PREDICTION OF TENSILE FAILURE STRAIN OF UNIDIRECTIONAL FIBRE COMPOSITES: INVESTIGATING THE EFFECT OF MATRIX YIELDING

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Abstract

A Monte Carlo simulation is carried out to predict the failure strain of a single layer of 20-fibre composite with the thickness of the fibre ineffective length. The tensile failure strain of a 3D sized composite is also predicted by statistical analysis of a chain of numerous composite layers.

The effect of matrix shear yielding is introduced to the model through load sharing factors between surviving fibres adjacent to a fibre-break. Strain concentration factors (SCF) of adjacent fibres are obtained by Finite Element Analysis (FEA) in a three dimensional multi-fibre unit cell where an elasto-plastic matrix encloses the fibres.

The strain distribution among the intact neighbouring fibres of broken fibre(s) is clearly affected by the yielding mechanism in the resin matrix. It is also demonstrated that the introduction of the modified SCF to the statistical model increases the tensile failure strain of the 3D fibre composite.

1 Introduction

In a unidirectional (UD) fibre composite, the statistical strength distribution along the fibres results in a large discrepancy between the failure strength of a real composite and the one predicted from a simple rule of mixtures. A fibre-break is a critical micro-event in a UD composite. The stress released by a fibre-break should be redistributed among the intact adjacent fibres to re-establish a local load equilibrium. Therefore, the stress is intensified in the adjacent fibres to a fibre-break. The strain (or stress) concentration factor (SCF) increases the probability of fracture of neighbouring fibres and hence controls the ultimate failure of the composite material. The SCF value is a key parameter used in the predictive computer models for the failure strength of a UD composite.

The matrix surrounding the fibres plays a significant role in transferring stress to the fibres adjacent to a broken fibre. It is believed that interfacial shear yielding of an elasto-plastic matrix governs the redistribution of the overload among the intact fibres. This leads to a decrease in the probability that a crack will propagate through the matrix as well as reducing the SCF in the adjacent fibres [1]. Therefore, the behaviour of an elasto-plastic matrix under load can alter the micromechanics of the fracture in a fibre composite.

Most of the leading predictive models of the composite failure are based on statistical methods which include the statistical scatter of strength along the fibres. These models employ the stress concentration factor, to estimate the length of the adjacent fibre onto which the overload is thrown. However, overlooking the nonlinear behaviour of the matrix in the analytical models [2-4], has led to invalid results and limited the capability of the models.

Among the numerous models proposed for the prediction of the strength of UD composites, two models are fundamental. Firstly, Rosen's model [2] which considers the Equal Load Sharing (ELS) rule between fibres and a matrix which only carries a shear load. This model predicted significantly higher failure strength than that of real composites. Secondly, Zweben's model [3] which used the SCF values suggested by Hedgepeth and Van Dyke [5]. This model reported a lower strength than that of a real composite. This could be attributed to the high values of SCF obtained by the analytical methods. Wada *et al* [6] suggested that the Rosen and the Zweben's models are able to predict the upper and the lower bounds of the composite failure strength.

Most of the statistical models [6-12] for composite failure strength are based on the linkage of several single fibre elements of the size of a given ineffective length to be incorporated in a Monte-Carlo simulation. Curtis [10] established an uncomplicated computer model which predicts the progressive growth of fibre breaks and the failure strain of UD composites under tension. This study has ignored the effect of a yielding matrix in the model.

Thus it is essential to establish a link between the matrix yield properties and the ultimate failure strength of UD composites through determining the strain concentration factors in neighbouring fibres at the vicinity of a fibre-break. In this paper, it is attempted to predict the failure strain of UD composite considering the effect of a yielding resin matrix in redistribution of overload using the Curtis model [10].

2 Materials

The epoxy resin used in this study is triglycidyl *p*-aminophenol (TGAP), Araldite MY0510®, which is one of the base monomers of most resin formulations used in aerospace composites. The epoxy resin was cured with 4-4'diaminodiphenyl sulphone, DDS hardener. In the literature, little is reported about the stress-strain behaviour of this resin.

