ON MICROMECHANICAL DEFORMATION BEHAVIOR OF FOAM WITH HIGH RELATIVE DENSITY

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Abstract

In this study, we employ the two-dimensional homogenization model based on molecular chain network theory to investigate the micro- to macroscopic mechanical behavior of the foam under macroscopic uniform compression. A parametric study is performed to quantify the effect of a characteristic value of polymer matrix, the distribution and the initial volume fraction of voids, and the macroscopic triaxiality of loading condition on the deformation behavior of the foam. The results suggest that the initial macroscopic elastic resistance and the yield stress of the foam have no dependence on the characteristic value of polymer matrix. The onset of localized shear band at the ligament between voids leads to the macroscopic yield of the foam. Furthermore, the microscopic localized shear deformation behavior is promoted in the case of high initial volume fraction of voids and high macroscopic triaxiality loading condition, which results in the early appearance of the macroscopic yield.

1 Introduction

It is well known that the foams such as polystyrene, polyurethane and polyethylene are widely used in packaging due to their high qualities in absorption of impact energy [1]. To study the mechanical behavior of the foam with low relative density which is smaller than 0.6, several models have been proposed [2-7]. However, due to the complicate deformation behavior such as microscopic buckling and plastic shear deformation occurring at the microscopic region, it is still a hard work to constitute a computational model to evaluate the mechanical behavior of the foam with high relative density which is larger than 0.6.

Therefore, in this study, we employ the two-dimensional homogenization model based on molecular chain network theory to investigate the micro- to macroscopic mechanical behavior of the foam with high relative density under macroscopic uniform compression. A parametric study is performed to quantify the effect of a characteristic value of polymer matrix, the distribution and the initial volume fraction of voids, and the macroscopic triaxiality of loading condition on the deformation behavior of the foam.

2 Constitutive Equation

In this study, we assume that the matrix of the foam behaves in the similar way to that of glassy polymer, which has the typical features such as small strain softening and large strain hardening. The complete constitutive equation for glassy polymer is given in ref. [8]; hereby we provide a brief explanation of the constitutive equation. The total strain rate is expressed by Hooke's law and plastic strain rate is modeled by using the affine eight-chain model. The final constitutive equation that relates the rate of Kirchhoff stress $\dot{S}_{ij}$ and strain rate $\dot{\varepsilon}_{ij}$ becomes

$$\dot{S}_{ij} = L_{ijkl}\dot{\varepsilon}_{ij} - P'_{ij}\,,\quad L_{ijkl} = D^e_{ijkl} - F_{ijkl}\,,$$

$$F_{ijkl} = \frac{1}{2}\left(\sigma_{ik}\delta_{jl} + \sigma_{il}\delta_{jk} + \sigma_{jl}\delta_{ik} + \sigma_{jk}\delta_{il}\right), \quad (1)$$

$$P'_{ij} = D^e_{ijkl}\frac{\tilde{\sigma}'_{kl}}{\sqrt{2}\tilde{\tau}}\dot{\gamma}^p\,,\quad \tilde{\tau} = \frac{1}{2}\tilde{\sigma}'_{ij}\tilde{\sigma}'_{ij}\,,\quad \tilde{\sigma}'_{ij} = \sigma_{ij} - B_{ij}\,,$$

where D^e_{ijkl} is the elastic stiffness tensor and σ_{ij} is the Cauchy stress. The shear strain rate $\dot{\gamma}^p$ in Eq. 1 is related to the applied shear stress $\tilde{\tau}$ as

$$\dot{\gamma}^p = \dot{\gamma}_0 \exp\left[-\frac{A\tilde{s}}{T}\left\{1-\left(\frac{\tilde{\tau}}{\tilde{s}}\right)^{5/6}\right\}\right], \quad (2)$$

where $\dot{\gamma}_0$ and A are constants, T is the absolute temperature, $\tilde{s} = s + \alpha p$ indicates shear strength, s is the shear strength which changes with plastic strain from the athermal shear strength $s_0 = 0.077\mu/(1-\upsilon)$ to a stable value s_{ss}, p is the pressure, α is a pressure-dependent coefficient, μ is the elastic shear modulus and υ is Poisson's ratio. Since s depends on strain rate, the evolution equation of s can be expressed as $\dot{s} = h(1 - s/s_{ss})\dot{\gamma}^p$ where h is the rate of resistance with respect to plastic strain rate. Furthermore, B_{ij} in Eq. 1 is the back-stress tensor of which the principal components are expressed by employing the eight-chain model as

$$B_i = C^R \frac{\sqrt{N}}{3} \frac{V_i^2 - \lambda^2}{\lambda} L^{-1}\left(\frac{\lambda}{\sqrt{N}}\right), \quad (3)$$

$$L(x) = \coth x - \frac{1}{x}, \quad \lambda^2 = \frac{1}{3}\left(V_1^2 + V_2^2 + V_3^2\right),$$

