CHARACTERIZATION OF CURING STRESS EFFECTS ON FRACTURE BEHAVIOUR OF FRP COMPOSITE LAMINATE WITH ELLIPTICAL CUTOUTS

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Keywords: *Composites, Delamination, Thermomechanical, FEA, MCCI, SERR.*

Abstract

This paper deals with the characterization of curing stress effects on fracture behaviour of FRP composite laminates with elliptical cutouts. Two sets of full three dimensional finite element analyses(one with residual thermal stresses developed during the curing of the laminate and the other without residual thermal stresses i.e. with mechanical loading only) have been performed to calculate the displacements and interlaminar stresses along the delaminated interfaces responsible for the delamination onset and propagation. Modified Crack Closure Integral (MCCI) method based on the concept of Linear Elastic Fracture Mechanics (LEFM) has been followed to evaluate the individual modes of Strain Energy Release Rates (SERR) along the delamination front. Numerical calculations are carried out for multiply cross-ply and angle-ply glass/epoxy laminates and the strain energy release rate plots demonstrate large asymmetries along the delamination front due to the interaction of residual curing stresses and superimposed mechanical loading. It is observed that residual thermal stresses have detrimental effect of enhancing the delamination growth behaviour. On subsequent loading this can be a potential source of causing premature failure due to superimposed thermomechanical effects.

1 Introduction

Delamination in fiber-reinforced composites has been a subject of intensive research for many years. Advanced composite materials are prone to delamination damage which may also arise out of low-velocity impact, and during typical manufacturing operations, such as drilling, reaming, and trimming. Cutouts or holes are commonly drilled in composite parts to facilitate fasteners, avionics, or weight savings. Often, drilling leaves defects in the region around the circumference of the cutout or hole. Drilling induced interlaminar delamination is shown in Fig. 1. Under mechanical loading, the propagations of these interlaminar delaminations have been one of the most serious problems leading to the failure of composite laminates. Further, thermal residual stresses resulting from the curing action during the preparation of the laminated FRP composites influence the delamination growth behaviour. Therefore an understanding on the delamination growth characteristics in the presence of thermoelastic stress field, which has often been not considered, is essential.

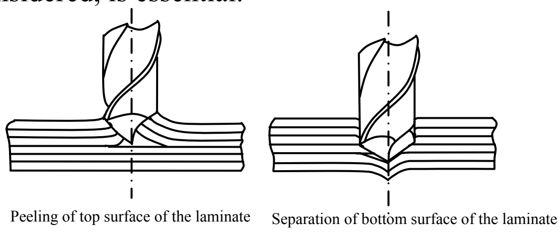

Fig.1. Drilling induced interlaminar delamination.

The problem of delamination growth have been widely studied especially for idealized situations such as the edge delamination of specially designed plate specimen. Stress analysis based study for the identification of the locations in composite structures at which delamination is likely to initiate have been made by Pipes and Pagano [1]. They have shown that high positive interlaminar normal stress is detrimental for the laminate strength. A high interlaminar shear stress may also cause delamination as described by Herakovich [2]. However, it has been pointed out by Wang [3] that a stress analysis may give an erroneous prediction of the delamination location, especially if the delamination is in mixed-mode. In fracture mechanics approach to delamination problems in composites [4-6], the strain energy release rate (G) is often evaluated since this is an experimental measurable quantity and has been mathematically well defined. Wang and Crossman [4] in their works assumed that material flaws exist randomly at any interface in the composite. A fracture mechanics approach based on the strain energy release rate is then used in order to determine the location most energetically favourable for delamination. Rybicki et al. [5] used SERR to study free-edge delamination problems. While characterizing the delamination onset and growth in a composite laminate, O'Brien [6] looked at the energy release rate at the interfaces at free edges of the laminate. The calculation of the strain energy release rate was based on the stiffness loss due to delamination, calculated by means of classical laminate theory. Hellen [7] used Virtual Crack Extension (VCE) method to calculate G. A disadvantage of this method is that it is not possible to separate G into its components, G_{I}, G_{II} and G_{III}. However, the research works of Wilkins et al. [8] and Hahn and Johannesson [9] indicate that delamination initiation and growth are very much dependent on a combination of the opening and shearing mode components of G. For example in the work of Hahn and Johannesson [9] G_{IIC} was found to be five times larger than G_{IC} for graphite/epoxy. This suggests that a separation of the strain energy release rater into its components may increase the predictive power of the fracture mechanics analysis. Wang [10] discussed the possible delamination crack surface closure which may occur in mixed-mode situations. Strain energy release rate calculations along an elliptical delamination front have been conducted by Pradhan and Chakraborthy [11] to assess the effect of stacking sequence, aspect ratio of embedded delamination at the interface and laminate thickness on the interlaminar delamination crack growth behaviour.