Epoxy resins typically exhibit brittle behaviour in the bulk with a low tensile failure strain. However, in a real fibre composite with a perfect interfacial fibre/matrix bond, the large difference between the elastic moduli of the components leads to a high shear stress within the thin layer of matrix at a fibre-beak. In order to measure full stress-strain curves of the resin matrix, uniaxial compression tests were carried out. Fig. 1 shows the stress-strain behaviour of the resin matrix at room temperature and a strain rate of 1.67×10^{-4} s^{-1}. It is reported in details elsewhere that the test temperature [13] and absorbed moisture [14] alter the yield behaviour of the resin matrix.

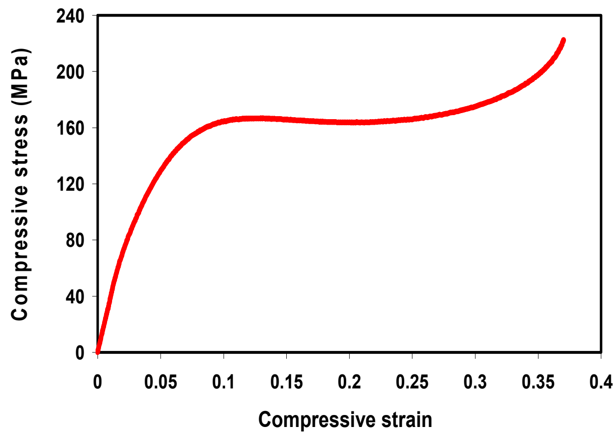

Fig. 1. Compressive true stress-strain curve of the resin matrix MY0510 at room temperature and the strain rate of 1.67×10^{-4} s^{-1}.

The properties of the transversely orthotropic reinforcing carbon fibres used in this study were taken from elsewhere [15] and listed in Table 1.

Table 1. Mechanical properties of the carbon fibre employed in the FE model [15].

Type	HTA5131	
Diameter (μm)	7.0	
Young's modulus (GPa)	E_z	235
	$E_x = E_y$	13.8
Poisson's ratio	ν_{yx}	0.35
	ν_{zy}	0.2
	ν_{zx}	0.2
Shear modulus (GPa)	G_{xy}	5.11
	$G_{yz} = G_{xz}$	18

3 Finite Element Model of UD composite

In this section, the relationship between the matrix yielding properties and SCF in the adjacent fibres to a fibre-break is studied. Therefore, a high volume fraction fibre composite containing a fibre-break was modelled using a unidirectional multi-fibre unit cell as a representative FE model. A strain analysis was carried out because of the known failure response of these composites.

A three dimensional (3D) model of a hexagonal array of fibres was generated using the ANSYS 8.1 software package. Because of symmetry, this model was limited to one-twelfth of three rings of fibres as shown in Fig. 3.

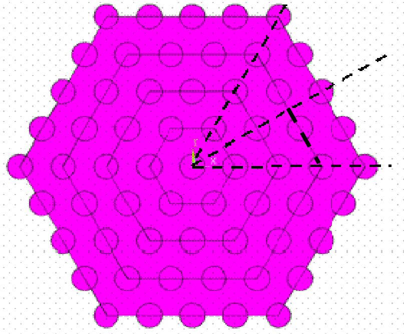

Fig. 3. Schematic of several rings of hexagonal packed fibres in a UD composite.

The model was built up with six 100 μm long carbon fibres of the radius of 3.5 μm each. The fibres were enclosed in a volume of the resin matrix. The fibre volume fraction of 50 % was fixed by adjusting the distance between the fibres. Perfect interfacial bonding was achieved between the fibres and the matrix by ensuring coincidence of nodes along the fibre/matrix interfaces in the FE model. The matrix properties were introduced into the model in the form of experimental compressive true stress-strain curves of the resin.

The compressive true stress-strain curve of the resin was digitized and input into the model. The multi-linear elasticity (MELAS) option was chosen to represent the elasto-plastic behaviour of the resin. The nonlinearity of the resin was defined by a series of straight lines fitted on the stress-strain curve of the material. The resin matrix was considered as an isotropic material with Poisson's ration of 0.35. The properties of the transversely orthotropic carbon fibres (Table 1) were also input into the model.