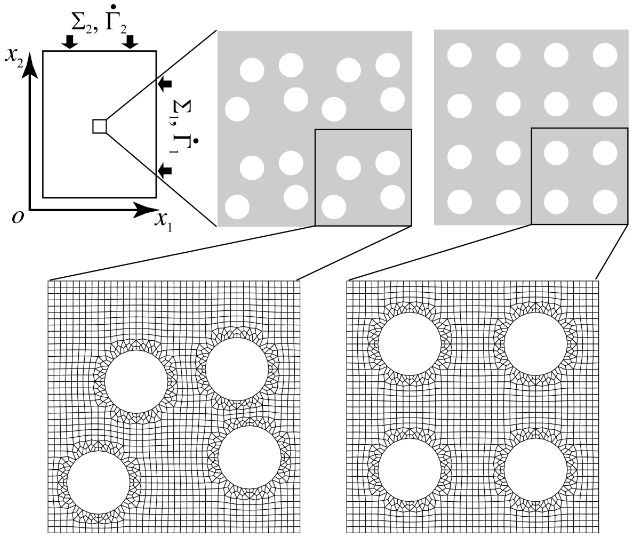
where V_i is the principal plastic stretch, N is the average number of segments in a single chain, $C^R = nkT$ is a constant, n is the number of chains per unit volume, k is Boltzmann's constant, and L is the Langevin function.

3 Computational Model

For the computational convenience, the foam is considered as a porous material and voids inside the foam are assumed to be distributed periodically as shown in Figure 1, in which one periodic unit consist of four same-sized voids. In order to discuss the effect of distribution of voids on the deformation behavior of the foam, two different patterns are shown in Figure 1. Moreover, as the constitutive equations of the polymer matrix have no material length scale, the dimensionless simulations are performed in which initial volume fraction of voids f_0 is the only parameter.

In order to estimate the micro- to macroscopic mechanical behavior of such porous material, the extended two-dimensional homogenization model is employed and formulated using the updated Lagrangian expression. The macroscopically homogeneous deformations are applied by prescribing the average strain rate $\dot{\Gamma}_1$, $\dot{\Gamma}_2$ or stress rate $\dot{\Sigma}_1$, $\dot{\Sigma}_2$ with respect to the coordinate directions x_1, x_2, respectively. The macroscopic compressive strain rate in x_2 direction is applied as $\dot{\Gamma}_2 = 10^{-5}/s$. Moreover, the surfaces of the macroscopic scale are shear-free.

The material parameters for the polymer matrix are the same as those for the polycarbonate which are described in Ref. [8].



(a) Irregular distribution (b) Regular distribution
Fig. 1. Simulation models of porous polymer

4 Results and Discussions

4.1 Effect of the characteristic value of polymer matrix and the distribution of voids

In glassy polymer, very long molecules entangle to form temporary physical cross-links and a number of the characteristic properties of bulk material are explicable in terms of the behavior of the deformed molecular chain network [9]. On the other hand, the physically entangled points are, in general, not permanent and may change depending on deformation. Therefore, in this study, we assume that the variation of the number of the physically entangled points during the formulation process of the foam leads to the variation of the stable value of the athermal shear strength of the polymer matrix s_{ss} and the effect of the value of s_{ss} on the compressive deformation behavior of the foam is investigated.

Figure 2 shows the stress-strain relation of polymer matrix under simple compression. In the case of $s_{ss} = 77.6\text{MPa}$, the characteristic responses of glassy polymer such as small strain softening and large strain hardening are observed. When the value of s_{ss} increases, strain softening disappears and the deformation resistance increases immediately after yield.

Figure 3 shows the macroscopic stress-strain relation of the foam under simple compression.

Figure 4 shows the corresponding equivalent plastic strain rate distributions in the matrix of the foam.