The thermal residual stresses while curing the laminate (cooling form an elevated temperature to thermosetting to the room temperature) are known as manufacturing induced stresses or curing stresses. They significantly affect the initiation and further propagation of delamination in cross-ply and angle-ply FRP composites. Therefore, proper consideration of residual thermal stress effects should be made in analyzing the delamination crack growth behaviour.

Issue such as residual thermal stress effects are beginning to be addressed in recent literature by Yin [12]. An experimental study by Fish and Malaznik [13] suggested the residual thermal stress effects are significant for delamination initiation but not for propagation along a 0/90 interface in laminated composites. Nairn [14] found that neglecting residual thermal stress in data reduction for DCB specimens may result in huge errors in the calculation of fracture toughness. Tay and Shen [15] obtained the distribution of the local strain energy release rate along the delamination front via the Virtual Crack Closure Technique (VCCT) to three-dimensional finite element model of circular delaminations embedded in woven and non-woven composite laminates considering residual thermal stresses. They have concluded that growth criteria based on components of the strain energy release rate predict the rate of delamination growth much better than those based on the total strain energy release rate. Pradhan and Panda [16] studied the thermoelastic effect of material anisotropy and curing stresses on interlaminar embedded delamination fracture characteristics in multiply laminated fiber-reinforced polymeric (FRP) composites, and they have concluded that curing stresses have a significant effect on the delamination behaviour. In order to gain a better understanding of the effects of residual thermal stresses on the delamination crack growth behaviour in laminated FRP composites, extensive study and analysis results are needed.

This paper presents two sets of full three-dimensional thermoelastic finite element analyses to study the curing stress effect on delamination fracture growth emanating from elliptical cutouts in laminated Fiber-Reinforced Polymeric (FRP) composites. Also the influence of residual thermal stresses and material anisotropy on the delamination fracture behaviour characteristics is addressed. Modified Crack Closure Integral (MCCI) methods based on the concepts of Linear Elastic Fracture Mechanics (LEFM) have been used as a meaningful tool to calculate the individual modes of Strain Energy Release Rates (SERR) from the thermoelastic stress and displacement fields due to a combined thermal and mechanical loading. Residual stresses developed due to the thermoelastic anisotropy of the laminae are found to strongly influence the delamination onset and propagation characteristics, which have been reflected by the asymmetries in the nature of energy release rate plots and their significant variation along the delamination front.

2 Laminate specimen geometry and thermoelastic material properties

The laminate configurations considered in the analyses have ply sequence of $[0_m/90_m]_T$ cross-ply and $[+45_m/-45_m]_T$ angle-ply with elliptical delamination in the resin layer centrally located at the interface around the edge of the elliptical cutout as shown in Fig. 2. The exploded view of which is shown in Fig. 3. The thermoelastic material properties of glass/epoxy composite and the material properties of the interface isotropic resin layer are given in Table 1. Referring to Fig. 2, the various dimensions of the laminate and the elliptical cutout for finite element modelling have been presented as follows in terms of geometric ratios w.r.t typical specimen length of L = 200mm. Laminate width: W/L = 1/2, Laminate thickness: t/L = 1/10, Elliptical cutout geometry: Aspect ratio e = b/a = 1/2 with b/L = 1/40, Size of elliptical delamination emanating form the cutout: p/a = 1.5 and q/b = 1.5.