The 8-noded solid brick structural SOLID45 element was employed to mesh both the fibres and the matrix in the FE model. The meshes were refined to increase the concentration of the elements towards the front face of the model for both the fibres and the matrix. Therefore, the computational effort was focused directly into the region adjacent to the induced fibre-break. The number of finite elements in this model was 64392.

In a static FE model, a fibre-break can be introduced into the model prior to the loading, as it is equivalent to the fracture of the fibre during loading. The top layer of the elements can be removed from the fibre-end as shown in Fig. 4.

An axial displacement of 1 % was applied on the front face of the model in z-direction while the back face was constrained with z = 0. The model was constrained from motion on its other faces to satisfy the model's stability. Moreover, the nodes at the rim of the fibre-break in the matrix are constrained at UX and UY to prevent rotation during deformation.

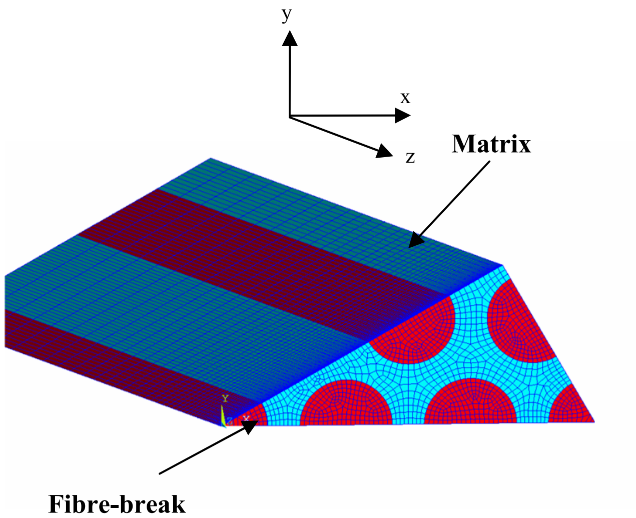

Fig. 4. The Image of the meshed FE model. A first row of elements was removed to represent a fibre-break on the broken fibre. The nearest neighbouring fibre is immediately adjacent.

4 The statistical Model

The model consisted of a layer of UD composite with 20 fibres of thickness equal to the critical or ineffective length of the fibre. The fibres are packed in a hexagonal array and embedded in the resin matrix.

The failure strain of each fibre in the layer is taken randomly from a normal distribution database corresponding to the mean failure strain of a single fibre with the size of the ineffective length. However, measuring the failure strain of single fibres of such a short length is experimentally impractical. Therefore, the mean fibre strength of an ineffective length was obtained by extrapolating the data of Bader and Priest [16] for longer lengths. The coefficient of variation (*cv*) of approximately 20% was used in this analysis.

Twenty uniform numbers in the range of (0,1) are generated. The Box-Muller method [17] was used to generate standard normal random numbers, z_i with a mean value of 0 and a standard deviation of 1. The normal random fibre failure strains, ε_i, are obtained by the following equation:

$$\varepsilon_i = \mu + z_i \sigma \qquad (1)$$

where μ and σ are the mean and standard variation of failure strains of the fibre at a length equal to an ineffective length, respectively.

To run the model, the computer programme finds the weakest fibre in the layer and increases the applied strain to match the failure strain of the fibre. As a result of the fibre fracture, an overload is shed equally among its intact nearest neighbouring fibres according to the strain concentration factor obtained by the FE model. If one fibre-break causes the next failure in the neighbouring fibres, the programme updates the strain of the immediate adjacent fibres to these two fibre breaks. Therefore, the number of fibres affected by the failures continuously changes with the occurrence of new fibre-breaks. This is schematically shown in Fig. 5.

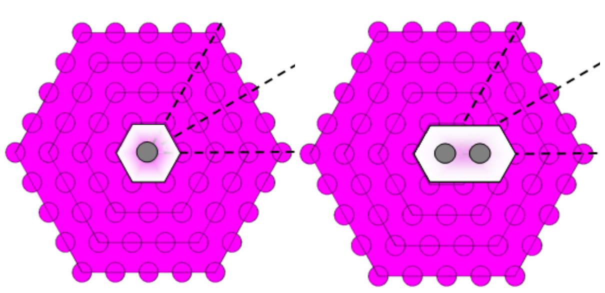

Fig. 5. The growth in the number of fibres participating in the load redistribution procedure at the vicinity of fibre fractures.