In the case of the foam with irregularly distributed voids, localized plastic deformation behavior onsets at the ligament between voids at the deformation stage $\Gamma_2 = 0.015$, which leads to the macroscopic yield. Contrastively, localized plastic deformation behavior onsets at the deformation stage $\Gamma_2 = 0.03$ in the case of the foam with regularly distributed voids and as a result, the onset of the macroscopic yield is delayed. With regard to the initial macroscopic elastic resistance and the yield stress, the foam with regularly distributed voids shows the higher value, whereas the effect of the value of s_{ss} on them is negligible.

After the macroscopic yield of the foam with irregularly distributed voids, newly developed plastic deformation region appears at different ligament between voids. Meanwhile, microscopic buckling is observed at the ligament, where the former plastic deformation region developed. In the case of the foam with regularly distributed voids, no newly developed plastic deformation region appears and the existing plastic deformation region propagates along the ligament between voids. With regard to the macroscopic response after yield, similar to that of the polymer matrix shown in Figure 2, the deformation resistance of the foam decreases together with the decrease of the value of s_{ss}. On the other hand, the increment of the deformation resistance of the foam is almost at the same level after the macroscopic yield and has no dependence on the value of s_{ss}.

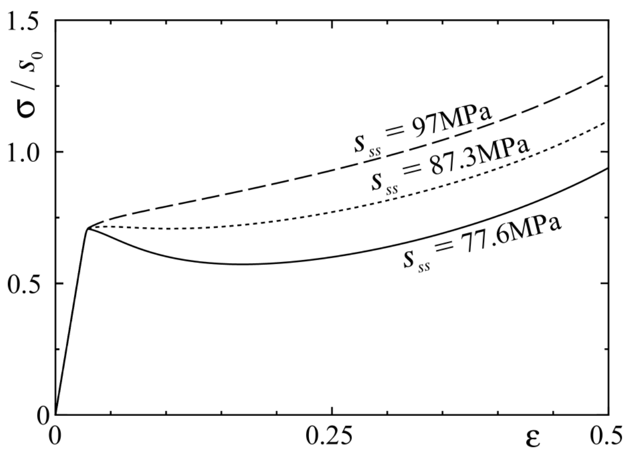


Fig. 2. Effect of the characteristic value s_{ss} on simple compressive stress-strain relation of polymer matrix

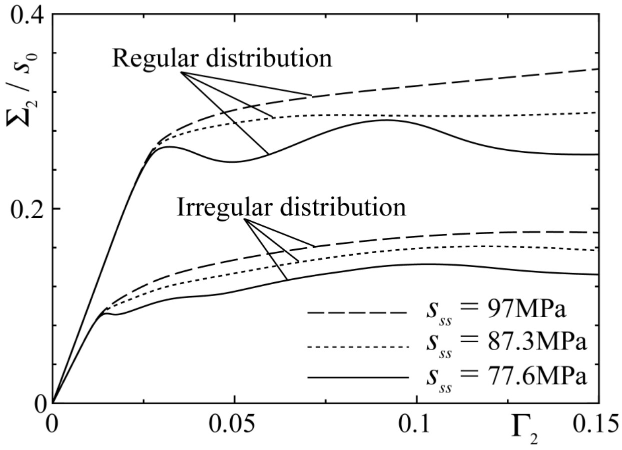


Fig. 3. Effect of the characteristic value s_{ss} and distribution of voids on macroscopic compressive stress-strain relation of the foam for $\Sigma_1 / \Sigma_2 = 0$, $f_0 = 40\%$

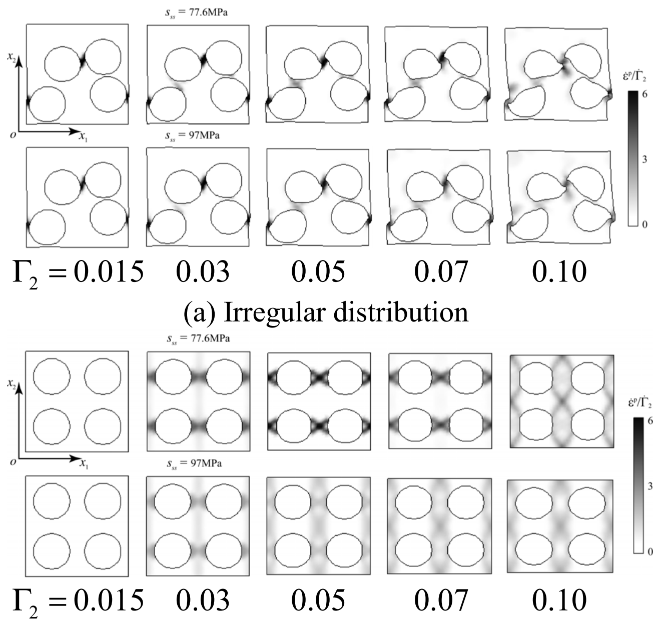


Fig. 4. Equivalent plastic strain rate distributions in the matrix of the foam for $\Sigma_1 / \Sigma_2 = 0$, $f_0 = 40\%$