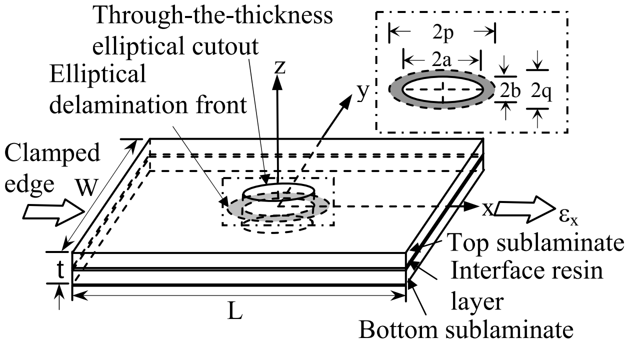

Fig. 2. Schematic of the laminate specimen with an elliptical interfacial delamination in the resin layer emanating form the edge of the elliptical cutout.

Boundary conditions:

Referring to the cross-ply or angle-ply laminate with a central elliptical cutout as shown in Fig. 2:

at x = 0; $u_x = u_y = u_z = 0$ and $\theta_x = \theta_y = \theta_z = 0$, and the uniform applied extension is given as

at x = L; $\varepsilon_x = 0.005$.

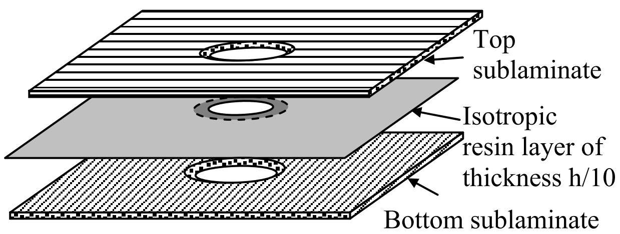

Fig. 3. Exploded view of the laminate specimen with an elliptical interfacial delamination in the resin layer emanating from the edge of the

The constraining effect of manufacturing residual thermal stresses has been simulated by uniform temperature drop form a stress-free state at curing temperature of 120°C to room temperature of 30°C over the delaminated composite laminate and then the composite laminate is subjected to uniform axial extension along the x-axis for subsequent mechanical loading. The interface resin layer, in which the delamination is assumed to be embedded, has the properties of a low modulus isotropic material.

3 Finite Element Analysis (FEA) of thermoelastic effects on delamination growth

The general problem of modelling the growth of delaminations of arbitrary shapes along with different orientations depicting the practical aspects of the effects of residual thermal stresses on the superimposed thermoelastic loading can be tackled efficiently by using a rigorous three-dimensional finite element analysis. This would be so necessary particularly for studying the delamination growth behaviour emanating from the edge of cutouts in cross-ply and angle-ply laminates. The reason, why a full three-dimensional finite element analysis is a must, in spite of its large computer storage space and memory requirements, is to keep the scope of modelling as general as possible to accommodate curvilinear delamination front propagation and composite laminates with varied lay-up schemes.

Table 1. Thermoelastic material properties (Adopted from Ref. [16])

Material	Glass/Epoxy	Resin layer
Elastic Properties	E_x (GPa) = 36.6.00	E = 3.89 GPa
	$E_y = E_z$ (GPa) = 8.27	
	$G_{xy} = G_{xz}$ (GPa) = 4.14	
	G_{yz} (GPa) = 4.00	
	$\upsilon_{xy} = \upsilon_{xz}$ = 0.25	ν= 0.37
	υ_{yz} = 0.27	
Thermal properties Coefficient of thermal expansion	$\alpha_x = 7\times10^{-6}$/ °C	$\alpha = 44 \times 10^{-6}$/°C
	$\alpha_y = \alpha_z = 21\times10^{-6}$/ °C	