It can be seen that the load carried earlier by seven fibres now re-distributes among six intact adjacent fibres. With two adjacent fibres broken, the overload shed by these fibres is now thrown onto the eight adjacent fibres and so on. Thus the local strain concentration should be updated after each fibre-break for all surrounding fibres.

The procedure is repeated until an unstable break occurs where the fibres start to fail successively without an increase in the applied strain and thus the layer fails. The applied strain at this stage is taken as the failure strain of the composite layer. Since the fibres are assigned to random failure strains, it is not required to set a failure initiation locus in the layer. The sequence of failure can also be obtained. The programme can be ended when 2-3% of fibres in a layer have failed [10].

In the next stage, a fixed number of single layers of the size of an ineffective length are stacked on top of each other to form a piece of UD composite of a known length. However, in this preliminary study, the interaction between the single layers is not considered. In the weakest link theory the failure is assumed to be reached when random fibre breaks have weakened one layer of composite so that it can no longer withstand the applied load. The average failure strain of a known size of composite (i.e. a chain of layers) can be found from the re-arranged form of Eq. 1:

$$\bar{\varepsilon}_{composite} = \bar{\varepsilon}_{lf} - z_l \sigma_l \qquad (2)$$

where $\bar{\varepsilon}_{composite}$, $\bar{\varepsilon}_{lf}$, z_l and σ_l are the average failure strain of the composite, the average failure strain of the single layers, the standard normal number (obtained from probability standard tables) corresponding to the probability of failure of the layers in the composite and the standard deviation of failure strain in the composite layers, respectively.

5 Results

5.1 Ineffective length

The compressive true stress-strain curve of the resin matrix (Fig. 2) was input into the FE model. Fig. 6 shows the development of the axial tensile strain along the centre of the broken fibre.

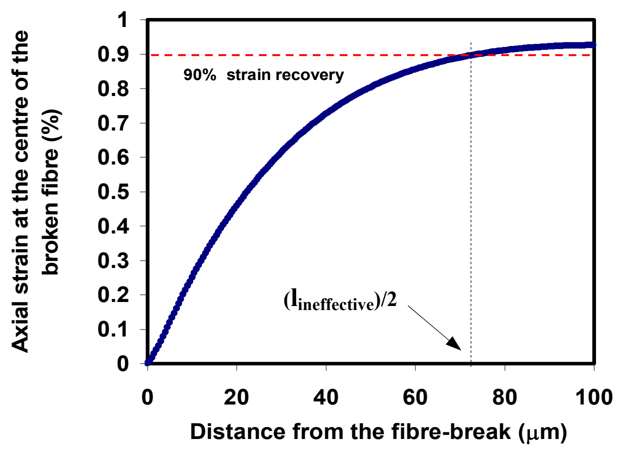

Fig. 6. The axial tensile strain profile at the centre of the broken fibre at an applied strain of 1 %.

It can be seen that the yielding of the matrix resulted in an exponential increase in axial tensile strain in the broken fibre after a fibre fracture. Because of the elasto-plastic behaviour of the matrix, the applied strain cannot be fully recovered even at the far-field from the broken fibre as shown in Fig. 6, so it falls below the applied strain of 1 %. The ineffective length of the fibre can be considered as twice the length of the fibre fragment over which 90% of strain recovery occurred. The ineffective length of 0.149 mm (= 0.0745 mm × 2) obtained from Fig. 6 is in the range of values reported by other work [18].

5.2 Strain Concentration Factor

In the absence of a fibre-break, the axial tensile strain would be the same as the applied strain all along the neighbouring fibre. However, the introduction of a break results in a considerable increase in the axial strain at the vicinity of the break. Fig. 7 shows the regions of intensified strain in the surrounding fibres which is located at the immediate distance from the fibre-break.