4.2 Effect of the macroscopic triaxiality of loading condition

Figure 5 shows the macroscopic stress-strain relation of the foam under different macroscopic triaxiality loading condition. Figure 6 shows the corresponding equivalent plastic strain rate distributions in the matrix of the foam.

In the case of hydrostatic pressure ($\dot{\Gamma}_1 / \dot{\Gamma}_2 = 1$), localized plastic deformation behavior onsets at the

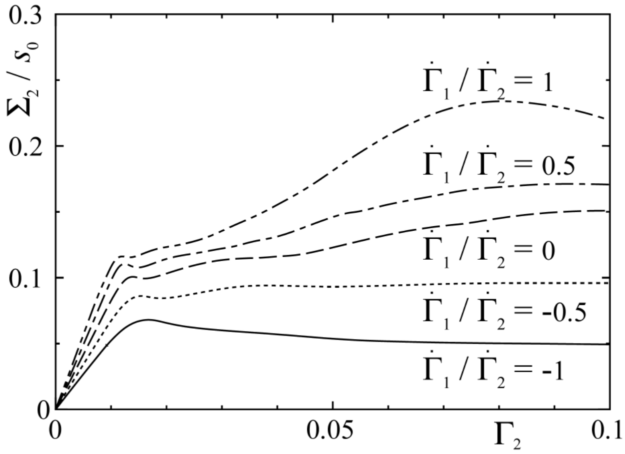


Fig. 5. Effect of macroscopic triaxiality of loading condition on macroscopic compressive stress-strain relation of the foam for $f_0 = 40\%$

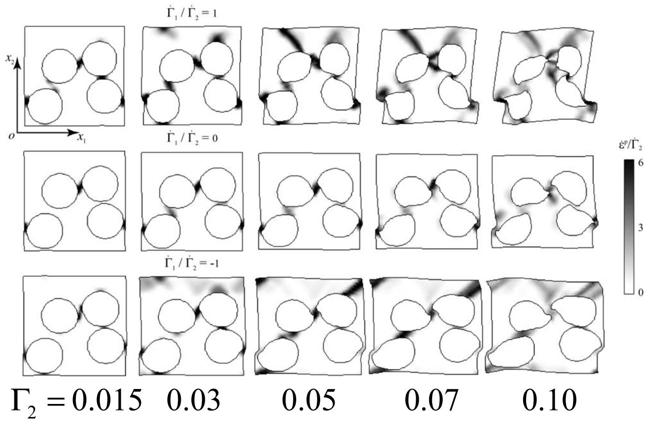


Fig. 6. Equivalent plastic strain rate distributions in the matrix of the foam for $f_0 = 40\%$ under different macroscopic triaxiality loading condition

ligament between voids at the deformation stage $\Gamma_2 = 0.01$ and subsequently develops to connect all of the adjacent voids. At later deformation stage, microscopic buckling occurs at all of the ligaments between adjacent voids. In the case of low macroscopic triaxiality loading condition ($\dot{\Gamma}_1 / \dot{\Gamma}_2 = -1$), localized plastic deformation behavior onsets at the deformation stage $\Gamma_2 = 0.015$ and the onset and propagation of localized shear band becomes the dominant deformation behavior at later deformation stage, whereas the microscopic buckling is somewhat suppressed.

With regard to the macroscopic response, the initial macroscopic elastic resistance of the foam under hydrostatic pressure is higher than that of the foam under low macroscopic triaxiality loading conditions. Moreover, after the macroscopic yield, the volume of the voids decreases in the case of hydrostatic pressure, which leads to a quickly increase of deformation resistance. Contrastively, in the case of low macroscopic triaxiality loading condition ($\dot{\Gamma}_1 / \dot{\Gamma}_2 = -1$), the volume of the voids remains almost constant after the macroscopic yield and no great change of the deformation resistance is observed.