3.1 SERR Evaluation Procedure

In the present work, the three components of strain energy release rate viz. G_I, G_{II} and G_{III} have been used as parameters for assessing delamination growth. Energy release rate procedure of evaluation is robust as it is based upon a sound energy balance principle and mode separation of SERR is possible. The energy release rates are evaluated using the Virtual Crack Closure Technique (VCCT) proposed by Rybicki and Kanninen [17]. The VCCT technique is based on Irwin's assumption [18] that when a crack extends by a small amount, the energy released in the process is equal to the work required to close the crack to its original length. In literature, it is also addressed as Modified Crack Closure Integral (MCCI). Thus the separate modes of SERR can be calculated by a single finite element analysis. The strain energy release rate components at any point on the delamination front under mechanical and thermal loading due to a uniform temperature drop from curing temperature to room temperature is obtained by superposing their respective effects based on the assumptions of linear elasticity.

A schematic representation of elliptical delamination front propagation is shown in Fig. 4. For a curved crack front the tangent and normal along the curvature varies from location to location as shown in Fig.4. Appropriate transformations are performed to obtain the displacements and stresses with respect to the [n t z] coordinate system from the finite element output, which is with respect to the global [x y z] coordinate system. The components of the strain energy release rate due to the propagation of the elliptical delamination front subjected to thermomechanical loading are calculated by the Eqs. (1), (2) and (3) as:

$$G_I(\phi) = \lim_{\Delta A \to 0} \frac{1}{2\,\Delta A} \int_{a}^{a+\Delta a} \int_{-\Delta a/2}^{\Delta a/2} [\sigma_{zzM}(n,t) + \sigma_{zzT}(n,t)] \times [\delta u_{zM}(n-\Delta a,t) + \delta u_{zT}(n-\Delta a,t)]\, dn dt \qquad (1)$$

$$G_{II}(\phi) = \lim_{\Delta A \to 0} \frac{1}{2\,\Delta A} \int_{a}^{a+\Delta a} \int_{-\Delta a/2}^{\Delta a/2} [\sigma_{znM}(n,t) + \sigma_{znT}(n,t)] \times [\delta u_{nM}(n-\Delta a,t) + \delta u_{nT}(n-\Delta a,t)]\, dn dt \qquad (2)$$

$$G_{III}(\phi) = \lim_{\Delta A \to 0} \frac{1}{2\,\Delta A} \int_{a}^{a+\Delta a} \int_{-\Delta a/2}^{\Delta a/2} [\sigma_{ztM}(n,t) + \sigma_{ztT}(n,t)] \times [\delta u_{tM}(n-\Delta a,t) + \delta u_{tT}(n-\Delta a,t)]\, dn dt \qquad (3)$$

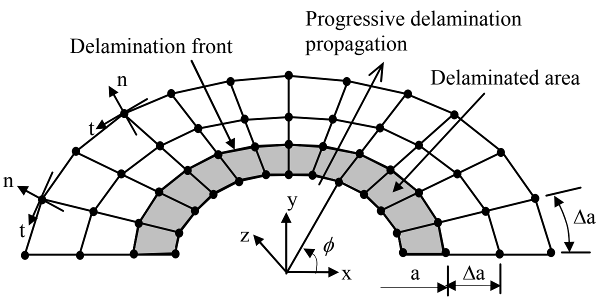

Fig.4. Interfacial elliptical delamination front

Here, $G_I(\phi)$, $G_{II}(\phi)$ and $G_{III}(\phi)$ are respectively the mode I, mode II and mode III strain energy release rates at any angular position ‘ϕ’ along the delamination front. The subscripts ‘T’ and ‘M’ in the above equations represent respectively the thermal and mechanical effects of the denoted parameters.