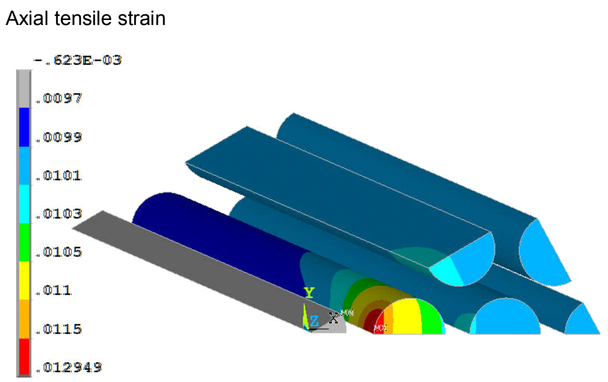

Fig. 7. The contour map of axial tensile strain in the hexagonally packed fibres with a fibre-break showing strain concentrations in the nearest and next nearest neighbouring fibres. The FE model was subjected to 1 % applied strain. The resin matrix is not shown.

It can be seen that the neighbouring fibres experience the maximum axial strain on their outer surfaces at the immediate distance from the broken fibre. The SCF is calculated by dividing the maximum axial strain in the cross section in the plane of the fibre-break by the applied strain. Table 2 shows the strain concentrations obtained for the surrounding fibres of a fibre-break. This shows that the SCF on the next-nearest neighbouring fibres (at 1.006) is almost one and therefore can be ignored.

Table 2. The strain concentration factor in the neighbouring fibres of a broken fibre in the FE model of an UD composite containing the resin matrix MY0510.

Fibre Geometry	Nearest Neighbour	Next-Nearest Neighbour
Hexagonal	1.097	1.006

5.3 Failure of a fibre composite layer

The statistical model is run for a layer of fibre composite of length of the ineffective length, 0.149 mm, with and without the effect of the yielding in the matrix. In the non-yield matrix case, where the broken fibre was assumed to shed its entire load equally across its six neighbours, the SCF value of 1.167 is applicable [10]. For each condition, fifteen iterations were executed by computer.

Fig. 8 shows an example of a fibre failure sequence in a composite layer. Initially stable groups of broken fibres form in the layer that can withstand the overload imposed by the broken fibres but they become unstable at a higher applied strain. At this strain the fibres fail one after another without an increase in the applied strain. Therefore, the failure strain of the layer was determined by that of the weakest fibre before an unstable failure begins. However, the output of the model shows that the layer fails with 3 to 4 fibre failures.

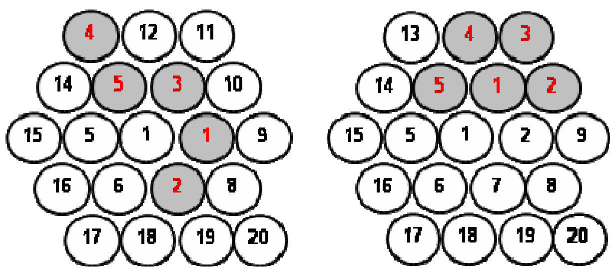

Fig. 8. The sequence of fibre failure in two examples of a fibre composite layer with different random fibre failure distributions.

Table 4 shows the mean failure strains of the composite layer with two different fibre SCF. It can be seen that although the mean failure strain of a fibre of the size of the ineffective length is 2.50 %, the first fibre failure occurred at the mean strain of about 1.67%. The introduction of a yieldable resin matrix increased the mean failure strain of the composite layer. The average size of the largest stable group of broken fibres is also shown in Table

4. It can be seen that more of the adjacent fibres can survive the applied strain when the matrix yields (with the SCF of 1.097 in the neighbouring fibres).

Table 4. The effect of SCF in the fibres on the layer failure strain and the size of stable group of broken fibres (the layer thickness is 0.149 mm).

SCF	1.167 (non-yield matrix)	1.097 (yielding matrix)
Fibre failure strain (%)[16]	2.50 ± 0.49	2.50 ± 0.49
Layer strain at the first fibre failure (%)	1.67 ± 0.16	1.67 ± 0.16
Layer failure strain (%)	1.80 ± 0.13	1.90 ± 0.12
Mean size of stable group of broken fibres	0.93	1.5

5.4 Strength of UD Composite

The effect of stacking the composite layers together to form a sensible size of UD composite was studied. It was assumed that a specific number of single layers with a random distribution of failure strain are stacked to make a fibre composite. The effect of SCF on the variation of composite failure strain was also studied. Fig. 9 shows the change in failure strain of 3D composite with logarithmic length. The failure strain decreases become length independent when the length exceeds a few centimetres (observed in a non-logarithmic scale). It can also be seen that the failure strain of the composite increases when the SCF is reduced through matrix yielding.