4.3 Effect of the initial volume fraction of voids

Figure 7 shows the macroscopic stress-strain relation of the foam with different initial volume fraction of voids. Figure 8 shows the corresponding equivalent plastic strain rate distributions in the matrix of the foam. Figure 9 shows the macroscopic volumetric strain-compressive strain relation of the foam, where V means the volume of the deformed foam and V_0 means that of the undeformed foam.

In the case of the foam with small initial volume fraction of voids, the distance between the voids is larger than that in the foam with high initial volume fraction of voids, which delays the onset of the localized plastic deformation behavior and the macroscopic yield. On the other hand, in this case, sharp shear band onsets and propagates along the direction, which is 45° to the macroscopic loading direction, and no microscopic buckling is observed. Moreover, due to the smaller reduction of the volume fraction of the voids, the deformation resistance remains almost constant after the macroscopic yield.

5 Conclusions

In this study, we employ the two-dimensional homogenization model based on molecular chain network theory to investigate the micro- to macroscopic mechanical behavior of the foam with high relative density under macroscopic uniform compression. The effect of the characteristic value of polymer matrix s_{ss} , the distribution and the initial volume fraction of voids, and the macroscopic triaxiality of loading condition on the deformation behavior of the foam are discussed. The following is a summary of the results:

(1) The effect of the value of s_{ss} on the initial macroscopic elastic resistance and the yield stress of the foam is negligible.

(2) After the macroscopic yield, the increment of the deformation resistance of the foam is almost at the same level and has no dependence on the value of s_{ss} .

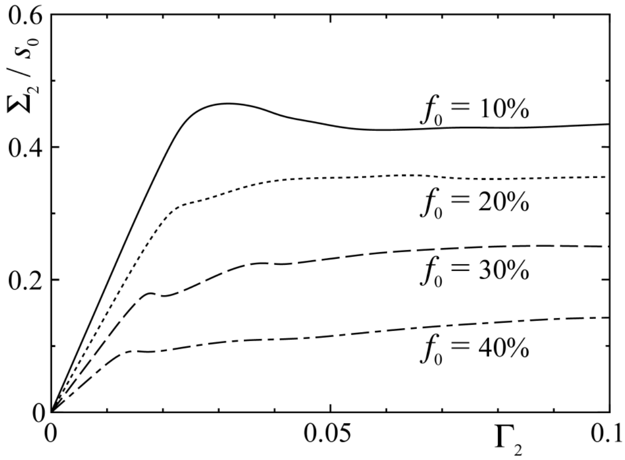


Fig. 7. Effect of initial volume fraction of voids on macroscopic compressive stress-strain relation of the foam for $\Sigma_1 / \Sigma_2 = 0$

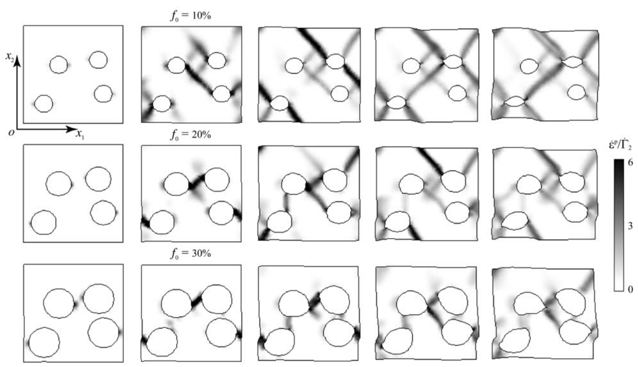


Fig. 8. Equivalent plastic strain rate distributions in the matrix of the foam for $\Sigma_1 / \Sigma_2 = 0$ with different initial volume fraction of voids

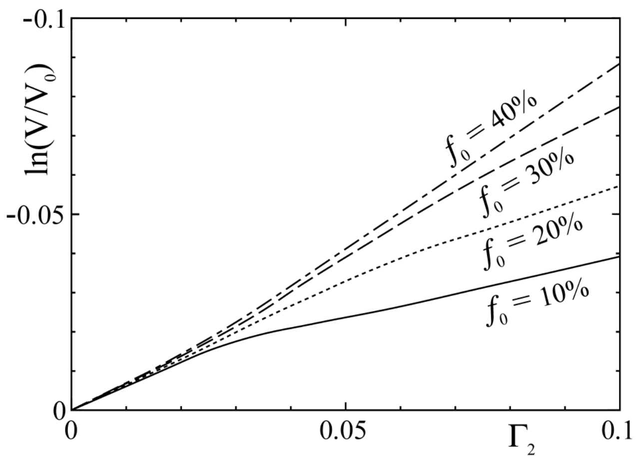


Fig. 9. Effect of initial volume fraction of voids on macroscopic volumetric strain-compressive strain relation of the foam for $\Sigma_1 / \Sigma_2 = 0$