In Eqs. (1-3), [σ_{zz} τ_{zn} τ_{zt}] are the elliptical local coordinate transformations of interlaminar stresses [σ_{zz} τ_{zx} τ_{zy}] in global coordinates and δu_z, δu_n, δu_t are respectively the relative opening, normal and tangential displacements of the upper delaminated surface to the lower one along the delamination boundary. The total energy release rate considering the thermal residual stress effects can then be expressed simply as the algebraic sum of the individual modes as follows:

$$G_{total} = G_I + G_{II} + G_{III} \qquad (4)$$

3.2 FE meshing of laminate specimen with annular shaped elliptical delamination emanating from the edge of a central elliptical cutout

Delamination propagations at different ply interfaces have been found to be inherently three-dimensional. This indicates that three-dimensional analyses are required when delaminations are to be modelled at different ply interfaces [19]. However 3D solid or brick elements used for this purpose makes the analysis computationally much expensive for modelling the multi-layered FRP composite laminates. In some delamination studies a global-local modelling approach was followed by using layered shell elements away from the delamination zone and brick or solid elements were used in the more sensitive delamination front [20]. In composite laminates, due to their inherent complications, exact closed form expressions for the energy release rates are not possible. This leads to the finite element method based evaluation of energy release rates. The stress state has been evaluated throughout the composite laminate using finite element analysis software ANSYS 8.0 on HP workstation. Eight-node layered brick elements (Solid 46) with three degrees of freedom (translations in the x, y and z directions) at each node have been used to model the analysis domain.

In this investigation, sublaminate techniques have been used to model the delamination emanating from the edge of circular hole in laminated FRP composites. The analysis domain has been divided into three sublaminates (Fig. 3). The top and bottom sublaminates comprising of several layers represent the composite type i.e glass/epoxy. The middle sublaminate models the resin layer, in which the delaminations have been assumed to be embedded. In the 3D finite element analyses, the mesh has been made progressively finer as it approaches the delamination front along the interface. The corresponding nodes of the top and bottom sublaminates over the undelaminated region are constrained by Multipoint Constraint Elements (MPC) so that the continuity condition is prevailed. By sequentially removing these constraints, progressive propagation of self-similar elliptical delamination has been realized as per Raju et al. [21]. To avoid delamination face interpenetration due to bending-stretching coupling along some portions of the delamination front, bi-linear 3-D node-to- node contact elements (CONTA 178) are used in the delaminated region. Fig. 5 shows the 3D finite element mesh developed for studying the thermoelastic effect on delamination growth behaviour. The displacements and stresses are obtained from the three dimensional finite element analyses. Under the moderate temperature variation from curing state of 140°C to room temperature of 30°C both thermophysical and thermomechanical properties are assumed to remain unchanged. For elliptical delamination, the mode definition changes along the edge of the delamination front. This results in a mixed-mode behaviour under any individual mode of loading. Relevant error analyses and mesh refinements have been carried out for an error limit of 0.001% on strain energy release rates along the delamination front. For this purpose the element size near the delamination front has been taken to be nearly equal to be one-quarter of the individual ply thickness along the plane of delamination [22]. To capture the delamination region stress field and avoid the oscillatory nature of the stresses very near to the crack front, this element size is found to be sufficient. Progressive mesh refinements have been made judiciously from delamination front to the laminate boundary. This scientifically graded mesh pattern significantly reduces the burden of computational effort necessary for an otherwise complex thermoelastic fracture analysis. Appropriate transformations are performed to obtain the displacements and stresses in the finite element output with respect to the [n t z] coordinate system from the global [x y z] coordinate system. The three components of strain energy release rate viz. G_I, G_{II} and G_{III} have been used as parameters for assessing delamination growth characteristics.

4 Results and Discussions

The interaction of initial residual thermal stress components and subsequent mechanical loading affect the nature of delamination damage propagation at the delaminated interface. Calculations of the interlaminar stresses σ_{zz}, τ_{zr}, and $\tau_{z\theta}$ along the edge of the elliptical delamination front are the important point of the strain energy release rate analysis to study the delamination crack growth behaviour. The annular shaped elliptical delamination is assumed to pre-exist along the centre of the 0/90 interface and +45/-45 interface in the two laminate cases considered. The residual stresses are induced when the laminate is cooled down to room temperature from the curing state of 120°C. When the residual stress effects are considered, several peculiar behaviours are observed along the delamination front, on subsequent mechanical loading of the laminate.