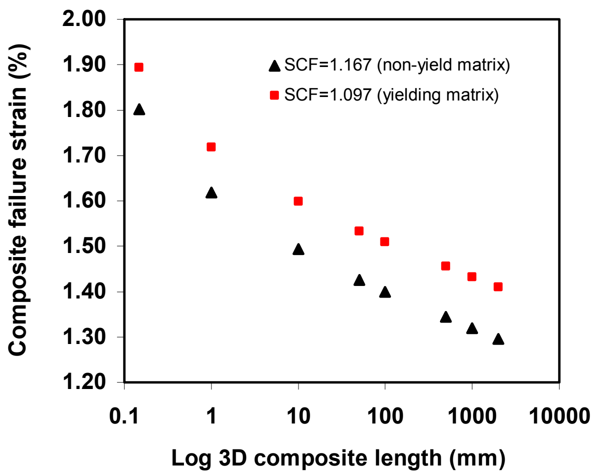

Fig. 9. The effect of SCF in the fibres on the failure strain of the UD composite as a function of length.

6 Discussion

In this paper, yielding was identified as a key property of the matrix and characterised to provide a link with the macroscopic mechanical behaviour of a UD fibre reinforced composite. It is generally believed that unidirectional composite materials mostly fail by in-plane fibre fracture. Therefore, the redistribution of the stress between the intact fibres adjacent to a broken fibre is a governing mechanism in the ultimate composite strength. The elasto-plasticity of the matrix has been incorporated into a FE model and led to a significant decrease in strain concentration factor in the fibres in comparison with the non-yield (elastic) matrix case [1,14]. As the elasto-plastic matrix at the vicinity of a broken fibre absorbs energy to deform plastically, it reduces the stress concentration on the intact adjacent fibres of a broken fibre [14]. This reduces the probability of failure of the adjacent fibres and eventually of the UD composite material.

In this study, the statistical model predicts the sequence of fibre failure in a composite layer consisting of 20 fibres which are surrounded by an elasto-plastic resin matrix which yields under the tensile load. The strain analysis which is carried out here enables us to compare the results obtained from different types of fibre while the behaviour of the resin matrix can also be monitored at a particular strain. The inclusion of the resin matrix effect through the reduced SCF of 1.097 on the surviving adjacent fibres of a fibre-break reduces the probability of their failures. This, in turn, leads to the formation of larger cluster of stable broken fibres in the layer compared to the elastic (non-yield) matrix simulation and therefore increases the failure strain of the composite layer. In this preliminary study of the layer composite, it is assumed that all neighbouring fibres around fractured fibres share an equal overload, ignoring their absolute distances from the crack. In a chain of composite layers, the failure occurs once the weakest layer fails. Therefore, the composite failure strain is determined by the failure strain of the weakest layer.

The failure strain of a composite increased with a decrease in the SCF as a result of the inclusion of a yield in the matrix. However, the composite failure strain predicted in this preliminary study is greater than typical experimental values (nearly 1.12%) for a given length of composite [10]. This may be associated with the choice of ineffective length in this study. It has been demonstrated [10] that a slight increase in the layer thickness decreases the composite failure strain and results in a better agreement with experiment. Interaction between layers may also be significant and has been ignored in this analysis.

In this study, it has been assumed that only nearest neighbouring fibres of a crack carry the overload shed by broken fibres. However, when a fibre fractures at the edge of the layer, the overload is shared between fewer intact adjacent fibres. This leads to an incorrect overstraining of these fibres which in turn decreases the failure strain of the layer.