(3) The localized plastic deformation behavior is promoted in the foam under high macroscopic triaxiality loading condition and in the foam with high initial volume fraction of voids, which leads to early appearance of the macroscopic yield.
(4) The expanded microscopic buckling and the great increase of the relative density of the foam is observed in the foam under high macroscopic triaxiality loading condition, which leads to a remarkable increase of the deformation resistance after the macroscopic yield.
(5) The smaller reduction of the volume fraction of the voids in the foam with small volume fraction of voids leads to a almost constant deformation resistance after the macroscopic yield.

The results shown here suggest that it is necessary to develop the constitutive equations for the foam with the consideration of microscopic buckling. We are doing such work now and will report it elsewhere.

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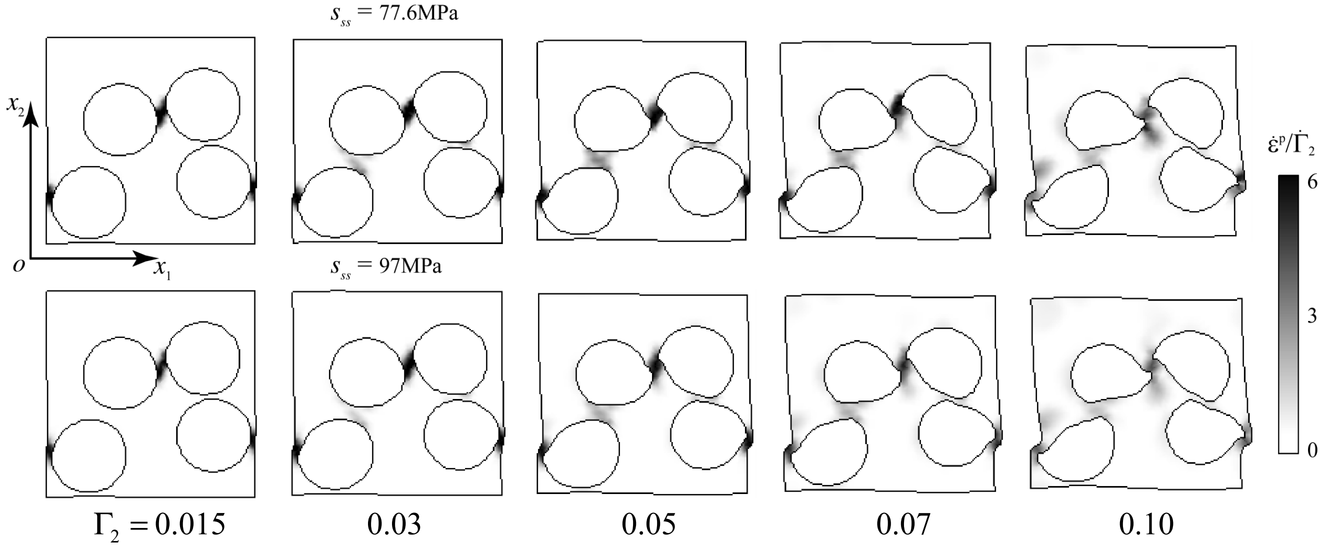
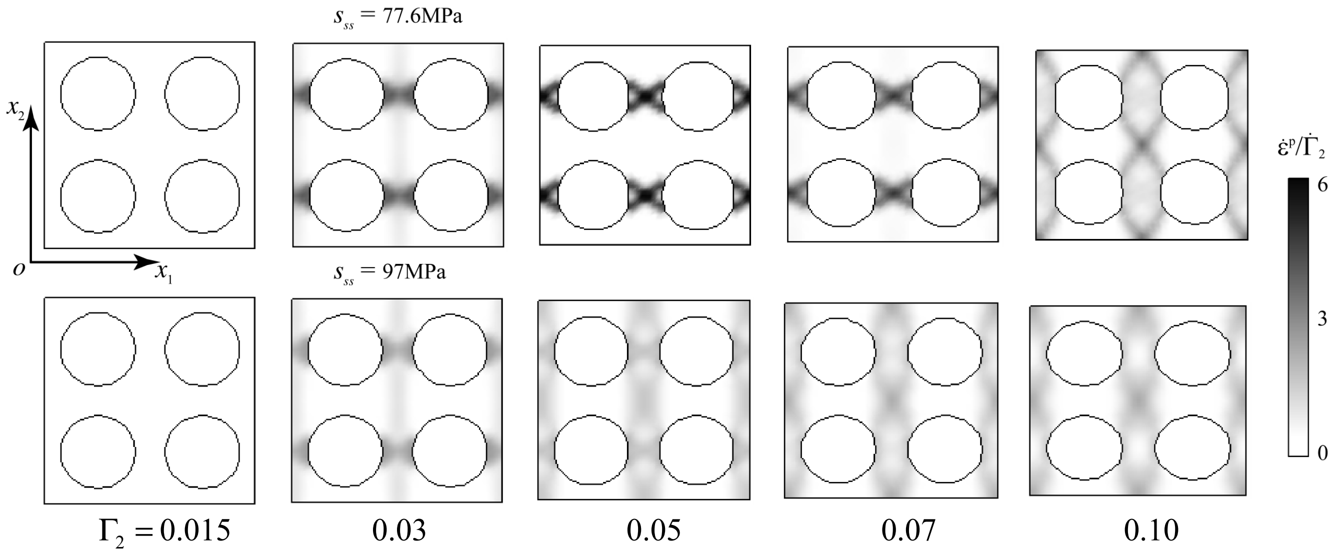
Magnified Figures



