_{44}c_n+e_{15}d_n+f_{15}e_n\right] \quad (36)$$

$$K_D(c)=\lim_{x\to c}\sqrt{2(x-c)}D_y(x,0)=-e^{-\beta c}\sqrt{c}\sum_{n=1}^{\infty}\left[e_{15}c_n-\varepsilon_{11}d_n-g_{15}e_n\right] \quad (37)$$

$$K_B(c)=\lim_{x\to c}\sqrt{2(x-c)}B_y(x,0)=-e^{-\beta c}\sqrt{c}\sum_{n=1}^{\infty}\left[f_{15}c_n-g_{11}d_n-\mu_{11}e_n\right] \quad (38)$$

From Eqs.(23, 36~37), it is easy to know that the SIFs, the EDIFs and the MIIFs are independent and that they can be obtained by solving Eqs.(29), respectively. Namely, the SIFs, the EDIFs and the MIIFs are only related to the corresponding mechanical, electrical and magnetical loading. So that it should be noted that for the magneto-electro-elastically impermeable cracks, as electrical and/or magnetical load are applied, the SIFs cannot perfectly describe the fracture characteristics as in the purely elastic case. Therefore, the energy release rates (ERRs) G are introduced by calculating the work done in closing the crack tip over an infinitesimal distance. In accordance with the definition of the energy release rate proposed by [12], after a similar deriving process carried out by Wang and Yu [13] and Feng and Su [6, 7], we can finally obtain

$$G(c)=\frac{1}{2}\left[K_{III}(c)\tilde{K}_w(c)+K_D(c)\tilde{K}_\phi(c)+K_B(c)\tilde{K}_\psi(c)\right] \quad (39)$$

where

$$\tilde{K}_w(c)=\frac{\left(\mu_{11}\varepsilon_{11}-g_{11}^2\right)K_{III}(c)+\left(e_{15}\mu_{11}-f_{15}g_{11}\right)K_D(c)+\left(f_{15}\varepsilon_{11}-e_{15}g_{11}\right)K_B(c)}{c_{44}\mu_{11}\varepsilon_{11}+e_{15}^2\mu_{11}+f_{15}^2\varepsilon_{11}-c_{44}g_{11}^2-2e_{15}f_{15}g_{11}} \quad (40a)$$

$$\tilde{K}_\phi(c)=\frac{\left(e_{15}\mu_{11}-f_{15}g_{11}\right)K_{III}(c)-\left(c_{44}\mu_{11}+f_{15}^2\right)K_D(c)+\left(c_{44}g_{11}+e_{15}f_{15}\right)K_B(c)}{c_{44}\mu_{11}\varepsilon_{11}+e_{15}^2\mu_{11}+f_{15}^2\varepsilon_{11}-c_{44}g_{11}^2-2e_{15}f_{15}g_{11}} \quad (40b)$$

$$\tilde{K}_w(c)=\frac{\left(f_{15}\varepsilon_{11}-e_{15}g_{11}\right)K_{III}(c)+\left(c_{44}g_{11}+e_{15}f_{15}\right)K_D(c)-\left(c_{44}\varepsilon_{11}+e_{15}^2\right)K_B(c)}{c_{44}\mu_{11}\varepsilon_{11}+e_{15}^2\mu_{11}+f_{15}^2\varepsilon_{11}-c_{44}g_{11}^2-2e_{15}f_{15}g_{11}} \quad (40c)$$

For magneto-electrically permeable case, the field intensity factors and ERRs are respectively

$$K_{III}(c)=\lim_{x\to c}\sqrt{2(x-c)}\tau_{yz}(x,0)=-e^{-\beta c}\sqrt{c}\sum_{n=1}^{\infty}c_{44}c_n \quad (41)$$

$$K_D(c)=\frac{e_{15}}{c_{44}}K_{III}(c) \quad (42)$$

$$K_B(c)=\frac{f_{15}}{c_{44}}K_{III}(c) \quad (43)$$

$$G(c)=\frac{K_{III}^2(c)}{2c_{44}} \quad (44)$$

6. Conclusions

A surface crack in a functionally magneto-electro-elastic strip bonded to an elastic layer is considered. Both impermeable and permeable crack assumption are considere. For the magneto-electrically impermeable cracks, the SIFs, the EDIFs and MIIFs are, respectively, related to applied mechanical loads, electrical loads and magnetical loads only. The ERRs depend on both applied loads including mechanical, electrical and magnetical loads and material parameters. For the magneto-electrically permeable cracks, both magnetical and electrical loads have no contribution to ERRs and field intensity factors.

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