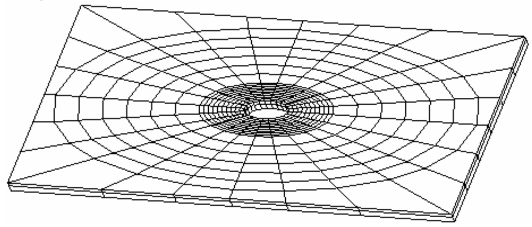

Fig. 5. Finite element meshing of the laminate specimen containing annular shaped elliptical delamination in the resin layer along at the interface along the edge of the central elliptical cutout.

4.1 Effect of thermoelastic interaction on the distribution of strain energy release rate

This section presents the strain energy release rate distribution due to the thermoelastic superimposed effect upon the interfacial delamination propagation. Subscript ‘M’ in the numerical results represents the effect due to purely mechanical loading, whereas ‘TM’ indicates the superimposed effect of residual thermal stresses and the mechanical loading.

4.1.1 Comparison of G_I, G_{II} and G_{III} distribution for cross-ply $[0_m/90_m]_T$ glass/epoxy laminates

The annular shaped elliptical delamination is embedded along the 0/90 interface in the $[0_m/90_m]_T$ cross-ply glass/epoxy FRP laminate. The distribution of individual modes of strain energy release rates G_I, G_{II} and G_{III} along the circular delamination front with respect to its angular location ‘ϕ’ are displayed in Figs 6-8, respectively. It is observed that contribution of Mode-I delamination front propagation is less in comparison to other two modes. This is due to the nature of loading, which causes crack closure over some portion of the delamination zone. Referring to Fig. 6, the effect of residual thermal stress on Mode I delamination front propagation is appreciable and can be seen from the mismatch in the strain energy release rate plots of G_{IM} and G_{ITM}. The mismatch is seen to be mostly non-uniform all along the delamination front. The maximum mismatch occurs at ϕ = 90° and 270° along the delamination front. The effect of residual thermal stresses is to enhance the delamination front propagation.

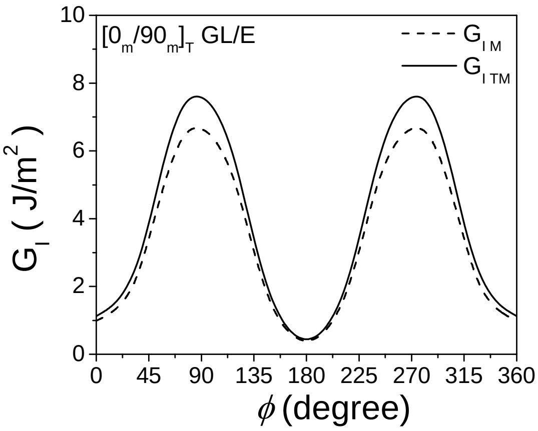

Fig.6. G_I distribution at 0/90 interface of $[0_m/90_m]_T$ GL/E composite laminate.

The comparison of Mode-II SERR along the delamination front with and without considering residual stresses is shown in Fig. 7. It is clearly seen that the mode II strain energy release rate G_{IITM} due to residual thermal stresses and mechanical loading is more than G_{IM}, which is due to the nature of loading and mismatch in anisotropic material properties at the interfaces. It is also observed that the mode II component of SERR dominates the overall delamination fracture behaviour. The difference between the G_{IIM} and G_{IITM} components are significant at angular position ϕ = 180°. Though both G_{IIM} and G_{IITM} follow similar asymptotic patterns at different angular positions around the delamination front, there is a significant rise in the maximum value of G_{IIM} due to the curing stress effects. Also a close look at the strain energy release rate plots reveals that the area between the curves for mechanical and thermomechanical analyses are significantly different indicating a premature failure of the composite laminate. The maximum value of G_{II} is three times the maximum value of G_I.

Referring to Fig. 8, mode III contribution (G_{III}) to delamination front propagation is considerable, However, the effect of residual thermal stress on G_{III} is insignificant as compared to the G_I and G_{II} strain energy release rate values. The maximum of G_{III} occurs at angular locations ϕ = 130° and 240° along the delamination front.