(a) Irregular distribution



(b) Regular distribution

Fig. 4. Equivalent plastic strain rate distributions in the matrix of the foam for $\Sigma_1 / \Sigma_2 = 0$, $f_0 = 40\%$

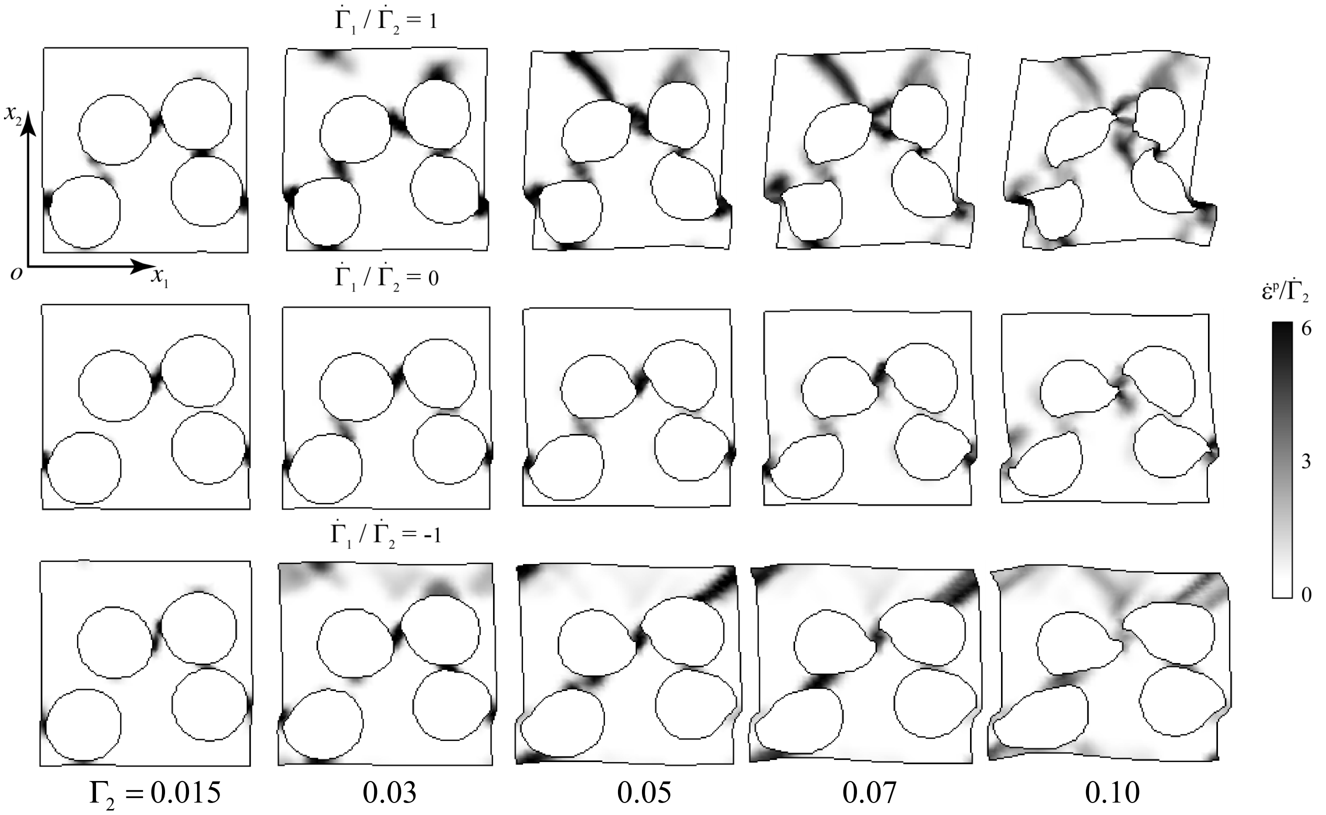


Fig. 6. Equivalent plastic strain rate distributions in the matrix