7 Conclusions

The yield behaviour of the resin matrix, which is a key mechanism for controlling the matrix-to-fibre strain transfer, was linked to the probability of failure of a UD fibre composite. The shear yielding of the matrix reduced the strain concentration factor in the fibres at the vicinity of a fibre fracture. Therefore, the probability of failure of the fibres decreased compared to ones in a non-yield (elastic) matrix. The failure strain of a 3D fibre composite is predicted based on the weakest link theory using the failure strain of the composite layers of the size of the fibre ineffective length. The presence of an elasto-plastic matrix surrounding the fibres in the composite led to an increase in the failure strain of a realistic size of the composite compared to the elastic matrix case. It is suggested that a better agreement between the predicted failure strain and experiment requires a detailed study of the influence of the ineffective length of the fibres.

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References

[1] Lane R., Hayes S. A., Jones F. R., "Fibre/matrix stress transfer through a discrete interphase: 2. High volume fraction systems", *Composites Science and Technology*, Vol. **61,** No. 4, pp 565-578, 2001.

[2] Rosen B. W., "Tensile failure of fibrous composites", *AIAA J.*, Vol. **2,** No. 11, pp 1985-1991, 1964.

[3] Zweben C., "Tensile failure analysis of fibrous composites", *AIAA J.*, Vol. **6,** No. 12, pp 2325-2331, 1968.

[4] Zweben C., Rosen W., "A statistical theory of material strength with application to composite materials", *J. of the Mechanics and Physics of Solids*, Vol. **18,** No. 3, pp 189-206, 1970.

[5] Hedgepeth J. M., van Dyke P., "Local stress concentrations in imperfect filamentary composite materials", *J. of Composite Materials*, Vol. **1,** No. 3, pp 294-309, 1967.

[6] Wada A., Fukuda H., "Approximate upper and lower bounds for the strength of unidirectional composites", *Composites Science and Technology*, Vol. **59,** No. 1, pp 89-96, 1999.

[7] LienKamp M., Schwartz P., "A Monte Carlo simulation of the failure of a seven fibre microcomposite", *Composites Science and Technology*, Vol. **46,** No. 2, pp 139-146, 1993.

[8] Fukuda H., Kawata K., "On the strength distribution of unidirectional fibre composites", *Fibre Science and Technology*, Vol. **10,** No. 1, pp 53-63, 1977.

[9] Manders P. W., Bader M. G., Chou T. W., "Monte Carlo simulation of the strength of composite fibre bundles", *Fibre Science and Technology*, Vol. **17,** No. 3, pp 183-204, 1982.

[10] Curtis P. T., "A computer-model of the tensile failure process in unidirectional fibre composites", *Composites Science and Technology*, Vol. **27,** No. 1, pp 63-86, 1986.

[11] Ochiai S., Osamura K., "Influences of interfacial bonding strength and scatter of fibre strength on tensile behaviour of unidirectional metal matrix composites", *J. of Materials Science*, Vol. **23,** No. 3, pp 886-893, 1988.

[12] Curtin W. A., Takeda N., "Tensile strength of fibre reinforced composites: II.Application to polymer matrix composites", *J. of Composite Materials*, Vol. **32,** No. 22, pp 2060-2081, 1998.

[13] Behzadi S., Jones F. R., "Yielding behaviour of the model epoxy matrices for fibre reinforced composites: Effect of strain rate and temperature", *J. of Macromolecular Science-Part B: Physics*, Vol. **44,** No. 6, pp 993-1005, 2005.

[14] Behzadi S., *Role of matrix yield in composite performance*, University of Sheffield, Sheffield, 2006, PhD thesis.

[15] Nedele M. R., *Micromechanical modelling of unidirectional composites subjected to external and internal loadings*, University of Bristol, Bristol, 1996, PhD.

[16] Bader M. G., Priest A. M., "*Statistical aspects of fibre and bundle strength in hybrid composites*", ICCM-IV, Tokyo, pp 1159-1136, 1982.

[17] Box G. E. P., Muller M. E., "A note on the generation of random normal deviates", *Annals of Mathematical Statistics*, Vol. **29,** No. 2, pp 610-611, 1958.

[18] Phoenix S. L., Schwartz P., Robinson H. H., "Statistics for the strength and lifetime in creep-rupture of model carbon/epoxy composites", *Composite Science and Technology*, Vol. **32,** No. 2, pp 81-120, 1988.