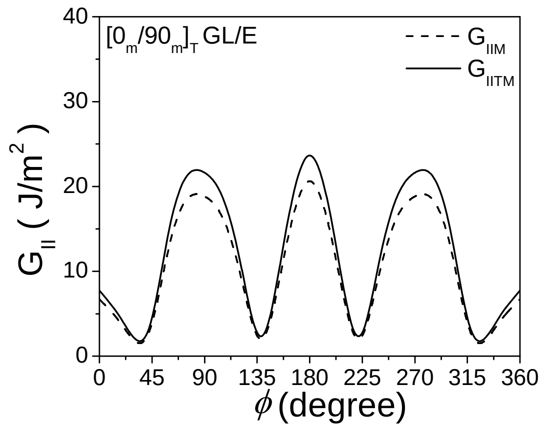

Fig.7. G_{II} distribution at 0/90 interface of $[0_m/90_m]_T$ GL/E composite laminate.

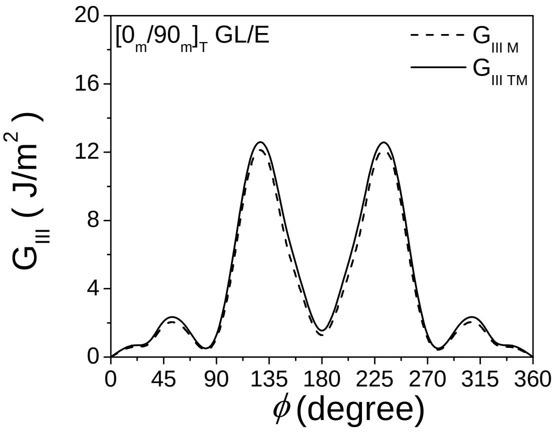

Fig.8. G_{III} distribution at 0/90 interface of $[0_m/90_m]_T$ GL/E composite laminate.

4.1.2 Comparison of G_I, G_{II} and G_{III} distribution for angle-ply $[+45_m/-45_m]_T$ glass/epoxy laminates

Thermoelastic analysis of an anti-symmetric $[+45_m/-45_m]_T$ angle-ply glass/epoxy laminate having a centrally located annular shaped elliptical delamination between the plies of different orientations i.e., (+45/-45) has been conducted. As discussed earlier the laminate is subjected to in-plane loading and delamination characteristics in terms of strain energy release rate parameters due to curing stresses have been studied. The SERR values are observed to be much higher in angle-ply laminates as compared to cross-ply laminate. This is due to the maximum mismatch of interlaminar stresses developed at the interface (i.e.+45/-45).

Referring to Fig. 9, the difference between the changes in the mode I strain energy release rates due to mechanical and thermomechanical loading is observed at different zones around the elliptical delamination front for the $[+45_m/-45_m]_T$ angle-ply glass/epoxy laminate. It is to be noted that residual thermal stresses enhance the delamination front propagation. The maximum mismatch between G_{IM} and G_{ITM} occurs at angular locations ϕ = 100° and 280° along the delamination front.

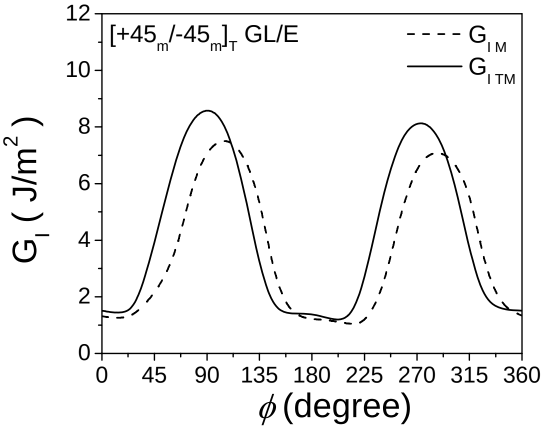

Fig.9. G_I distribution at +45/-45 interface of $[+45_m/-45_m]_T$ GL/E composite laminate.