of the foam for $f_0 = 40\%$ under different macroscopic triaxiality loading condition

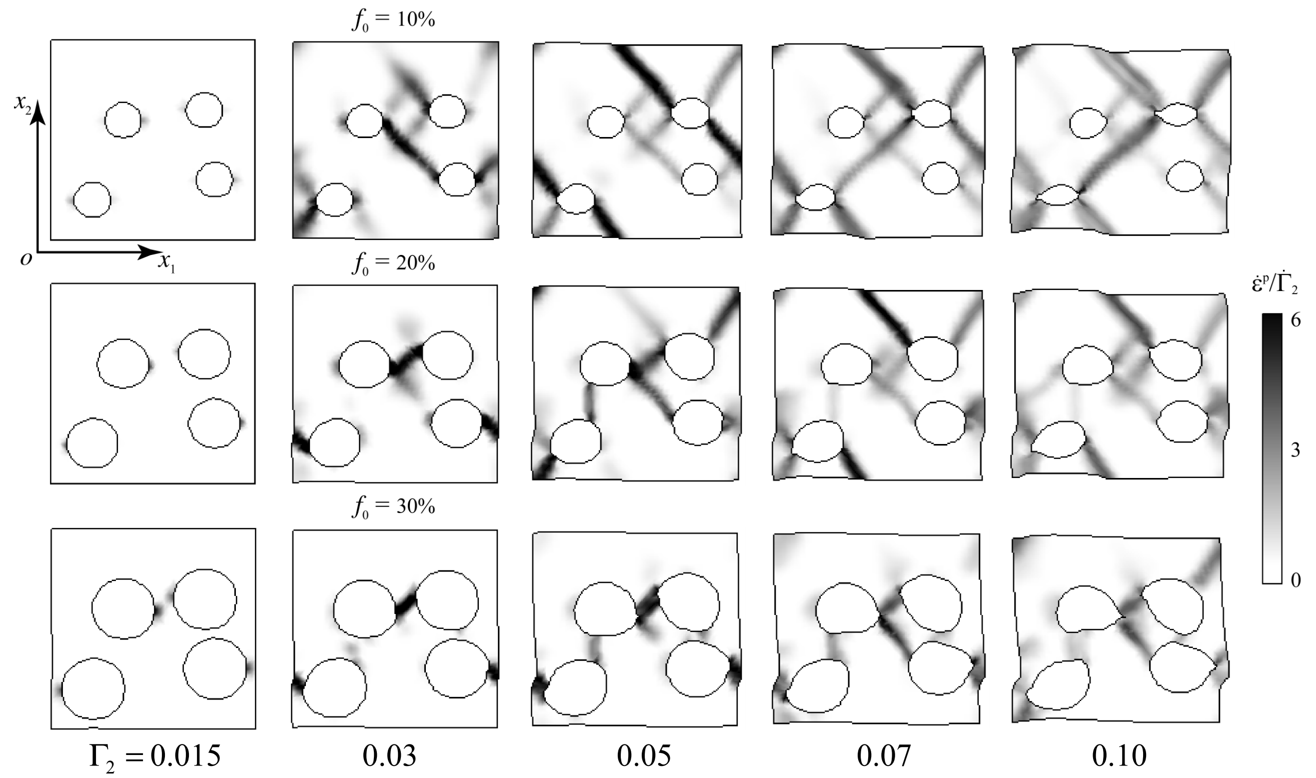


Fig. 8. Equivalent plastic strain rate distributions in the matrix of the foam for $\Sigma_1 / \Sigma_2 = 0$ with different initial volume fraction of voids