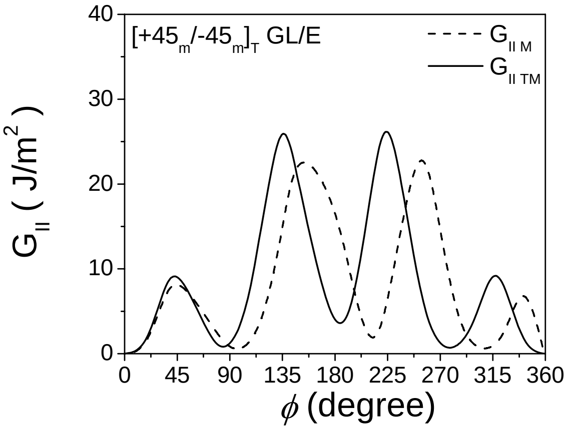

Fig.10. G_{II} distribution at +45/-45 interface of $[+45_m/-45_m]_T$ GL/E composite laminate.

Fig. 10 shows the distribution of strain energy release rate for mode II delamination front propagation with and without considering residual thermal stresses. The contribution of G_{II} component of SERR is significant as compared to G_I component of SERR. The effect of residual thermal stresses is to enhance the delamination front propagation. The maximum mismatch between G_{IIM} and G_{IITM} occurs at angular locations ϕ = 135° and 225° along the delamination front. The asymptotic distribution of individual of individual modes of SERRs around the delamination front for various angular locations ϕ exhibits much asymmetric and irregularity of the pattern in comparison to those in cross-ply laminate.

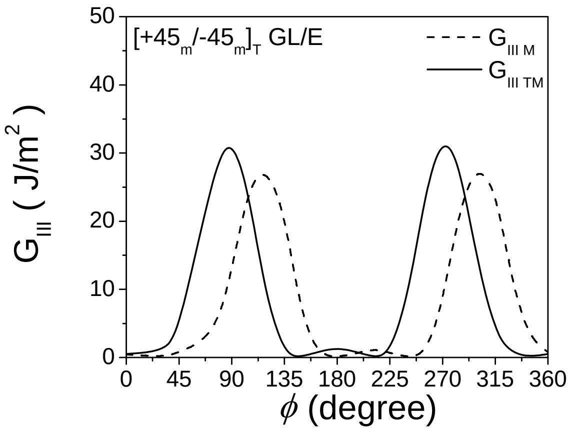

Fig.11. G_{III} distribution at +45/-45 interface of $[+45_m/-45_m]_T$ GL/E composite laminate.

Fig. 11 shows the distribution of mode-III SERR along the delamination front. It is clearly seen that mode-III component of delamination front propagation is dominating the other two modes of fracture. The maximum value of G_{III} is four times the maximum value of G_I. The mismatch between G_{IIIM} and G_{IIITM} is seen to be maximum at the angular locations ϕ = 90° and 270° along the delamination front. The thermomechanical loading effect has been considered, in comparison to G_{IIIM}, obtained due to a purely mechanical loading without considering residual thermal stress effects. It has been observed that the interface bonding phenomena and delamination crack growth mechanisms are altered due to the initial residual stress field considerations. The mismatch between the SERR plots and the asymmetries in energy release rate values along the delamination front necessitates a thermal analysis to be conducted before stress analysis of the mechanically loaded composite laminate to take into account the thermoelastic interaction along the delamination interface.

5 Conclusions

A detailed investigation of the curing residual thermal stress effects on the interlaminar delamination behaviour of FRP composite laminates with elliptical delamination emanating from the edge of elliptical cutouts has been presented. Strain energy release rate calculations are carried out for the three individual basic fracture modes following the modified crack closure integral procedures based on the concepts of linear elastic fracture mechanics. The results obtained are summarized below:

- The residual stresses are responsible for the asymmetric growth of delamination behaviour of composites when subjected to various mechanical loadings.
- The delamination growth characteristic is significantly affected by the presence of thermal residual stresses. It enhances the mechanism of delamination growth depending on the loading type, fiber orientation, and delamination shape.
- In most of the cases the residual thermal stresses add damages to the laminate in addition to those caused by mechanical loading. This additional damage degrades the structural integrity of the laminate more.

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